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ON $m\Delta$-OPEN SETS IN MICRO TOPOLOGICAL SPACES

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ABSTRACT. The aim of this article, we introduced and studied $m\Delta$-open sets in micro topological spaces. We offer a new class of sets called $m\Delta$-closed sets in micro topological spaces and we study some of its basic properties. we introduce $m\Delta$-interior and $m\Delta$-closure and study some of its basic properties. We introduce $m\Delta$-continuous maps , $m\Delta$-irresolute maps and study some of its basic properties.

1. INTRODUCTION

M. Lellis Thivagar and Carmel Richard[3] introduced and studied nano forms of weakly open sets S. Chandrasekar [1] introduced and studied micro-open and micro continuous in micro topological spaces. A set in a topological space is called Δ-open if it is the symmetric difference of two open sets. The notion of Δ-open sets appeared in [4] and in [2]. However, it was pointed out in [4] and in [2] that the notion of Δ-open sets is due to a preprint by M. Veera Kumar. The complement of a Δ-open set is Δ-closed. Hereafter, I. Rajasekaran [5] introduced and studied nano Δ-open set in nano topological spaces The main objective of this study is to introduce a new hybrid intelligent structure called micro $m\Delta$ -open sets in micro topology spaces. The significance of introducing hybrid structures is that the computational techniques, based on any one of these structures alone, will not always yield the best results but a fusion of two or more of them can often give better results. The rest of this article is organized as follows. Some preliminary concepts required in our work are briefly recalled in section 2. In section 3, the concept of $m\Delta$-open sets in micro topology is investigated with some properties on micro Δ-interior, micro Δ-closure, micro Δ-continuous and micro Δ-irresolute.

2. PRELIMINARIES

Definition 2.1. [3] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X $\subseteq$ U.

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(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by L$_R(X)$.
$Thatis, L_R(X) = \bigcup x\epsilon U\{R(X) : R(X)$⊆ X$\}$ where R(x) denotes the equivalence class determined by X.
(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by U$_R(X)$.
$Thatis, U_R(X) = \bigcup x\epsilon U\{R(X) : R(X) \cap$ X $\neq \phi\}$

(3) The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by B$_R(X).Thatis, B_R(X) = U_R(X) - L_R(X)$

Definition 2.2. [3]
If (U, $\tau_R(X)$) is the nano topological space with respect to X where X ⊆ U and if A ⊆ U, then

(1) The nano interior of the set A is defined as the union of all nano open subsets contained in A and it is denoted by nint(A). That is, nint(A) is the largest nano open subset of A.
(2) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and it is denoted by ncl(A). That is, ncl(A) is the smallest nano closed set containing A.

Definition 2.3. [1] Let (U, $\tau_R(X)$) be a nano topological space. Then, $\mu_R(X)$ = {N ∪ ($\acute{N} \cap \mu$) : N, $\acute{N} \in \tau_R(X)$ and $\mu \notin \tau_R(X)$} is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4. [1] The Micro topology $\mu_R(X)$ satisfies the following axioms

(1) U, $\phi \in \mu_R(X)$.
(2) The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.
(3) The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological spaces and The elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.5. [1] The Micro interior of a set A is denoted by Micro-int(A) (briefly, m-int(A)) and is defined as the union of all Micro open sets contained in A. i.e., Mic-int(A) = ∪ {G : G is Micro open and G ⊆ A }.

Definition 2.6. [1] The Micro closure of a set A is denoted by Micro-cl(A) (briefly, m-cl(A)) and is defined as the intersection of all Micro closed sets containing A. i.e., Mic-cl(A) = ∩ {F : F is Micro closed and A ⊆ F}.

Definition 2.7. [4, 2] A subset A of a space (X, τ) is called Δ-open if A = (B − C) ∪ (C − B), where B and C are open subsets of X. The complement of Δ-open sets is called Δ-closed sets.

Definition 2.8. [5] A subset S of a space (U, $\tau_R(X)$) is said to be nano Δ-open set (in short, $n\Delta$-open) if S = (A − B) ∪ (B − A), where A and B are nano-open subsets in U. The complement of nano-Δ-open sets is called nano -Δ-closed sets.

