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NANO Δ GENERALIZED-CLOSED SETS IN NANO TOPLOGICAL SPACES

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ABSTRACT. In the present article, we define the concept of $n\Delta g$-closed and $n\Delta g$-open sets by using in nano topological spaces, and several properties of that set are studied. Besides, based on the concepts of $n\Delta g$-closed and $n\Delta g$-open sets we investigate the notions of $n\Delta g$-interior, $n\Delta g$-closure. We also studied a practical example of a $n\Delta g$-closed set and a $n\Delta g$-open set.

1. INTRODUCTION

Several notions of open-like and closed-like sets in nano topological spaces were introduced and studied. The beginning was with M. Lellis Thivagar and Carmel Richard who initiated the notion of nano forms of weakly open sets, [7]. The concept of closed sets in nano topological spaces was extended to nano generalized closed sets, [1]. However, another extension of closed sets in nano topological spaces called nano $\ddot{g}$-closed sets was obtained in [10].

Recently, the idea of introducing new classes of open-like and closed-like sets in nano topological spaces is still attracting many researchers, for example [2, 3, 4, 5, 11] and [12].

A set in a topological space is called Δ-open if it is the symmetric difference of two open sets. The notion of Δ-open sets appeared in [8] and in [6]. However, it was pointed out in [8] and in [6] that the notion of Δ-open sets is due to a preprint by M. Veera Kumar. The complement of a Δ-open set is Δ-closed.

A set in a nano topological space is called $n\Delta$-open if it is the symmetric difference of two nano open sets were initiated, [9].

Preliminary concepts required in our work are briefly recalled in section 2. In section 3, the concept of $n\Delta g$-closed sets in nano topology is investigated with some properties on $n\Delta g$-interior, $n\Delta g$-closure.

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2. PRELIMINARIES

Definition 2.1. [7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X $\subseteq$ U.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by L$_R(X)$. $Thatis, L_R(X)$ =$\bigcup x\epsilon U\{R(X) : R(X)$$\subseteq$ X$\}$ where R(x) denotes the equivalence class determined by X.
(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by U$_R(X)$. $Thatis, U_R(X)$ =$\bigcup x\epsilon U\{R(X) : R(X)$$\cap$ X $\neq \phi\}$

(3) The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by B$_R(X).Thatis, B_R(X) = U_R(X) - L_R(X)$

Definition 2.2. [7]
If (U, $\tau_R(X)$) is the nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then

(1) The nano interior of the set A is defined as the union of all nano open subsets contained in A and it is denoted by nint(A). That is, nint(A) is the largest nano open subset of A.
(2) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and it is denoted by ncl(A). That is, ncl(A) is the smallest nano closed set containing A.

Definition 2.3. [8, 6] A subset A of a space (X, τ) is called Δ-open if A = (B $-$ C) $\cup$ (C $-$ B), where B and C are open subsets of X. The complement of Δ-open sets is called Δ-closed sets.

Definition 2.4. [9] A subset S of a space (U, $\tau_R(X)$) is said to be nano Δ-open set (in short, $n\Delta$-open) if S = (A $-$ B) $\cup$ (B $-$ A), where A and B are nano-open subsets in U. The complement of nano-Δ-open sets is called nano -Δ-closed sets.

Definition 2.5. [9] The nano interior of a set A is denoted by nano Δ-int(A) (briefly, $n\Delta$-int(A)) and is defined as the union of all $n\Delta$ open sets contained in A. i.e., $n\Delta$-int(A) = $\cup$ $\{$G : G is $n\Delta$-open and G $\subseteq$ A $\}$.

Definition 2.6. [9] The nano closure of a set A is denoted by nano Δ-cl(A) (briefly, $n\Delta$-cl(A)) and is defined as the intersection of all $n\Delta$-closed sets containing A. i.e., $n\Delta$-cl(A) = $\cap$ $\{$F : F is $n\Delta$-closed and A $\subseteq$ F$\}$.

3. $n\Delta g$-CLOSED SETS

Definition 3.1. A subset A of a space (U, $\tau_R(X)$) is called a $n\Delta$-generalized-closed (briefly, $n\Delta g$-closed) set if $n\Delta$cl(A) $\subseteq$ T whenever A $\subseteq$ T and T is $n\Delta$-open in (U, $\tau_R(X)$).
The complement of $n\Delta g$-closed set is called $n\Delta g$-open set.

