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STABILITY OF 4-VARIABLE QUADRATIC FUNCTIONAL EQUATION IN BANACH SPACES

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ABSTRACT. In this work, authors investigate the generalized Hyers-Ulam stability of the 4-variable quadratic functional equation of the form

$$\begin{aligned}\phi\left(2a_1 \pm a_2 \pm a_3 \pm a_4\right) = \phi\left(a_1 \pm a_3 \pm a_4\right) + \phi\left(a_1 \pm a_2 \pm a_3\right) + \phi\left(a_1 \pm a_2 \pm a_4\right) \\ + \phi\left(\pm a_1\right) - \phi\left(\pm a_2\right) - \phi\left(\pm a_3\right) - \phi\left(\pm a_4\right)\end{aligned}$$

in Banach spaces.

1. INTRODUCTION

The notion presented by Rassias's theorem expressively inclined a numeral mathematicians to examine the stability problems for numerous functional equations inspected in paranormed spaces. The well-known functional equation in the field of stability is the quadratic functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y). \tag{1.1}$$

The function $f(x) = x^2$ is the solution of (1.1). The stability issues of numerous functional equations have been broadly examined through a numeral researchers and there are various stimulating consequences regarding this problem (see [1, 3, 5, 7, 9]).

Choonkil Park and Jung Rye Lee showed the Hyers-Ulam stability of an additive, quadratic, cubic, quartic functional equation in paranormed spaces. Rassias result which permits the Cauchy difference to be controlled by a general unbounded function in the spirit of Rassias method.

Stability problem for functional equations araises from the famed conversation of Ulam and the partial solution of Hyers to the Ulam's problem. The idea presented by Russias theorem meaningfully partial a numeral mathematicians to inspect the stability problems for numerous functional equations. The stability problem initiated from a question of Ulam regarding the stability of group homomorphisms.

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In this paper, we examine the generalized Hyers-Ulam stability of the 4-variable quadratic functional equation of the form

$$\phi(2a_1 \pm a_2 \pm a_3 \pm a_4) = \phi(a_1 \pm a_3 \pm a_4) + \phi(a_1 \pm a_2 \pm a_3) + \phi(a_1 \pm a_2 \pm a_4) + \phi(\pm a_1) - \phi(\pm a_2) - \phi(\pm a_3) - \phi(\pm a_4) \quad (1.2)$$

in Banach spaces using direct and fixed point methods.

Also, some of the research papers connected to many normed spaces are very valuable to grow this article such as [4, 8, 10, 12, 14, 17] and some of the additional papers are utilized to shape this paper (see [2, 6, 11, 13, 15, 16]).

Theorem (The Alternative of fixed point): Suppose that for a complete generalized metric space (E, d) and a strictly contractive mapping $\Omega : E \to E$ with Lipschitz constant L. Then, for each $a \in E$ either

(B1) $d(\Omega^i a, \Omega^{i+1} a) = +\infty, \forall i \geq 0$, or

(B2) There exists natural number i_0 such that

i) $d(\Omega^i a, \Omega^{i+1} a) < \infty \ \forall i \geq i_0$;

ii) The sequence $(\Omega^i a)$ is convergent to a fixed point α^* of Ω;

iii) α^* is the unique fixed point of Ω in the set $F = \left\{\alpha \in E; d(\Omega^{i_0} a, \alpha) < \infty\right\}$;

iv) $d(\alpha^*, \alpha) \leq \frac{1}{1-L} d(\alpha, \Omega\alpha) \, \forall \alpha \in F$.

We consider E and F are normed space and Banach space, respectively. For notational handiness, we define $\phi : E \to F$ by

$$D\phi(a_1, a_2, a_3, a_4) = \phi(2a_1 \pm a_2 \pm a_3 \pm a_4) - \phi(a_1 \pm a_3 \pm a_4) - \phi(a_1 \pm \phi_2 \pm \phi_3) - \phi(a_1 \pm a_2 \pm a_4) - \phi(\pm a_1) + \phi(\pm a_2) + \phi(\pm a_3) + \phi(\pm a_4)$$

for all $a_1, a_2, a_3, a_4 \in E$.