3. $m\Delta$-OPEN SETS

Definition 3.1. A subset A of a space (O, $\tau_R(X)$, $\mu_R(X)$) is said to be micro Δ-open set (in short, $m\Delta$-open) if A = (S $-$ G) $\cup$ (G $-$ S), where S and G are micro-open subsets in (O, $\tau_R(X)$, $\mu_R(X)$) . The complement of micro-Δ-open sets is called micro -Δ-closed sets.

It is evident that every micro-open set is $m\Delta$-open and also every $n\Delta$-open set is $m\Delta$-open. But the converse implications are not true in general. Following is an example.

Example 3.2. Let O = $\{1, 2, 3\}$ with O/ R= $\{\{1\}, \{2, 3\}\}$ and X = $\{1\}$. The nano topology $\tau_R(X)$ = $\{\phi$, $\{1\}$, O$\}$. If μ = $\{1, 3\}$ then the micro topology $\mu_R(X)$ = $\{\phi$, $\{1\}$, $\{1, 3\}$, O$\}$. Then $n\Delta$-open sets are $\{\phi$, $\{1\}$, $\{2, 3\}$, O$\}$ and $m\Delta$-open sets are $\{\phi$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 3\}$, $\{2, 3\}$, O$\}$. Here, J = $\{2, 3\}$ is $m\Delta$-open but not micro-open. Also H = $\{1, 3\}$ is $m\Delta$-open but not $n\Delta$-open.

Remark. *(1) The union of any two $m\Delta$-open sets is not a $m\Delta$-open set.*
(2) The intersection of any two $m\Delta$-closed sets is a $m\Delta$-closed set.

Example 3.3. In the Example 3.2. (i) Here M = $\{1\}$ and N = $\{2\}$ are $m\Delta$-open sets, but M $\cup$ N = $\{1, 2\}$ is not a $m\Delta$-open set.

Definition 3.4. Let (O, $\tau_R(X)$, $\mu_R(X)$) bea micro topological space and let A $\subseteq$ O . Then the union of all $m\Delta$-open sets contained in A, denoted by $m\Delta$-int(A), is called the $m\Delta$-interior of A.

Theorem 3.1. *Let (O, $\tau_R(X)$, $\mu_R(X)$) bea micro topological space and A $\subset$ O. Then, A is $m\Delta$-open if and only if A = $m\Delta$-int(A).*

Proof. Let A be a $m\Delta$-open set. Then, A $\subseteq$ A and this implies that A $\in$ $\{$U $|$ U is $m\Delta$-open and U $\subset$ A$\}$. Since union of this collection is in A. Therefore, A = $m\Delta$-int(A).
Conversely, suppose that A = $m\Delta$-int(A). Hence, A is $m\Delta$-open. □

Definition 3.5. Let A be a subset of a space (O, $\tau_R(X)$, $\mu_R(X)$) then the $m\Delta$-closure of A, denoted by $m\Delta$cl(A) is defined as the intersection of all $m\Delta$-closed subsets of X containing A.

Theorem 3.2. *Let (O, $\tau_R(X)$, $\mu_R(X)$) bea micro topological space and A, B $\subseteq$ O. Then, the following statements hold:*

(1) *A $\subseteq$ $m\Delta$cl(A).*
(2) *$m\Delta$cl(A) is the smallest $m\Delta$-closed set containing A, that is $m\Delta$cl(A) = $\bigcap$ $\{$ W $|$ W is $m\Delta$-closed ans A $\subseteq$ W$\}$.*
(3) *A is $m\Delta$-closed if and only if A = $m\Delta$cl(A).*
(4) *If A $\subseteq$ B, then $m\Delta$cl(A) $\subseteq$ $m\Delta$cl(B).*
(5) *$m\Delta$cl(A) $\cup$ $m\Delta$cl(B) $\subseteq$ $m\Delta$cl(A $\cup B$).*
(6) *$m\Delta$cl(A $\cap B$) $\subseteq$ $m\Delta$cl(A) $\cap$ $m\Delta$cl(B).*

