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FUZZY SOFT TRI-IDEALS OVER $\Gamma-$SEMIRINGS

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ABSTRACT. In this paper, we introduce the notion of a fuzzy soft tri-ideal over $\Gamma-$semiring. We characterize the regular $\Gamma-$semiring in terms of fuzzy soft tri-ideals, and study some of the properties. M is a regular $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft left tri-ideal over M, then (μ, A) is a fuzzy soft right ideal over M.

1. INTRODUCTION

In 1995, Murali Krishna Rao[10] introduced the notion of a $\Gamma-$semiring as a generalization of a $\Gamma-$ring, a ring, a ternary semiring, and a semiring. A universal algebra $(S, +, \cdot)$ is called a semiring if and only if $(S, +), (S, \cdot)$ are semigroups connected by distributive laws, $i.e.$, $a(b+c) = ab+ac,\ (a+b)c = ac+bc,$ for all $a, b, c \in S$. Though semiring is a generalization of a ring, ideals of semiring do not coincide with ring ideals. Semirings are useful for studying automata, coding theory, and formal languages. Dedekind introduced ideals for the theory of algebraic numbers, and E. Noether generalized these for associative rings. The one and two-sided ideals she introduced are still central concepts in ring theory. We know that the notion of a one-sided ideal of any algebraic structure is a generalization of the notion of an ideal. M. Murali Krishna Rao[11, 12, 13] introduced the notion of bi-interior ideals, quasi-interior ideals, bi-quasi ideals, bi-quasi interior ideals, tri-ideals, and tri-quasi interior ideals as a generalization of a quasi-ideal, a bi-ideal and an interior ideal of semirings, semigroups, $\Gamma-$semirings, $\Gamma-$semigroups, and studied their properties.

Due to various uncertainties, many real-world problems are complicated. In addressing them, classical methods may not be the best option. Uncertainty in many real-world problems arise in economics, engineering, and medical sciences. Many researchers proposed several theories to address uncertainity such as probability, randomness, fuzzy set, intuitionistic fuzzy set, and rough set. L. A. Zadeh[18] developed the fuzzy set theory in 1965. Many papers on fuzzy sets appeared, showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, and measure theory. N. Kuroki [7] studied fuzzy interior ideals in semigroups. Fuzzy set theory has certain limitations in assigning membership degrees.

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Molodtsov[9] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties, which only partially resolves the problem that objects in the universal set often do not precisely satisfy the parameters associated with each element in the set. Acar et al.[1] gave the basic concept of a soft ring. Feng et al.[4] studied soft semirings using the soft set theory. Soft set theory has many applications in game theory, operations research, Riemann integration, computer science, economics, data analysis, medical science, decision-making, and measurement theory. Maji et al.[8] extended soft set theory to fuzzy soft set theory and applied it in decision-making problems. Aktas and Cagman[2] defined the soft set and soft groups. Jayanth Ghosh et al.[5] initiated the study of fuzzy soft rings and fuzzy soft ideals. M.Murali Krishna Rao introduced and studied fuzzy soft ideals, fuzzy soft Γ-semiring homomorphism, fuzzy soft $\Gamma-$semiring, and fuzzy soft quasi-interior ideals over $\Gamma-$Semirings[3]. Soft set theory and fuzzy soft set theories have applications in fields like forecasting, simulation, evaluation of sound quality, and smoothness of functions. This paper aims to introduce the notion of fuzzy soft tri-ideals of a $\Gamma-$ semiring and characterizes a regular $\Gamma-$semiring in terms of fuzzy soft tri-ideals.

2. Preliminaries

In this section, we recall some of the fundamental concepts and definitions which are necessary for this paper.

Definition 2.1. [17] Let M and Γ be non-empty subsets. Then we call M a $\Gamma-$semigroup, if there exists a mapping $M \times \Gamma \times M \rightarrow M$ (images of (x, α, y) will be denoted by $x\alpha y, x, y \in M, \alpha \in \Gamma$) such that it satisfies $x\alpha(y\beta z) = (x\alpha y)\beta z$. for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.2. [11] Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M a $\Gamma-$semiring if there exists a mapping $M \times \Gamma \times M \rightarrow M$,(images of (x, α, y) will be denoted by $x\alpha y; x, y \in M; \alpha \in \Gamma,$) satisfying the following conditions for all $x, y, z \in M; \alpha, \beta \in \Gamma$,

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$,
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$,
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$,
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Definition 2.3. [11] An element a of a $\Gamma-$semiring M is called a regular element if there exist an element b of $M, \alpha, \beta \in \Gamma$ such that $a = a\alpha b\beta a$. M is called a regular $\Gamma-$semiring if every element of M is a regular element.

Definition 2.4. [11] A non-empty subset B of a $\Gamma-$semiring M is called

(i) a $\Gamma-$subsemiring of M if B is an additive $\Gamma-$subsemigroup of M and $B\Gamma B \subseteq B$.
(ii) a left(right) bi- quasi ideal of M if B is a $\Gamma-$subsemiring of M and $M\Gamma B \cap M\Gamma M\Gamma B(B\Gamma M \cap M\Gamma B\Gamma M) \subseteq B$.
(iii) a bi-quasi ideal of M if B is a left bi-quasi ideal and a right bi- quasi ideal of M
(iv) a left(right) quasi-interior ideal of M, if B is a $\Gamma-$subsemiring of M and $M\Gamma B\Gamma M\Gamma B(B\Gamma M\Gamma B\Gamma M) \subseteq B$.
(v) a right(left) tri-ideal of M, if B is a $\Gamma-$subsemiring of M and $B\Gamma B\Gamma M\Gamma B \subseteq B(B\Gamma M\Gamma B\Gamma B \subseteq B)$.

(vi) a tri-ideal of M, if B is a $\Gamma-$subsemiring of M and B is a left tri-ideal and a right tri-ideal of M.