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Proposition 3.1. *Every $n\Delta$-closed set is $n\Delta g$-closed.*

Proof. Let A be a $n\Delta$-closed set and T be any $n\Delta$-open set containing A. Since A is $n\Delta$-closed, we have $n\Delta$cl(A) = A $\subseteq$ T. Hence A is $n\Delta g$-closed. □

The converse of Proposition 3.1 need not be true as seen from the following example.

Example 3.2. Let U = {a, b, c} with U/ R= {{a}, {b, c}} and X = {a}. The nano topology $\tau_R(X)$ = {ϕ, {a}, U}. Then $n\Delta$-closed sets are ϕ, {a}, {b, c}, U} and $n\Delta g$-closed sets are ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, U. Here, H = {a, b} is $n\Delta g$-closed set but it is not $n\Delta$-closed.

Table 1 Explicate of nano Δ in Example 3.2

A	$n\Delta$-cl(A)	T is a $n\Delta$-open
{a}	{a}	{a}
{b}	{b, c}	{b, c}
{c}	{b, c}	{b, c}
{a, b}	U	U
{a, c}	U	U
{b, c}	{b, c}	{b, c}
ϕ	ϕ	ϕ
U	U	U

Proposition 3.2. *If P and Q are $n\Delta g$-closed sets, then P $\cup$ Q is also a $n\Delta g$-closed set.*

Proof. Let P and Q are $n\Delta g$-closed sets. Then $n\Delta$cl(P) $\subseteq$ T where P $\subseteq$ T and T is $n\Delta$-open and $n\Delta$cl(Q) $\subseteq$ T where Q $\subseteq$ T and T is $n\Delta$-open. Since P and Q are subsets of T, (P $\cup$ Q) is a subset of T and T is $n\Delta$-open. Then $n\Delta$cl(P $\cup$ Q) = $n\Delta$cl(P) $\cup$ $n\Delta$cl(Q) which implies that (P $\cup$ Q) is $n\Delta g$-closed. □

Remark. *If K and L are $n\Delta g$-closed sets, then K $\cap$ L is a $n\Delta g$-closed set.*

Example 3.3. Let U and $\tau_R(X)$ as in the Example 3.2. Here, k = {a, c} and L = {b, c} are $n\Delta g$-closed sets but K $\cap$ L = {c} is a $n\Delta g$-closed set.

Proposition 3.3. *If A subset A of (U, $\tau_R(X)$) is a $n\Delta g$-closed if and only if $n\Delta$cl(A) $-$ A does not contain any nonempty $n\Delta$-closed set.*

Proof. Necessity. Suppose that A is $n\Delta g$-closed. Let S be a $n\Delta$-closed subset of $n\Delta$cl(A) $-$ A. Then A $\subseteq$ S^c . Since A is $n\Delta g$-closed, we have $n\Delta$cl(A) $\subseteq$ S^c . Consequently, S $\subseteq$ (nΔcl(A))^c . Hence, S $\subseteq$ $n\Delta$cl(A) $\cap$ (nΔcl(A))^c = ϕ. Therefore S is empty.

Sufficiency. Suppose that $n\Delta$cl(A) $-$ A contains no nonempty $n\Delta$-closed set. Let A $\subseteq$ G and G be $n\Delta$-closed If $n\Delta$cl(A) $\neq$ G, then $n\Delta$cl(A) $\subseteq$ G^c $\neq \phi$. Since $n\Delta$cl(A) is a $n\Delta$-closed set and G^c is a $n\Delta$-closed set, $n\Delta$cl(A) $\cap$ G^c is a nonempty $n\Delta$-closed subset of $n\Delta$cl(A) $-$ A. This is a contradiction. Therefore, $n\Delta$cl(A) $\subseteq$ G and hence A is $n\Delta g$-closed. □

Proposition 3.4. *If A is $n\Delta g$-closed in (U, $\tau_R(X)$) such that A $\subseteq$ B $\subseteq$ $n\Delta$cl(A), then B is also a $n\Delta g$-closed set of (U, $\tau_R(X)$).*