Proof. (1) Let x $\in$ A and suppose that x $\notin$ $m\Delta$cl(A). Then, there exits $m\Delta$-open set V containing x such that V $\cap$ A = $\emptyset$ and this is a contradiction. Therefore, x $\in$ $m\Delta$cl(A).
(2) Let x $\in$ $m\Delta$cl(A). Then, V $\cap$ A $\neq$ $\emptyset$ for every $m\Delta$-open set V containing x. Now, suppose the contrary, that x $\notin$ $\bigcap$ $\{$ W $|$ W is $m\Delta$-closed ans A $\subseteq$ W$\}$. Then, x $\notin$ W for some $m\Delta$-closed set W, so x $\in$ X $-$ W for some $m\Delta$-open set X $-$ W. So, (X$-$ W) $\cap$ A = $\emptyset$ for some $m\Delta$-open set X $-$ W containing x and this is a contradiction. Therefore, x $\notin$ $\bigcap$ $\{$ W $|$ W is $m\Delta$-closed ans A $\subseteq$ W$\}$. Conversely, let y $\in$ x $\notin$ $\bigcap$ $\{$ W $|$ W is $m\Delta$-closed

ans A $\subseteq$ W}. Then, y $\in$W for all $m\Delta$-closed set W containing A. Now, suppose that y $\notin$ $m\Delta$cl(A). Then, there exits $m\Delta$-open set V containing y such that V $\cap$ A = $\emptyset$. Therefore, X $-$ V is $m\Delta$-closed set containing A and y $\notin$ X $-$ V and this is a contradiction. Therefore, y $\in$ $m\Delta$cl(A).
The proof of (3) and (4) are followed directly from the Definition 3.5. (5) and (6) are followed by applying part (4) of this Theorem. □

Theorem 3.3. *Let (O, $\tau_R(X)$, $\mu_R(X)$) bea micro topological space and A, B $\subseteq$ O. Then, the following statements hold:*

(1) If A $\subseteq$ B, then $m\Delta$-int(A) $\subseteq$ $m\Delta$-int(B).
(2) $m\Delta$-int(A) $\cup$ $m\Delta$-int(B) $\subseteq$ $m\Delta$-int(A $\cup B$).
(3) $m\Delta$-int(A $\cap B$) $\subseteq$ $m\Delta$-int(A) $\cap$ $m\Delta$-int(B).

Proof. The proof of (1) is followed directly from the Definition 3.4. (2) and (3) are followed by applying part (1) of this Theorem. □

Theorem 3.4. *Let (O, $\tau_R(X)$, $\mu_R(X)$) be a micro topological space and A $\subseteq$ O. Then, the following statements hold:*

(1) $m\Delta$-int(X $\setminus A) = X\setminus m\Delta cl$(A).
(2) $m\Delta cl$(X $\setminus A) = X\setminus m\Delta$-int(A).
(3) X $\setminus m\Delta cl$(X $\setminus A$) =$m\Delta$-int(A).
(4) X $\setminus m\Delta$-int(X $\setminus A$) =$m\Delta cl$(A).
(5) x $\in$ $m\Delta$-int(A) if and only if there exists a $m\Delta$-open set M such that x $\in$ M $\subseteq$ A.

Proof. Omitted. □

Theorem 3.5. *Let A be a subset of a micro topological space (O, $\tau_R(X)$, $\mu_R(X)$) . Then, x $\in$ $m\Delta cl$(A) if and only if for every $m\Delta$-open subset M of X containing x, A $\cap$ M $\neq \emptyset$.*

Proof. Let x $\in$ $m\Delta$cl(A) and suppose that M $\cap$ A = $\emptyset$ for some $m\Delta$-open set M which contains x. Then, (X $\setminus$ M) is $m\Delta$-closed and A $\subset$ (X $\setminus$ M), thus $m\Delta$cl(A) $\subset$ (X $\setminus$ M). But this implies that x $\in$ (X $\setminus$ M), a contradiction. Thus, A $\cap$ M $\neq \emptyset$.
Conversely, let A $\subseteq$ X and x $\in$ X such that for each $m\Delta$-open set M_1 which contains x, M_1 $\cap \neq \emptyset$. If x $\notin$ $m\Delta$cl(A), there is a $m\Delta$-closed set F such that A $\subseteq$ F and x $\notin$ F. Then, (X $\setminus$ F) is a $m\Delta$-open set with x $\in$ (X $\setminus$ F), and thus (X $\setminus$ F) $\cap$ A $\neq \emptyset$, which is a contradiction. □