Definition 2.5. [12] Let M be a non-empty set. A mapping $f : M \rightarrow [0,1]$ is called a fuzzy subset of M.

Definition 2.6. [12] Let f be a fuzzy subset of a non-empty set of a Γ-semiring M, for $t \in [0,1]$ the set $f_t = \{x \in M \mid f(x) \geq t\}$ is called a level subset of M with respect to f.

Definition 2.7. [13] Let M be a Γ-semiring. A fuzzy subset μ of M is called

(i) a fuzzy left (right) tri-ideal, if
   (a) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$,
   (b) $\mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in M, \alpha \in \Gamma$,
   (c) $\mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu\big(\mu \circ \mu \circ \chi_M \circ \mu \subseteq \mu\big)$,

(ii) a fuzzy tri-ideal, if it is a left tri-ideal and a right tri-ideal of M.

Definition 2.8. [3] Let f and g be fuzzy subsets over a $\Gamma-$semiring M. Then $(f \circ g)$ is defined as

$$(f \circ g)(z) = \begin{cases} \underset{z=x\alpha y}{Sup} \{min\{f(x), g(y)\}\}, \\ 0, \ otherwise, \end{cases}$$

$$f \cup g(z) = max\{f(z), g(z)\} \ ; \ f \cap g(z) = min\{f(z), g(z)\},$$

for all $x, y, z \in M, \alpha \in \Gamma$.

Definition 2.9. [12] Let μ be a fuzzy subset of a nonempty set M, for $k \in [0,1]$ the set $\mu_k = \{x \in M \mid \mu(x) \geq k\}$ is called a level subset of M with respect to μ.

Definition 2.10. [12] Let M be a $\Gamma-$semiring and A be a nonempty subset of M. The characteristic set $\chi_A(x) : A \rightarrow [0,1]$ is defined as

$$\chi_A(x) = \begin{cases} 1, & if x \in A, \\ 0, & if x \notin A. \end{cases}$$

Definition 2.11. [3] Let U be an initial universe set and E be the set of parameters. Let $P(U)$ denotes the power set of U. A pair (μ, E) is called a soft set over U where μ is a mapping given by $\mu : E \rightarrow \ P(U)$.

Definition 2.12. [3] Let U be an initial universe set, E be the set of parameters and $A \subseteq E$. A pair (μ, A) is called a fuzzy soft set over U where μ is a mapping given by $\mu : A \rightarrow \ I^U$ where I^U denotes the collection of all fuzzy subsets of U. $\mu(a), a \in A$, be a fuzzy subset and it is denoted by μ_a

Definition 2.13. [3] Let $(\mu, A), (\lambda, B)$ be fuzzy soft sets over U then (μ, A) is said to be a fuzzy soft subset of (λ, B) denoted by $(\mu, A) \subseteq (\lambda, B)$ if $A \subseteq B$ and $\mu_a \subseteq \lambda_a$ (μ_a, λ_a are fuzzy subsets) for all $a \in A$.

Definition 2.14. [3] Let M be a $\Gamma-$semiring and E be a parameters set and
$A \subseteq E$. Let μ be a mapping given by $\mu : A \rightarrow \ [0,1]^M$ where $[0,1]^M$ denotes the collection of all fuzzy subsets of M. Then (μ, A) is called a fuzzy soft $\Gamma-$semiring over M, if for each $a \in A$, $\mu(a) = \mu_a$ is the fuzzy $\Gamma-$subsemiring of M, ie.,
(i) $\mu_a(x+y) \geq \ \min\{\mu_a(x), \mu_a(y)\}$
(ii) $\mu_a(x\alpha y) \geq \ \min\{\mu_a(x), \mu_a(y)\}$, for all $x, y \in M, \alpha \in \Gamma$.
And (μ, A) is called a *fuzzy soft left* ($right$) *ideal over* M, if for each $a \in A$, the corresponding fuzzy subset $\mu_a : M \rightarrow \ [0,1]$ is a fuzzy soft left(right) ideal of M, i.e., for all $x, y \in M, \alpha \in \Gamma$, (i) $\mu_a(x+y) \geq \ \min\{\mu_a(x), \mu_a(y)\}$,

(ii) $\mu_a(x\alpha y) \geq \mu_a(y)(\mu_a(x))$.
(μ, A) is called a *fuzzy soft ideal* over M, if
(i) $\mu_a(x+y) \geq \min\{\mu_a(x), \mu_a(y)\}$,
(ii) $\mu_a(x\alpha y) \geq \max\{\mu_a(x), \mu_a(y)\}$.

Theorem 2.1. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. Let (μ, A) be a fuzzy soft set over M, and χ_M be the characteristic fuzzy set. If (μ, A) is a fuzzy soft right(left) ideal over M, then $\mu_a \circ \chi_M \subseteq \mu_a(\chi_M \circ \mu_a \subseteq \mu_a)$, for all $a \in A$.*

3. Generalization of fuzzy soft ideals:

In this section, we introduce the notion of fuzzy soft tri-ideals of a $\Gamma-$semiring and study some of their properties.

Definition 3.1. Let M be a $\Gamma-$semiring and E be a parameters set and $A \subseteq E$. Let μ be a mapping given by $\mu : A \to [0,1]^M$ where $[0,1]^M$ denotes the collection of all fuzzy subsets of M. Then (μ, A) is called a fuzzy soft left (right) tri-ideal over M if and only if for each $a \in A$, the corresponding fuzzy subset satisfies:

(i) $\mu_a(x+y) \geq \min\{\mu_a(x), \mu_a(y)\}$,
$\mu_a(x\alpha y) \geq \min\{\mu_a(x), \mu_a(y)\}$, for all $x, y \in M, \alpha \in \Gamma$.
(ii) $\mu_a \circ \chi_M \circ \mu_a \circ \mu_a \subseteq \mu_a\big(\mu_a \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a\big)$.