Proof. Let W be a $n\Delta$-open set of (U, $\tau_R(X)$) such that B $\subseteq$ W. Then A $\subseteq$ W. Since A is $n\Delta g$-closed, we get, $n\Delta$cl(A) $\subseteq$ W. Now $n\Delta$cl(B) $\subseteq$ $n\Delta$cl($n\Delta$cl(A)) = $n\Delta$cl(A) $\subseteq$ W. Therefore, B is also a $n\Delta g$-closed set of (U, $\tau_R(X)$). □

Definition 3.4. The intersection of all $n\Delta$-open subsets of (U, $\tau_R(X)$) containing A is called the $n\Delta$-kernel of A and denoted by $n\Delta$-ker(A).

Table 2 in Example 3.2

A	$n\Delta$-ker(A)	$n\Delta$-cl(A)
{a}	{a}	{a}
{b}	{b, c}	{b, c}
{c}	{b, c}	{b, c}
{a, b}	U	U
{a, c}	U	U
{b, c}	{b, c}	{b, c}
ϕ	ϕ	ϕ
U	U	U

Lemma 3.5. *A subset A of (U, $\tau_R(X)$) is $n\Delta g$-closed if and only if $n\Delta$cl(A) $\subseteq$ $n\Delta$-ker(A).*

Proof. Suppose that A is $n\Delta g$-closed. Then $n\Delta$cl(A) $\subseteq$ T whenever A $\subseteq$ T and T is $n\Delta$-open. Let u $\in$ $n\Delta$cl(A). If u $\notin$ $n\Delta$-ker(A), then there is a $n\Delta$-open set T containing A such that u $\notin$ T. Since T is a $n\Delta$-open set containing A, we have u $\notin$ $n\Delta$cl(A) and this is a contradiction.

Conversely, let $n\Delta$cl(A) $\subseteq$ $n\Delta$-ker(A). If T is any $n\Delta$-open set containing A, then $n\Delta$cl(A) $\subseteq$ $n\Delta$-ker(A) $\subseteq$ T. Therefore, A is $n\Delta g$-closed. □

Definition 3.5. A subset A of a space U is said to be $n\Delta g$-open if A^C is $n\Delta g$-closed.

The class of all $n\Delta g$-open subsets of U is denoted by $n\Delta g$o($\tau_R(X)$).

Proposition 3.6. *Every $n\Delta$-open set is $n\Delta g$-open set but not conversely.*

Proof. Omitted. □

Proposition 3.7. *A subset A of a nano topological space U is said to $n\Delta g$-open if and only if P $\subseteq$ $n\Delta$int(A) whenever A $\supseteq$ P and P is $n\Delta$-closed in U.*

Proof. Suppose that A is $n\Delta g$-open in U and A $\supseteq$ P, where P is $n\Delta$-closed in U. Then A^c $\subseteq$ P^c , where P^c is $n\Delta$-open in U. Hence we get $n\Delta$cl (A^c) $\subseteq$ P^c implies (nΔint(A))^c $\subseteq$ P^c . Thus, we have $n\Delta$int(A) $\supseteq$ P.
conversely, suppose that A^c $\subseteq$ T and T is $n\Delta$-open in U then A $\supseteq$ T^c and T^c is $n\Delta$-closed then by hypothesis $n\Delta$int(A) $\supseteq$ T^c implies (nΔint(A))^c $\subseteq$ T. Hence $n\Delta$cl (A^c) $\subseteq$ T gives A^c is $n\Delta g$-closed. □

Proposition 3.8. *In a nano topological space U, for each u $\in$ U, either $\{u\}$ is $n\Delta$-closed or $n\Delta g$-open in U.*

Proof. Suppose that {u} is not $n\Delta$-closed in U. Then $\{u\}^c$ is not $n\Delta$-open and the only $n\Delta$-open set containing $\{u\}^C$ is the space U itself. Therefore, $n\Delta$cl ($\{u\}^C$) $\subseteq$ U and so $\{u\}^C$ is $n\Delta g$-closed gives {u} is $n\Delta g$-open. □

Definition 3.6. For any A $\subseteq$ U, $n\Delta g$-int(A) is defined as the union of all $n\Delta g$-open sets contained in A. i.e., $n\Delta g$-int(A) = $\cup$ {G : G $\subseteq$ A and G is $n\Delta g$-open}.