Theorem 3.6. *Let (O, $\tau_R(X)$, $\mu_R(X)$) bea micro topological space A $\subseteq$ O. Then A is $m\Delta$-open if and only if for each s $\in$ A, there exists a $m\Delta$-open set D such that s $\in$ D $\subseteq$ A.*

Proof. It is obvious. □

Definition 3.6. A map f : (O, $\tau_R(X)$, $\mu_R(X)$) $\to$ (P, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) is said to be $m\Delta$-continuous if f^{-1}(A) is $m\Delta$-open set in (O, $\tau_R(X)$, $\mu_R(X)$) for every micro-open set A of (P, $\tau_R(X)^{'}$, $\mu_R(X)^{'}$) .

Theorem 3.7. *A map f : O $\to$ P is $m\Delta$-continuous if and only if the inverse image of every micro-open set in P is $m\Delta$-open in O.*

Proof. Let f be $m\Delta$-continuous and K be any micro-open set in P. If f^{-1}(K) = $\emptyset$, then f^{-1}(K) is a $m\Delta$-open set in O but if f^{-1}(K) $\neq \emptyset$, then there exists o $\in$ f^{-1}(K) which implies f(o) $\in$ K. Since f is $m\Delta$-continuous, then there exists a $m\Delta$-open set L in O containing o such that f(L) $\subseteq$ K. This implies that o $\in$ L $\subseteq$ f^{-1}(K) and hence f^{-1}(K) is $m\Delta$-open.

Conversely, let K be any micro-open set in P containing f(o), then o $\in$ f^{-1}(K) and by hypothesis f^{-1}(K) is a $m\Delta$-open set in O containing o, so f(f^{-1}(K)) $\subseteq$ K. Thus, f is $m\Delta$-continuous. □

Theorem 3.8. *For a map f : O $\rightarrow$ P , the following statements are equivalent:*

(1) f is $m\Delta$-continuous.
(2) f^{-1}(K) is a $m\Delta$-open set in O, for each micro-open subset K of P.
(3) f^{-1}(F) is a $m\Delta$-closed set in O, for each micro-closed subset F of P.
(4) f($m\Delta$cl(A)) $\subseteq$ m-cl(f(A)), for each subset A of O.
(5) $m\Delta$cl(f^{-1}(B)) $\subseteq$ f^{-1}(m-cl(B)), for each subset B of P.
(6) f^{-1}(m-int(B)) $\subseteq$ $m\Delta$int(f^{-1}(B)), for each subset B of P.

Proof. (1) $\Rightarrow$ (2): Directly from Theorem 3.7.
(2) $\Rightarrow$ (3): Let F be any micro-closed subset of P. Then, P $\setminus$ F is a micro-open subset of P. By (2), f^{-1}(P $\setminus$ F) = O $\setminus$ f^{-1}(F) is a $m\Delta$-open set in O and hence f^{-1}(F) is a $m\Delta$-closed set in O.
(3) $\Rightarrow$ (4): Let A be any subset of O. Then, f(A) $\subseteq$ m-cl(f(A)) and m-cl(f(A)) is a micro-closed set in P. Hence, A $\subseteq$ f^{-1}(m-cl(f(A))). By (3), we have f^{-1}(m-cl(f(A))) is a $m\Delta$-closed set in O. Therefore, $m\Delta$cl(A) $\subseteq$ f^{-1}(m-cl(f(A))). Hence, f($m\Delta$cl(A)) $\subseteq$ m-cl(f(A)).
(4) $\Rightarrow$ (5): Let B be any subset of P. Then, f^{-1}(B) is a subset of O. By (4), we have f($m\Delta$cl(f^{-1}(B))) $\subseteq$ m-cl(f(f^{-1}(B))) $\subseteq$ m-cl(B). Hence, $m\Delta$cl(f^{-1}(B)) $\subseteq$ f^{-1}(m-cl(B)).
(5) $\Leftrightarrow$ (6): Let B be any subset of P. Then, apply (5) to P $\setminus$ B we obtain $m\Delta$cl(f^{-1}(P $\setminus$ B))$\subseteq$ f^{-1}(m-cl(P $\setminus$ B)) $\Leftrightarrow$ $m\Delta$cl(O $\setminus$ f^{-1}(B))$\subseteq$ f^{-1}(P $\setminus$ m-int(B)) $\Leftrightarrow$ O $\setminus$ $m\Delta$int(f^{-1}(B))$\subseteq$ O $\setminus$ f^{-1}(m-int(B)) $\Leftrightarrow$ f^{-1}(m-int(B)) $\subseteq$ $m\Delta$int(f^{-1}(B)). Thus, f^{-1}(m-int(B))) $\subseteq$ $m\Delta$int(f^{-1}(B)).
(6) $\Rightarrow$ (1): Let o $\in$ O and K be any micro-open subset of P containing f(o). By (6), we have f^{-1}(m-int(K)) $\subseteq$ $m\Delta$int(f^{-1}(K)) implies that f^{-1}(K) $\subseteq$ $m\Delta$int(f^{-1}(K)). Hence, f^{-1}(K) is a $m\Delta$-open set in O which contains o and clearly f(f^{-1}(K)) $\subseteq$ K. Thus, f is $m\Delta$-continuous. □