A fuzzy soft subset(μ, A) of a $\Gamma-$semiring M is called a fuzzy soft tri-ideal if it is both a fuzzy soft left tri-ideal and a fuzzy soft right tri-ideal of M.

Example 3.2. Let $M = \{0, a, b, c\}$, and $\Gamma = \{\alpha, \beta\}$. The binary operation $+$ is defined by,

$+$	0	a	b	c
0	0	a	b	c
a	a	a	c	a
b	b	c	b	b
c	c	a	b	c

$+$	α	β
α	α	α
β	α	β

Then $(M, +)$ and $(\Gamma, +)$ are commutative semigroups.

The ternary operation $(x\ \alpha\ y) \to x\alpha y$ $(M \times \Gamma \times M \to M)$ is defined by

$\alpha\|\beta$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	b	c
c	0	c	c	c

Then M is a $\Gamma-$semiring.
Let $B = \{0, b\}$. Then B is not an ideal, not a left ideal, not a right ideal, and not an interior-ideal.
B is a left tri-ideal. Let $E = \{e_1, e_2, e_3\}$. And $f_{e_i}(i = 1,2,3) : M \to [0,1]$.

Defined by

	0	a	b	c
f_{e_1}	1	0.5	0.2	0.1
f_{e_2}	0.8	0.4	0.3	0.6
f_{e_3}	0.9	0.2	0.5	0.3

$\{f_{e_i}\}, i = 1,2,3$.
Then (F, E) is a fuzzy soft left tri-ideal over M.

Theorem 3.1. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. Then (I, A) is a soft right tri-ideal of a $\Gamma-$semiring M if and only if (χ_I, A) is a fuzzy soft right tri-ideal of a $\Gamma-$semiring M.*

Proof. Suppose (I, A) is a soft right tri-ideal over M. Then for each $a \in A$, I_a is a right tri-ideal of M. Obviously, χ_I is a fuzzy $\Gamma-$subsemiring of M. We have $I_a\Gamma I_a\Gamma M\Gamma I_a \subseteq I_a$. Then

$$\chi_{I_a} \circ \chi_{I_a} \circ \chi_M \circ \chi_{I_a} = \chi_{I_a\Gamma I_a\Gamma M\Gamma I_a} \subseteq \chi_{I_a}.$$

Therefore $\chi_{I_a}$is a fuzzy right tri-ideal of M. Hence (χ_I, A) is a fuzzy soft right tri- ideal over M.
Conversely, suppose that (χ_I, A) is a fuzzy soft right tri-ideal of M.
Then I is a $\Gamma-$subsemiring of M. For each $a \in A$, We have
$\chi_{I_a} \circ \chi_{I_a} \circ \chi_M \circ \chi_{I_a} \subseteq \chi_{I_a},$ *then* $\chi_{I_a\Gamma I_a\Gamma M\Gamma I_a} \subseteq \chi_{I_a}$
Therefore $I_a\Gamma I_a\Gamma M\Gamma I_a \subseteq I_a$. Therefore I_a is a right tri-ideal of M. Hence (I, A) is a soft right tri-ideal over M. □

Theorem 3.2. *Let M be a $\Gamma-$semiring. E be a parameters set, $A \subseteq E$ and (μ, A) be a non-empty fuzzy soft over M. Then (μ, A) is a fuzzy soft right tri-ideal over M if and only if the level subset $(\mu_a)_k$ of (μ, A) is a right tri-ideal over $M, a \in A$, for every $k \in [0, 1]$, where $(\mu_a)_k \neq \phi$.*

Proof. Let M be a $\Gamma-$semiring and (μ, A) be a non-empty fuzzy soft subset over M. Let μ_a be a fuzzy right tri-ideal over U, $(\mu_a)_k \neq \phi, k \in [0, 1]$ and $l, m \in M$. Then

$$\mu_a(l) \geq k, \mu_a(m) \geq k, \text{ then } \mu_a(l + m) \geq \min\{\mu_a(l), \mu_a(m)\} \geq k,$$
$$\text{so } \mu_a(l\alpha m) \geq \min\{\mu_a(l), \mu_a(m)\} \geq k, \alpha \in \Gamma, \text{ hence } l + m \in (\mu_a)_k, l\alpha m \in (\mu_a)_k.$$

Therefore $(\mu_a)_k$ is a $\Gamma-$subsemiring.
Let $z \in (\mu_a)_k\Gamma M\Gamma(\mu_a)_k\Gamma(\mu_a)_k$. Then $z = l\alpha m\beta n\gamma p$, where
$m \in M, l, n, p \in (\mu_a)_k, \alpha, \beta$ and $\gamma \in \Gamma$,Then

$$\mu_a \circ \mu_a \circ \chi_M \circ \mu_a(z) \geq k$$
$$\Rightarrow \mu_a(z) \geq \mu_a \circ \mu_a \circ \chi_M \circ \mu_a(z)$$
$$\text{Therefore } z \in (\mu_a)_k.$$

Hence $(\mu_a)_k$ is a right tri-ideal over M.