2. Stability Result for (1.2) in Banach space using Direct Method

Theorem 2.1. *Let $\zeta \in \{-1, 1\}$. Let $\beta : E^4 \to [0, \infty)$ be a function such that $\sum_{l=0}^{\infty} \frac{\beta(5^{l\zeta} a_1, 5^{l\zeta} a_2, 5^{l\zeta} a_3, 5^{l\zeta} a_4)}{5^{2l\zeta}}$ converges in $\mathbb{R}$ and*

$$\lim_{l \to \infty} \frac{\beta(5^{l\zeta} a_1, 5^{l\zeta} a_2, 5^{l\zeta} a_3, 5^{l\zeta} a_4)}{5^{2l\zeta}} = 0 \quad \forall a_1, a_2, a_3, a_4 \in E. \quad (2.1)$$

If a function $\phi : E \longrightarrow F$ be a function fulfils

$$||D\phi(a_1, a_2, a_3, a_4)|| \leq \beta(a_1, a_2, a_3, a_4) \quad \forall a_1, a_2, a_3, a_4 \in E, \quad (2.2)$$

then there exist $Q_1 : E \to F$ be an unique quadratic mapping which fulfils (1.2) and

$$||\phi(a) - Q_1(a)|| \leq \frac{1}{25} \sum_{l=\frac{1-\zeta}{2}}^{\infty} \frac{\sigma(5^{l\zeta} a)}{5^{2l\zeta}} \quad (2.3)$$

where

$$\sigma(a) = \beta(a, a, a, a) \quad \forall a \in E. \quad (2.4)$$

The function Q_1 is given by

$$Q_1(a) = \lim_{l \to \infty} \frac{\phi(5^{l\zeta} a)}{5^{2l\zeta}} \quad \forall a \in E. \quad (2.5)$$

Proof. Taking $\zeta = 1$. Switching $a_1 = a_2 = a_3 = a_4 = a$ in (2.2), we reach

$$\|25\phi(a) - \phi(5a)\| \leq \beta(a, a, a, a) \quad \forall a \in E. \quad (2.6)$$

It follows from (2.6) that

$$\|\frac{\phi(5a)}{5^2} - \phi(a)\| \leq \frac{1}{5^2}\beta(a,a,a,a) \quad \forall a \in E. \tag{2.7}$$

Now, setting a by $5a$ and dividing by 5^2 in (2.7), we arrive

$$\|\frac{\phi(5^2a)}{5^4} - \frac{\phi(5a)}{5^2}\| \leq \frac{1}{5^4}\beta(5a,5a,5a,5a) \quad \forall a \in E. \tag{2.8}$$

Utilizing (2.7) and (2.8), we have

$$\|\frac{\phi(5^2a)}{5^2} - \phi(a)\| \leq \frac{1}{5^2}\left(\sigma(a) + \frac{\sigma(5a)}{5^2}\right) \quad \forall a \in E.$$

In general for any non-negative integer p , one can easy to verify that

$$\|\frac{\phi(5^pa)}{5^{2p}} - \phi(a)\| \leq \frac{1}{5^2}\sum_{l=0}^{\infty}\frac{\sigma(5^la)}{5^{2l}} \quad \forall a \in E. \tag{2.9}$$

In order to verify the sequence $\{\frac{\phi(5^pa)}{5^{2p}}\}$ is convergence , replacing a by 5^qa and dividing 5^{2q} in (2.9), for $p,q>0$, we get

$$\|\frac{\phi(5^{p+q}a)}{5^{2(p+q)}} - \frac{\phi(5^qa)}{5^{2q}}\| \leq \frac{1}{5^2}\sum_{l=0}^{p-1}\frac{\sigma(5^{l+q}a)}{5^{2(l+q)}} \to 0 \;\; as \;\; q \to \infty \tag{2.10}$$

for each $a \in E$. Therefore, $\{\frac{\phi(5^pa)}{5^{2p}}\}$ is a Cauchy sequence. As, F is complete, there exists a mapping $Q_1 : E \to F$ such that