Definition 3.7. A map f : (O, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (P, $\tau_R(X)'$, $\mu_R(X)'$) is said to be $m\Delta$-irresolute if f^{-1}(V) is $m\Delta$-open set in (O, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set V of (P, $\tau_R(X)'$, $\mu_R(X)'$) .

Theorem 3.9. *Let f : (O, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (P, $\tau_R(X)'$, $\mu_R(X)'$) be a map, then the following statements are equivalent:*

(1) f is $m\Delta$-irresolute.
(2) f($m\Delta$cl(A)) $\subseteq$ $m\Delta$cl(f(A)) holds for every subset A of O.
(3) f^{-1}(B) is $m\Delta$-closed set in O, for every $m\Delta$-closed subset B of P.

Proof. (2) $\Rightarrow$ (3): Let B be a $m\Delta$-closed set in P, then $m\Delta$cl(B) = B. By using (2), we have f($m\Delta$clf^{-1}(B))$\subseteq$ $m\Delta$cl(B) = B. Thus, ($m\Delta$clf^{-1}(B))$\subseteq$ f^{-1}(B) and hence f^{-1}(B) is $m\Delta$-closed in O.
(3) $\Rightarrow$ (2): If A $\subseteq$ O, then $m\Delta$cl(f(A)) is $m\Delta$-closed in P and by (3) f^{-1}($m\Delta$cl(f(A))) is $m\Delta$-closed in O. Furthermore, A $\subseteq$ f^{-1}(f(A)) $\subseteq$ f^{-1}($m\Delta$cl(f(A))). Thus, $m\Delta$cl(A) $\subseteq$ f^{-1}($m\Delta$cl(f(A))), consequently, f($m\Delta$cl(A)) $\subseteq$ f(f^{-1}($m\Delta$cl(f(A)))) $\subseteq$ $m\Delta$cl(f(A)).
(3) $\Leftrightarrow$ (1): Obvious. □

4. Conclusions

Every year different type of topological spaces are introduced by many topologist. Now a days available topologies are ideal topology, nano topology, grll topology, micro topology, micro ideal topology, micro grill topology and so on. In this article, we introduced $m\Delta$-open sets in micro topological spaces and study some of its basic properties. Hence this micro Δ-closed sets in micro topological spaces can also be extended to a neutrosophic spatial region. The results of this study may be help in many researches.

References

[1] S. Chandrasekar, On micro topological spaces, Journal of New Theory, (26), (2019), 23-31.
[2] S. Ganesan, On α-Δ-open sets and generalized Δ-closed sets in topological spaces, International Journal of Analytical and Experimental Model Analysis, Volume 12, Issue 10, October (2020), 213-239. (ISSN NO:0886-9367). DOI:18.0002.IJAEMA.2020.V12I10.200001.01568590054681
[3] M. Lellis Thivagar and Carmel Richard, On nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1(1), (2013), 31-37.
[4] T. M. Nour and Ahmad Mustafa Jaber, semi Δ-open sets in topological spaces, International Journal of Mathematics Trends and Technology (IJMTT), 66(8), Aug (2020), 139-143.
[5] I. Rajasekaran, On some nano Δ-open set in nano topological spaces, Asia Mathematika, 5(2), (2021), 103 -106.

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