Conversely, suppose that $(\mu_a)_k$ is a right tri-ideal of the $\Gamma-$semiring $M, a \in A$, for all $k \in [0, 1]$. Let $x, y \in M$, $\alpha \in \Gamma, \mu_a(x) = k_1, \mu_a(y) = k_2$ and $k_1 \geq k_2$.
Then $x, y \in (\mu_a)_{k2}$.

$$\Rightarrow x + y \in (\mu_a)_{k_2}$$
$$\Rightarrow \mu_a(x + y) \geq k_2 = \min\{k_1, k_2\} = \min\{\mu_a(x), \mu_a(y)\}$$
$$\text{Therefore } \mu_a(x\alpha y) \geq k_2 = \min\{\mu_a(x), \mu_a(y)\}.$$

We have $(\mu_a)_l\Gamma(\mu_a)_l\Gamma M\Gamma(\mu_a)_l \subseteq (\mu_a)_k$, for all $l \in Im\{(\mu_a)\}$.
Suppose $k = \min Im\{(\mu_a)\}$. Then $(\mu_a)_k\Gamma(\mu_a)_k\Gamma M(\mu_a)_k \subseteq (\mu_a)_k$.
Therefore $(\mu_a) \circ (\mu_a) \circ \chi_M \circ (\mu_a) \leq (\mu_a)$.
Hence (μ, A) is a fuzzy soft right tri-ideal over M. □

Definition 3.3. [3] Let (μ, A), (λ, B) be fuzzy soft sets. The $intersection$ of (μ, A) and (λ, B), denoted by $(\mu, A) \cap (\lambda, B) = (\gamma, C)$, where $C = A \cup B$, for all $c \in C, \gamma(c) = \gamma_c$, fuzzy subset of M is defined as:

$$\gamma_c = \begin{cases} \mu_c, & \text{if } c \in A \setminus B; \\ \lambda_c, & \text{if } c \in B \setminus A; \\ \mu_c \cap \lambda_c, & \text{if } c \in A \cap B. \end{cases}$$

Theorem 3.3. *Let M be a $\Gamma-$semiring, E be a parameters set and $A,\ B \subseteq E$. If (μ, A) and (λ, B) are fuzzy soft right tri-ideals over M, then $(\mu, A) \cap (\lambda, B)$ is a fuzzy soft right tri-ideal over M.*

Proof. Let (μ, A) and (λ, B) are fuzzy soft right tri-ideals of M. By Definition 3.3, we have that $(\mu, A) \cap (\lambda, B) = (\gamma, C)$ where $C = A \cup B$.
$Case\ (i)$: If $c \in A \setminus B$, then $\gamma_c = \mu_c$. Thus γ_c is a fuzzy right tri-ideal of M, since (μ, A) is a fuzzy soft right tri-ideal over M.
$Case\ (ii)$: If $c \in B \setminus A$, then $\gamma_c = \lambda_c$. Therefore γ_c is a fuzzy right tri-ideal of M, since (λ, B) is a fuzzy soft right tri-ideal over M.
$Case\ (iii)$: If $c \in A \cap B$, and $x, y \in M$, then $\gamma_c = \mu_c \cap \lambda_c$.
Let μ_c and λ_c be fuzzy right tri-ideals of M, $x, y \in M$ and $\alpha \in \Gamma$. Then

$$\begin{aligned} (\mu_c \cap \lambda_c)(x+y) &= \min\{\mu_c(x+y), \lambda_c(x+y)\} \\ &\geq \min\Big\{\min\{\mu_c(x), \mu_c(y)\}, \min\{\lambda_c(x), \lambda_c(y)\}\Big\} \\ &= \min\Big\{\min\{\mu_c(x), \lambda_c(x)\}, \min\{\mu_c(y), \lambda_c(y)\}\Big\} \\ &= \min\{\mu_c \cap \lambda_c(x), \mu_c \cap \lambda_c(y)\} \end{aligned}$$

$$\begin{aligned} \mu_c \cap \lambda_c(x\alpha y) &= \min\{\mu_c(x\alpha y), \lambda_c(x\alpha y)\} \\ &\geq \min\{\min\{\mu_c(x), \mu_c(y)\}, \min\{\lambda_c(x), \lambda_c(y)\}\} \\ &= \min\{\min\{\mu_c(x), \lambda_c(x)\}, \min\{\mu_c(y), \lambda_c(y)\}\} \\ &= \min\{\mu_c \cap \lambda_c(x), \mu_c \cap \lambda_c(y)\} \end{aligned}$$

Then $\mu_c \cap \lambda_c$ is a fuzzy subsemiring. And

$$\begin{aligned} \chi_M \circ (\mu_c \cap \lambda_c)(z) &= \sup_{z=l\alpha m} \min\{\chi_M(l), \mu_c \cap \lambda_c(m)\} \\ &= \sup_{z=l\alpha m} \min\Big\{\chi_M(l), \min\{\mu_c(m), \lambda_c(m)\}\Big\} \\ &= \sup_{z=l\alpha m} \min\Big\{\min\{\chi_M(l), \mu_c(m)\}, \min\{\chi_M(l), \lambda_c(m)\}\Big\} \\ &= \min\Big\{\sup_{z=l\alpha m} \min\{\chi_M(l), \mu_c(m)\}, \sup_{z=l\alpha m} \min\{\chi_M(l), \lambda_c(m)\}\Big\} \\ &= \min\{\chi_M \circ \mu_c(z), \chi_M \circ \lambda_c(z)\} \\ &= \chi_M \circ \mu_c \cap \chi_M \circ \lambda_c(z) \end{aligned}$$

$$\begin{aligned}((\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c))(x) &= \sup_{x=l\alpha m} \min\{(\mu_c \cap \lambda_c)(l), (\mu_c \cap \lambda_c)(m)\} \\ &= \sup_{x=l\alpha m} \min\{\min\{\mu_c(l), \lambda_c(l)\}, \{\min\{\mu_c(m), \lambda_c(m)\}\} \\ &= \sup_{x=l\alpha m} \min\{\min\{\mu_c(l), \lambda_c(l)\}, \sup_{x=l\alpha m} \min\{\min\{\mu_c(m), \lambda_c(m)\} \\ &= \min\{\sup_{x=l\alpha m} \{\min\{\mu_c(l), \mu_c(m)\}, \sup_{x=l\alpha m} \{\min\{\lambda_c(l), \lambda_c(m)\}\} \\ &= \min\{\mu_c \circ \mu_c(x), \lambda_c \circ \lambda_c(x)\} \\ &= (\mu_c \circ \mu_c) \cap (\lambda_c \circ \lambda_c)(x).\end{aligned}$$