$$Q_1(a) = \lim_{p\to\infty}\frac{\phi(5^pa)}{5^{2p}}$$

for all $a \in E$. Taking $p \to \infty$ in (2.9) we see that (2.4) holds for $a \in E$. Next, we show that Q_1 fulfils (1.2), interchanging (a_1,a_2,a_3,a_4) by $(5^qa,5^qa,5^qa,5^qa)$ and dividing 5^{2q} in (2.2), we reach

$$\frac{1}{5^{2q}}\|Q_1(5^qa,5^qa,5^qa,5^qa)\| \leq \frac{1}{5^{2q}}\beta(5^qa,5^qa,5^qa,5^qa)$$

for all $a_1,a_2,a_3,a_4 \in E$. Taking $q \to \infty$ in above inequality and utilizing the definition of $Q_1(a)$, we get $Q_1(a_1,a_2,a_3,a_4) = 0$. Hence Q_1 fulfils (1.2). Next, we verify that Q_1 is unique. Let Q_2 be the another quadratic mapping fulfilling (1.2) and (2.4), then

$$\begin{aligned}\|Q_1(a) - Q_2(a)\| &\leq \frac{1}{5^{2q}}\{\|Q_1(5^qa) - \phi(5^q)\| + \|\phi(5^qa) - Q_2(5^qa))\|\} \\ &\leq \frac{1}{5^2}\sum_{l=0}^{\infty}\frac{\sigma(5^{l+q}a)}{5^{2(l+q)}} \to 0 \;\; as \;\; q \to \infty\end{aligned}$$

for all $a \in E$. Hence Q_1 is unique. For $\zeta = -1$, we can prove a similar stability result. This completes the proof. $\square$

Corollary 2.2. *Let δ and s be a positive real numbers. Let a function $\phi : E \to F$ fulfilling*

$$||D\phi(a_1,a_2,a_3,a_4)|| \leq \begin{cases}\delta \\ \delta(||a_1||^\rho + ||a_2||^\rho + ||a_3||^\rho + ||a_4||^\rho) \\ \delta\Big(||a_1||^\rho \cdot ||a_2||^\rho \cdot ||a_3||^\rho \cdot ||a_4||^\rho \\ \qquad\qquad + ||a_1||^{4\rho} + ||a_2||^{4\rho} + ||a_3||^{4\rho} + ||a_4||^{4\rho}\Big)\end{cases}$$

for all $a_1, a_2, a_3, a_4 \in E$. Then there exists $Q_1 : E \to F$ is an unique quadratic mapping such that

$$||\phi(a) - Q_1(a)|| \leq \begin{cases} \frac{\delta}{|24|}, \\ \frac{4\delta||a||^{\rho}}{|5^2-5^{\rho}|}; & \rho \neq 2, \\ \frac{5\delta||a||^{4\rho}}{|5^2-5^{4\rho}|}; & \rho \neq \frac{2}{4}, \end{cases}$$

for all $a \in E$.

3. Stability Result for (1.2) in Banach space using Fixed Point Method

Theorem 3.1. *Let $\phi : E \to F$ be a mapping for which there exists a function $\chi : E^4 \to [0, \infty)$ with the condition*

$$lim_{l\to\infty} \frac{\chi(\nu_i^l a_1, \nu_i^l a_2, \nu_i^l a_3, \nu_i^l a_4)}{\nu_i^{2l}} = 0 \tag{3.1}$$

where

$$\nu_i = \begin{cases} 5, \ if & i = 0 \\ \frac{1}{5}, \ if & i = 1 \end{cases}$$

such that the functional inequality

$$||D\phi(a_1, a_2, a_3, a_4)|| \leq \chi(a_1, a_2, a_3, a_4) \quad \forall a_1, a_2, a_3, a_4 \in E. \tag{3.2}$$

If there exist $L = L(i)$ such that the function

$$a \to \gamma(a) = \chi\left(\frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}\right)$$

has the property,

$$\frac{1}{\nu_i^2}\gamma(\nu_i a) = L\gamma(a) \quad \forall a \in E. \tag{3.3}$$

Then there exists a unique quadratic function $Q_1 : E \to F$ fulfilling (1.2) and

$$||\phi(a) - Q_1(a)|| \leq \frac{L^{1-i}}{1-L}\gamma(a) \tag{3.4}$$

holds for all $a \in E$.