Lemma 3.9. *For any A $\subseteq$ U, int(A) $\subseteq$ $n\Delta g$-int(A) $\subseteq$ A.*

Proof. The proof follows from Proposition 3.6. □

Proposition 3.10. *For any A $\subseteq$ U, the following holds.*

(1) $n\Delta g$-*int(A) is the largest* $n\Delta g$-*open set contained in A.*
(2) *A is* $n\Delta g$-*open if and only if* $n\Delta g$-*int(A) = A.*

Proposition 3.11. *For any subsets A and B of (U,* $\tau_R(X)$*), the following holds.*

(1) $n\Delta g$-*int(A* $\cap$ *B) =* $n\Delta g$-*int(A)* $\cap$ $n\Delta g$-*int(B).*
(2) $n\Delta g$-*int(A* $\cup$ *B)* $\supseteq$ $n\Delta g$-*int(A)* $\cup$ $n\Delta g$-*int(B).*
(3) *If A* $\subseteq$ *B, then* $n\Delta g$-*int(A)* $\subseteq$ $n\Delta g$-*int(B).*
(4) $n\Delta g$-*int(U) = U and* $n\Delta g$-*int(*ϕ*) =* ϕ.

Definition 3.7. For every set A $\subseteq$ U, we define the $n\Delta g$-closure of A to be the intersection of all $n\Delta g$-closed sets containing A. i.e., $n\Delta g$-cl(A) = $\cap$ $\{$F : A $\subseteq$ F $\in$ $n\Delta g$-closed).

Lemma 3.12. *For any A* $\subseteq$ *U, A* $\subseteq$ $n\Delta g$-*cl(A)* $\subseteq$ $n\Delta$*cl(A).*

Proof. The proof follows from Proposition 3.1. □

Remark. *Both containment relations in Lemma 3.12 may be proper as seen from the following example.*

Example 3.8. Let U and $\tau_R(X)$ as in the Example 3.2. Let A = $\{$a, b$\}$. Here $n\Delta g$-cl($\{$a, b$\}$) = U and so A $\subseteq$ $n\Delta g$-cl(A) $\subseteq$ $n\Delta$cl(A).

Proposition 3.13. *For any A* $\subseteq$ *U, the following holds.*

(1) $n\Delta g$-*cl(A) is the smallest* $n\Delta g$-*closed set containing A.*
(2) *A is* $n\Delta g$-*closed if and only if* $n\Delta g$-*cl(A) = A.*

Proposition 3.14. *For any two subsets A and B of (U,* $\tau_R(X)$*), the following holds.*

(1) *If A* $\subseteq$ *B, then* $n\Delta g$-*cl(A)* $\subseteq$ $n\Delta g$-*cl(B).*
(2) $n\Delta g$-*cl(A* $\cap$ *B)* $\subseteq$ $n\Delta g$-*cl(A)* $\cap$ $n\Delta g$-*cl(B).*

Proposition 3.15. *Let A be a subset of a space U, then the following are true.*

(1) *(*$n\Delta g$-*int(A))*^{c} *=* $n\Delta g$-*cl(A*^{c}*).*
(2) $n\Delta g$-*int(A) = (*$n\Delta g$-*cl(A*^{c}*))*^{c}.
(3) $n\Delta g$-*cl(A) = (*$n\Delta g$-*int(A*^{c}*))*^{c}.

Proof. (1) Clearly follows from definitions.
(2) Follows by taking complements in (1).
(3) Follows by replacing A by A^c in (1). □

4. CONCLUSIONS

In this article, we provide characterizations for $n\Delta g$-closed sets and $n\Delta g$-open sets. We define and investigate the notions of $n\Delta g$-interior, $n\Delta g$-closure. However, counter examples are given to show distinction between the $n\Delta g$ notions and the usual nano topological notions. For a future work we are going to define and investigate the concepts of $n\Delta g$-continuous functions and $n\Delta g$-irresolute functions in nano topological spaces. The results of this study may be help in many researches.

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