Then, $(\chi_M \circ \mu_c) \cap (\chi_M \circ \lambda_c) = \chi_M \circ (\mu_c \cap \lambda_c)$. And

$$\begin{aligned}&\Big((\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c) \circ \chi_M \circ (\mu_c \cap \lambda_c)\Big)(z) \\ &= \sup_{z=l\alpha m} \min\Big\{(\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c)(l), \chi_M \circ (\mu_c \cap \lambda_c)(m)\Big\} \\ &= \sup_{z=l\alpha m} \min\Big\{\min\{(\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c)(l), \min\{\chi_M \circ (\mu_c \cap \lambda_c)(m)\}\Big\} \\ &= \sup_{z=l\alpha m} \min\Big\{\min\{(\mu_c \circ \mu_c)(l), (\lambda_c \circ \lambda_c)(l)\}, \\ &\min\{(\chi_M \circ \mu_c)(m), (\chi_M \circ \lambda_c)(m)\}\Big\} \\ &= \sup_{z=l\alpha m} \min\Big\{\min\{(\mu_c \circ \mu_c)(l), (\chi_M \circ \mu_c)(m)\}, \\ &\min\{(\lambda_c \circ \lambda_c)(l), (\chi_M \circ \lambda_c)(m)\}\Big\} \\ &= \min\Big\{\sup_{z=l\alpha m} \min\{(\mu_c \circ \mu_c)(l), (\chi_M \circ \mu_c)(m)\}, \\ &\sup_{z=l\alpha m} \min\{(\lambda_c \circ \lambda_c)(l), (\chi_M \circ \lambda_c)(m)\}\Big\} \\ &= \min\{\mu_c \circ \mu_c \circ \chi_M \circ \mu_c(z), \lambda_c \circ \lambda_c \circ \chi_M \circ \lambda_c(z)\} \\ &= (\mu_c \circ \mu_c \circ \chi_M \circ \mu_c \cap \lambda_c \circ \lambda_c \circ \chi_M \circ \lambda_c)(z).\end{aligned}$$

$(\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c) \circ \chi_M \circ (\mu_c \cap \lambda_c)(z) = (\mu_c \circ \mu_c \circ \chi_M \circ \mu_c \cap \lambda_c \circ \lambda_c \circ \chi_M \circ \lambda_c)(z)$.
Therefore $(\mu_c \cap \lambda_c)$ is a fuzzy right tri-ideal of M.
γ_c is a fuzzy right tri-ideal of M.
Hence $(\mu, A) \cap (\lambda, B)$ is a fuzzy soft right tri-ideal over M. □

Corollary 3.4. *If (μ, A) and (λ, B) are fuzzy soft left tri-ideals over Γ-semiring M, then $(\mu, A) \cap (\lambda, B)$ is a fuzzy soft left tri-ideal over M.*

Definition 3.4. [3] Let $(\mu, A),\ (\lambda, B)$ be fuzzy soft sets. The *union* of (μ, A) and (λ, B), denoted by $(\mu, A) \cup (\lambda, B) = (\gamma, C)$, where $C = A \cup B$, for all $c \in C, \gamma(c) = \gamma_c$, fuzzy subset of M is defined as:

$$\gamma_c = \begin{cases} \mu_c, & \text{if } c \in A \setminus B; \\ \lambda_c, & \text{if } c \in B \setminus A; \\ \mu_c \cup \lambda_c, & \text{if } c \in A \cap B. \end{cases}$$

Theorem 3.5. *Let M be a Γ-semiring, E be a parameters set and $A,\ B \subseteq E$. If (μ, A) and (λ, B) are fuzzy soft right tri-ideals over M, then $(\mu, A) \cup (\lambda, B)$ is a fuzzy soft right tri-ideal over M.*

Proof. Let (μ, A) and (λ, B) are fuzzy soft right tri-ideals over M. By Definition 3.4, we have that $(\mu, A) \cup (\lambda, B) = (\gamma, C)$ where $C = A \cup B$.
$Case\ (i)$: If $c \in A \setminus B$, then $\gamma_c = \mu_c$. Thus γ_c is a fuzzy right tri-ideal of M, since (μ, A) is a fuzzy soft right tri-ideal over M.
$Case\ (ii)$: If $c \in B \setminus A$, then $\gamma_c = \lambda_c$. Therefore γ_c is a fuzzy right tri-ideal of M, since (λ, B) is a fuzzy soft right tri-ideal over M.
$Case\ (iii)$: If $c \in A \cap B$, and $x, y \in M, \alpha \in \Gamma$ then $\gamma_c = \mu_c \cup \lambda_c$.

$$\begin{aligned}
(\mu_c \cup \lambda_c)(x+y) &= \max\{\mu_c(x+y), \lambda_c(x+y)\} \\
&\geq \max\Big\{ \min\{\mu_c(x), \mu_c(y)\}, \min\{\lambda_c(x), \lambda_c(y)\}\Big\} \\
&= \min\Big\{ \max\{\mu_c(x), \lambda_c(x)\}, \max\{\mu_c(y), \lambda_c(y)\}\Big\} \\
&= \min\{\mu_c \cup \lambda_c(x), \mu_c \cup \lambda_c(y)\} \\
\mu_c \cup \lambda_c(x\alpha y) &= \max\{\mu_c(x\alpha y), \lambda_c(x\alpha y)\} \\
&\geq \max\{\min\{\mu_c(x), \mu_c(y)\}, \min\{\lambda_c(x), \lambda_c(y)\}\} \\
&= \min\{\max\{\mu_c(x), \lambda_c(x)\}, \max\{\mu_c(y), \lambda_c(y)\}\} \\
&= \min\{\mu_c \cup \lambda_c(x), \mu_c \cup \lambda_c(y).\}
\end{aligned}$$