Proof. Suppose $d = \{s/s : E \to F, s(0) = 0)\}$ and take the generalized metric on Δ. $d(s,t) = inf\{l \in (0,\infty) : ||s(a) - t(a)|| \leq l\gamma(a), a \in E\}$. It is easy to show that (Δ, d) is complete. Let us define $\Lambda : \Delta \to \Delta$ by

$$\Lambda s(a) = \frac{1}{\nu_i^2} s(\nu_i a) \quad \forall a \in \Delta.$$

Now $s, t \in \Delta$,

$$\begin{aligned} d(s,t) \leq l &\Rightarrow ||s(a) - t(a)|| \leq l\gamma(a) \quad \forall a \in E. \\ &\Rightarrow ||\frac{1}{\nu_i^2}s(\nu_i a) - \frac{1}{\nu_i^2}t(\nu_i a)|| \leq \frac{1}{\nu_i^2} l\gamma(\nu_i a) \quad \forall a \in E. \\ &\Rightarrow ||\Lambda s(a) - \Lambda t(a)|| \leq l\gamma(a) \quad \forall a \in E. \\ &\Rightarrow d(\Lambda s, \Lambda t) \leq lL. \end{aligned}$$

From the above $d(\Lambda s, \Lambda t) \leq Ld(s,t) \quad \forall\, s, t \in \Delta$. (i,e.,) Λ is strictly contractive mapping on with Lipschtiz constant L. Replacing (a_1, a_2, a_3, a_4) by (a, a, a, a) in (3.2), we obtain

$$||\phi(5a) - 25\phi(a)|| \leq \chi(a, a, a, a) \tag{3.5}$$

for all $a \in E$. It is follows from (3.5) that

$$||\phi(a) - \frac{\phi(5a)}{25}|| \leq \frac{\chi(a,a,a,a)}{25} \tag{3.6}$$

for all $a \in E$. Utilizing (3.3) for $i = 0$, we have

$$||\phi(a) - \frac{\phi(5a)}{25}|| \leq \gamma(a)$$

for all $a \in E$.

$$i.e., d(\phi, \Lambda\phi) \leq 1 \Rightarrow d(\phi, \Lambda\phi) \leq 1 = L = L^1 < \infty.$$

Again interchanging $a = \frac{a}{5}$ in (3.5) and (3.6), we obtain

$$||\phi(a) - 25\phi\left(\frac{a}{5}\right)|| \leq \chi\left(\frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}\right)$$

and

$$||\phi(a) - 25\phi\left(\frac{a}{5}\right)|| \leq \chi\left(\frac{a}{2}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}\right) \tag{3.7}$$

for all $a \in E$. Utilizing (3.3) for $i = 0$, we obtain

$$||\phi(a) - \frac{\phi(5a)}{25}|| \leq L\gamma(a) \tag{3.8}$$

for all $a \in E$. (i.e.,) $d(\phi, \Lambda\phi) \leq 1 \Rightarrow d(\phi, \Lambda\phi) \leq 1 = L^0 < \infty$. In above case, we arrive

$$d(\phi, \Lambda\phi) \leq L^{1-i}.$$

Therefore, $(B_2(i))$holds. By $(B_2(ii))$, it follows that there exists a fixed point Q_1 of Λ in E, such that

$$Q_1(a) = lim_{l\to\infty} \frac{\phi(\nu_i^l a)}{\nu_i^{2l}} \quad \forall a \in E. \tag{3.9}$$

In order to show $Q_1 : E \to F$ is quadratic. Interchanging (a_1, a_2, a_3, a_4) by $(\nu_i^l a_1, \nu_i^l a_2, \nu_i^l a_3, \nu_i^l a_4)$ in (3.2) and dividing by ν_i^{2l}, it follows from (3.1) and (3.9), we see that Q_1 fulfils (1.2) for all $a_1, a_2, a_3, a_4 \in E$. Hence Q_1 fulfils (1.2). By $(B_2(iii))$, Q_1 is the unique fixed point of Λ in $F = \{\phi \in \Delta; d(\Lambda\phi, Q_1) < \infty\}$. Utilizing the fixed point alternative result, Q_1 is the unique function such that,

$$||\phi(a) - Q_1(a)|| \leq l\gamma(a) \quad \forall\, a \in E \; and \; l > 0.$$