Then $\mu_c \cup \lambda_c$ is a fuzzy subsemiring. And

$$\begin{aligned}
\chi_M \circ (\mu_c \cup \lambda_c)(z) &= \sup_{z=l\alpha m} \min\{\chi_M(l), \mu_c \cup \lambda_c(m)\} \\
&= \sup_{z=l\alpha m} \min\Big\{\chi_M(l), \max\{\mu_c(m), \lambda_c(m)\}\Big\} \\
&= \sup_{z=l\alpha m} \max\Big\{ \min\{\chi_M(l), \mu_c(m)\}, \min\{\chi_M(l), \lambda_c(m)\}\Big\} \\
&= \max\Big\{ \sup_{z=l\alpha m} \min\{\chi_M(l), \mu_c(m)\}, \sup_{z=l\alpha m} \min\{\chi_M(l), \lambda_c(m)\}\Big\} \\
&= \max\{\chi_M \circ \mu_c(z), \chi_M \circ \lambda_c(z)\} \\
&= \chi_M \circ \mu_c(z) \cup \chi_M \circ \lambda_c(z).
\end{aligned}$$

$$\begin{aligned}
&((\mu_c \cup \lambda_c) \circ (\mu_c \cup \lambda_c))(x) \\
&= \sup_{x=l\alpha m} \min\{(\mu_c \cup \lambda_c)(l), (\mu_c \cup \lambda_c)(m)\} \\
&= \sup_{x=l\alpha m} \min\{\max\{\mu_c(l), \lambda_c(l)\}, \{\max\{\mu_c(m), \lambda_c(m)\}\} \\
&= \sup_{x=l\alpha m} \min\{\max\{\mu_c(l), \lambda_c(l)\}, \sup_{x=l\alpha m} \min\{\max\{\mu_c(m), \lambda_c(m)\}\} \\
&= \max\{ \sup_{x=l\alpha m} \{\min\{\mu_c(l), \mu_c(m)\}, \sup_{x=l\alpha m} \{\min\{\lambda_c(l), \lambda_c(m)\}\} \\
&= \max\{\mu_c \circ \mu_c(x)\}, \{\lambda_c \circ \lambda_c(x)\} \\
&= ((\mu_c \circ \mu_c) \cup (\lambda_c \circ \lambda_c))(x).
\end{aligned}$$

Thus, $(\chi_M \circ \mu_c) \cup (\chi_M \circ \lambda_c) = \chi_M \circ (\mu_c \cup \lambda_c)$. On the other hand,

$$\begin{aligned}
&(\mu_c \cup \lambda_c) \circ (\mu_c \cup \lambda_c) \circ \chi_M \circ (\mu_c \cup \lambda_c)(z) \\
&= \sup_{z=l\alpha m} \min\Big\{(\mu_c \cup \lambda_c) \circ (\mu_c \cup \lambda_c)(l), \chi_M \circ (\mu_c \cup \lambda_c)(m)\Big\} \\
&= \sup_{z=l\alpha m} \min\{\mu_c \circ \mu_c \cup \lambda_c \circ \lambda_c(l), \chi_M \circ \mu_c \cup \lambda_c(m)\}
\end{aligned}$$

$= \sup_{z=l\alpha m} \max\Big\{ \min\{\mu_c \circ \mu_c(l), \chi_M \circ \lambda_c(m)\}, \min\{\lambda_c \circ \lambda_c(a), \chi_M \circ \lambda_c(m)\}\Big\}$
$= \max\Big\{ \sup_{z=l\alpha m} \min\{\mu_c \circ \mu_c(l), \chi_M \circ \lambda_c(m)\},$
$\sup_{z=l\alpha m} \min\{\lambda_c \circ \lambda_c(l), \chi_M \circ \lambda_c(m)\}\Big\}$
$= \max\{\mu_c \circ \mu_c \circ \chi_M \circ \mu_c(z), \lambda_c \circ \lambda_c \circ \chi_M \circ \lambda_c(z)\}$
$= \mu_c \circ \mu_c \circ \chi_M \circ \mu_c \cup \lambda_c \circ \lambda_c \circ \chi_M \circ \lambda_c(z).$
So $\mu_c \cup \lambda_c \circ \mu_c \cup \lambda_c \circ \chi_M \circ \mu_c \cup \lambda_c = \mu_c \circ \mu_c \circ \chi_M \circ \mu_c \cup \lambda_c \circ \lambda_c \circ \chi_M \circ \lambda_c$. Hence

$$\mu_c \cup \lambda_c \circ \mu_c \cup \lambda_c \circ \chi_M \circ \mu_c \cup \lambda_c = \mu_c \circ \mu_c \circ \chi_M \circ \mu_c \cup \lambda_c \circ \lambda_c \circ \chi_M \circ \lambda_c \subseteq \mu_c \cup \lambda_c.$$

Therefore $\mu_c \cup \lambda_c$ is a fuzzy right tri-ideal of the Γ-semiring M.
γ_c is a fuzzy right tri-ideal ideal of M.
Hence $(\mu, A) \cup (\lambda, B)$ is a fuzzy soft right tri-ideal over M. □

Corollary 3.6. *If (μ, A) and (λ, B) are fuzzy soft left tri-ideals over Γ-semiring M, then $(\mu, A) \cup (\lambda, B)$ is a fuzzy soft left tri-ideal over M.*

Theorem 3.7. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft right ideal over M, then (μ, A) is a fuzzy soft left tri-ideal over M.*

Proof. Let (μ, A) be a fuzzy soft right ideal of the $\Gamma-$semiring M. Then, for each $a \in A$, μ_a is a fuzzy right ideal of the $\Gamma-$semiring. Let $z \in M$.

$$\begin{aligned} \mu_a \circ \chi_M(z) &= \sup_{z=l\alpha m} \min\{\mu_a(l), \chi_M(m)\}\ l, m \in M, \alpha \in \Gamma. \\ &= \sup_{z=l\alpha m} \min\{\mu_a(l), 1\} \\ &= \sup_{z=l\alpha m} \{\mu_a(l)\} \\ &\leq \sup_{z=l\alpha m} \mu_a(l\alpha m) \\ &= \mu_a(z). \end{aligned}$$

Therefore $\mu_a \circ \chi_M \subseteq \mu_a$.

$$\begin{aligned} \mu_a \circ \chi_M \circ \mu_a \circ \mu_a(z) &= \sup_{z=l\alpha m\beta q} \min\{\mu_a \circ \chi_M(l\alpha m), \mu_a \circ \mu_a(q)\} \\ &\leq \sup_{z=l\alpha m\beta q} \min\{\mu_a(l\alpha m), \mu_a(q)\} \\ &= \mu_a(z). \end{aligned}$$

Hence (μ, A) is a fuzzy soft left tri-ideal over M. □

Corollary 3.8. *Every fuzzy soft (left) ideal of a $\Gamma-$semiring M is a fuzzy soft (right) tri-ideal over M.*

Theorem 3.9. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft left-ideal over M, then (μ, A) is a fuzzy soft left quasi interior ideal over M.*

Proof. Suppose (μ, A) is a fuzzy soft left-ideal over M. Then for each $a \in A, \mu_a$ is a left-ideal of M. Let $x \in M$,

$$\begin{aligned}\chi_M \circ \mu_a \circ \chi_M \circ \mu_a(z) &= \sup_{z=l\alpha m} \{\min\{\chi_M \circ \mu_a(l), \chi_M \circ \mu_a(m)\}\} \\ &= \sup_{z=l\alpha m} \{\min\{\mu_a(l), \mu_a(m)\}\} \\ &\leq \sup_{z=l\alpha m} \{\min\{\mu_a(l\alpha m)\}\} \\ &= \mu_a(z)\end{aligned}$$

$$\Rightarrow \chi_M \circ \mu_a \circ \chi_M \circ \mu_a(z) \leq \mu_a(z)$$

Therefore $\chi_M \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a$ for all $a \in A$. Hence (μ, A) is a fuzzy soft soft left quasi interior ideal over M. □

Corollary 3.10. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E$.*
If (μ, A) is a fuzzy soft (right) quasi interior ideal over M, then (μ, A) is a fuzzy soft (right) tri-ideal over M.

Theorem 3.11. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E$ and μ be a nonempty fuzzy subset of M. If (μ, A) is a fuzzy soft left quasi interior ideal over M, then (μ, A) is a fuzzy soft right tri-ideal over M.*

Proof. Suppose (μ, A) is a fuzzy soft left quasi interior ideal over M.
Then $\chi_M \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a$ for all $a \in A$.
$\mu_a \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \chi_M \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a$ for all $a \in A$.
Hence (μ, A) is a fuzzy soft right tri-ideal over M. □

Theorem 3.12. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft left-ideal over M, then (μ, A) is a fuzzy soft bi-deal over M.*

Proof. Suppose (μ, A) is a fuzzy soft left-ideal over M. Then for each $a \in A, \mu_a$ is a left-ideal of M. Let $x \in M$,

$$\begin{aligned}\chi_M \circ \mu_a(z) &= \sup_{z=l\alpha m} \{\min\{\chi_M(l), \mu_a(m)\}\} \\ &= \sup_{z=l\alpha m} \{\min\{1, \mu_a(m)\}\} \\ &= \sup_{z=l\alpha m} \{\min\{\mu_a(m)\}\} \\ &\leq \sup_{z=l\alpha m} \{\min\{\mu_a(l\alpha m)\}\} \\ &= \mu_a(z)\end{aligned}$$

$$\mu_a \circ \chi_M \circ \mu_a(z) = \sup_{z=l\alpha m} \{\min\{\mu_a(l), \chi_M \circ \mu_a(m)\}\}$$

$$l, m, p \in M, \alpha \in M.$$

$$\begin{aligned}&\leq \sup_{z=l\alpha m} \{\min\{\mu_a(l), \mu_a(m)\}\} \\ &= \mu_a(z)\end{aligned}$$

$$\Rightarrow \mu_a \circ \chi_M \circ \mu_a(z) \leq \mu_a(z)$$

Therefore $\mu_a \circ \chi_M \circ \mu_a \leq \mu_a$ for all $a \in A$. Hence (μ, A) is a fuzzy soft bi-deal over M. □

Corollary 3.13. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft right ideal over M, then (μ, A) is a fuzzy soft bi-ideal over M.*

Theorem 3.14. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. Then M is regular if and only if $\mu_a = \mu_a \circ \chi_M \circ \mu_a \circ \mu_a$ for $a \in A$ and (μ, A) is a fuzzy soft left tri-ideal of M.*

Proof. Let (μ, A) be a fuzzy soft left tri-ideal of the regular $\Gamma-$semiring M and $x, y \in M$. Then to each $a \in A$, $\mu_a \circ \chi_M \circ \mu_a \circ \mu_a \subseteq \mu_a$. Thus

$$\begin{aligned}\mu_a \circ \chi_M \circ \mu_a \circ \mu_a(z) &= \sup_{z=z\alpha p\beta z}\{\min\{\mu_a \circ \chi_M(z), \mu_a \circ \mu_a(p\beta z)\}\}\\ &\geq \sup_{z=z\alpha p\beta z}\{\min\{\mu_a(z), \mu_a(p\beta z)\}\}\\ &\geq \sup_{z=z\alpha p\beta z}\{\min\{\mu_a(z\alpha p\beta z)\}\}\\ &= \mu_a(z).\end{aligned}$$

Therefore $\mu_a \subseteq \mu_a \circ \chi_M \circ \mu_a \circ \mu_a$. Hence $\mu_a \circ \chi_M \circ \mu_a \circ \mu_a = \mu_a$.
Therefore (μ, A) is a fuzzy soft left tri-ideal of M.
Conversely, suppose that $\mu_a = \mu_a \circ \chi_M \circ \mu_a \circ \mu_a$, for any fuzzy soft left tri-ideal (μ, A) over M and $a \in A$. Let T be a left tri-ideal of the $\Gamma-$semiring M.
Then χ_T be a fuzzy left tri-ideal of M.

$$\begin{aligned}\text{Therefore } \chi_T &= \chi_T \circ \chi_M \circ \chi_T \circ \chi_T\\ &= \chi_{T\Gamma M\Gamma T\Gamma T}\\ T &= T\Gamma M\Gamma T\Gamma T.\end{aligned}$$

M is a regular $\Gamma-$semiring . □

Corollary 3.15. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. Then M is regular if and only if $\mu_a = \mu_a \circ \mu_a \circ \chi_M \circ \mu_a$ for $a \in A$ and (μ, A) is a fuzzy soft right tri-ideal of M.*

Theorem 3.16. *Let M be a regular $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft left tri-ideal over M, then (μ, A) is a fuzzy soft right ideal over M.*

Proof. Suppose, (μ, A) is a fuzzy soft left tri-ideal over regular $\Gamma-$semiring M, and $z \in M, z = z\alpha x\beta z, x \in M, \alpha, \beta \in M, x \in M$ is regular and $a \in A$,then

$$\begin{aligned}&\mu_a \circ \chi_M(z) \circ \mu_a \circ \mu_a \leq \mu_a(z)\\ &= \sup_{z=z\alpha x\beta z}\{\min\{\mu_a \circ \chi_M(z), \mu_a \circ \mu_a(x\beta z)\}\} \leq \mu_a(z)\\ &= \sup_{z=z\alpha x\beta z}\{\min\{\mu_a(z), \mu_a(z)\}\} \leq \mu_a(z)\\ &\Rightarrow \mu_a \circ \chi_M(z) \leq \mu_a(z).\end{aligned}$$

Therefore $\mu_a \circ \chi_M \leq \mu_a$.

Hence μ_a is a fuzzy soft right ideal over M. □

Corollary 3.17. *Let M be a regular $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft (right) tri-ideal over M, then (μ, A) is a fuzzy soft (left) ideal over M.*

Theorem 3.18. *Let M be a $\Gamma-$semiring, E be a parameters set and $A \subseteq E, B \subseteq E$. Then M is a regular if and only if $\mu_a \cap \lambda_b \subseteq \mu_a \circ \lambda_b \circ \mu_a \circ \mu_a$, for every fuzzy soft left tri-ideal (μ, A) and every fuzzy soft ideal (λ, B) over M, $a \in A, b \in B$.*

Proof. Let M be a regular $\Gamma-$semiring and $z \in M$. Then there exist $p \in M, \alpha, \beta \in \Gamma$ such that $z = z\alpha p\beta z$. Thus

$$\begin{aligned}\mu_a \circ \lambda_b \circ \mu_a \circ \mu_a(z) &= \sup_{z=z\alpha p\beta z}\{\min\{\mu_a \circ \lambda_b(z\alpha p), \mu_a \circ \mu_a(z)\}\} \\ &= \min\Big\{\sup_{z\alpha p=z\alpha p\beta z\alpha p}\{\min\{\mu_a(z), \lambda_b(p\beta z\alpha p)\}, \\ &\quad \sup_{z=z\alpha p\beta z}\{\min\{\mu_a(z), \mu_a(p\beta z)\}\}\Big\} \\ &\geq \min\{\min\{\mu_a(z), \lambda_b(z)\}, min\{\mu_a(z), \lambda_b(z)\}\} \\ &= \min\{\mu_a(z), \lambda_b(z)\} = \mu_a \cap \lambda_b(z).\end{aligned}$$

Hence $\mu_a \cap \lambda_b \subseteq \mu_a \circ \lambda_b \circ \mu_a \circ \mu_a$.
Conversely suppose $\mu_a \cap \lambda_b \subseteq \mu_a \circ \lambda_b \circ \mu_a \circ \mu_a$. Let (μ, A) be a fuzzy soft left tri-ideal of M. Then to each $a \in A$, $\mu_a \cap \chi_M \subseteq \mu_a \circ \chi_M \circ \mu_a \circ \mu_a$,
$\mu_a \subseteq \mu_a \circ \chi_M \circ \mu_a \circ \mu_a$. Therefore M is a regular $\Gamma-$semiring . □

Theorem 3.19. *Let M be a regular $\Gamma-$semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft set then the following are equivalent,*
(i) (μ, A) is a fuzzy soft ideal over M.
(ii) (μ, A) is a fuzzy soft quasi-interior ideal over M.
(iii) (μ, A) is a fuzzy soft tri-ideal over M.

Proof. By Corollary[3.8], $(i) \Rightarrow (ii)$.
By Corollary[3.10], $(ii) \Rightarrow (iii)$.
By Corollary[3.17],$(iii) \Rightarrow (i)$.

□

4. Conclusion

In this paper, we introduced the notion of fuzzy soft left (right) tri-ideal of a $\Gamma-$semiring. Characterized regular $\Gamma-$semiring in terms of fuzzy soft left (right) tri-ideals and studied some of the properties. We proved that if (μ, A) is a fuzzy soft (right) tri-ideal over M, then (μ, A) is a fuzzy soft (right) ideal over M, and M is regular if and only if $\mu_a \cap \lambda_b \subseteq \mu_a \circ \lambda_b \circ \mu_a \circ \mu_a$, for every fuzzy soft left tri-ideal (μ, A) and every fuzzy soft ideal (λ, B) over M, $a \in A, b \in B$. Further one can extend this work by studying other algebraic structures.

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