Finally, by $(B_2(iv))$, we reach

$$d(\phi, Q_1) \leq \frac{1}{1-L} d(\phi, \Lambda\phi)$$

$$(i.e.,)\; d(\phi, Q_1) \leq \frac{L^{1-i}}{1-L}.$$

Hence, we conclude that

$$||\phi(a) - Q_1(a)|| \leq \frac{L^{1-i}}{1-L}\gamma(a) \quad \forall a \in E.$$

□

Corollary 3.2. *Let $\phi : E \to F$ be a mapping and there exists a real numbers δ and ρ such that*

$$||D\phi(a_1,a_2,a_3,a_4)|| \leq \begin{cases} \delta \\ \delta(||a_1||^\rho + ||a_2||^\rho + ||a_3||^\rho + ||a_4||^\rho) \\ \delta\Big(||a_1||^\rho \cdot ||a_2||^\rho \cdot ||a_3||^\rho \cdot ||a_4||^\rho \\ \qquad\qquad + ||a_1||^{4\rho} + ||a_2||^{4\rho} + ||a_3||^{4\rho} + ||a_4||^{4\rho}\Big) \end{cases}$$

for all $a_1, a_2, a_3, a_4 \in E$. Then there exist a unique quadratic function $Q_1 : E \to F$ such that

$$||\phi(a) - Q_1(a)|| \leq \begin{cases} \frac{\delta}{|24|} \\ \frac{4\delta||a||^\rho}{|5^2-2^\rho|} & ; \quad \rho \neq 2 \\ \frac{5\delta||a||^{4\rho}}{|5^2-2^{4\rho}|} & ; \quad \rho \neq \frac{2}{4} \end{cases}$$

for all $a \in E$.

Proof. Setting

$$\chi(a_1,a_2,a_3,a_4) \leq \begin{cases} \delta \\ \delta(\sum_{i=1}^4 ||a_i||^\rho) \\ \delta(\prod_{i=1}^4 ||a_i||^\rho + \sum_{i=1}^4 ||a_i||^{4\rho}) \end{cases}$$

for all $a_1, a_2, a_3, a_4 \in E$. Now

$$\frac{\chi(\nu_i^l a_1, \nu_i^l a_2, \nu_i^l a_2, \nu_i^l a_2)}{\nu_i^{2l}} \leq \begin{cases} \frac{\delta}{\nu_i^l} \\ \frac{\delta}{\nu_i^l}\left\{\sum_{i=1}^4 ||\nu_i^l a_i||^\rho\right\} \\ \frac{\delta}{\nu_i^l}\left\{\prod_{i=1}^4 ||\nu_i^l a_i||^{4\rho} + \sum_{i=1}^4 ||\nu_i^l a_i||^{4\rho}\right\} \end{cases}$$

$$= \begin{cases} \longrightarrow \quad 0 \quad as \quad l \longrightarrow \infty \\ \longrightarrow \quad 0 \quad as \quad l \longrightarrow \infty \\ \longrightarrow \quad 0 \quad as \quad l \longrightarrow \infty \end{cases}$$

i.e., (3.5) is holds. But we have $\gamma(a) = \chi(\frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5})$.
Hence

$$\gamma(a) = \chi(\frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}) = \begin{cases} \delta \\ \frac{4\delta||a||^\rho}{5^{s\rho}} \\ \frac{5\delta||a||^{4\rho}}{5^{4\rho}} \end{cases}$$

$$\frac{1}{\nu_i^2}\gamma(\nu_i a) = \begin{cases} \frac{\delta}{\nu_i^2} \\ \frac{1}{\nu_i^2}\frac{4\delta||a||^\rho}{5^\rho} \\ \frac{1}{\nu_i^2}\frac{5\delta||a||^{4\rho}}{5^{4\rho}} \end{cases}$$

$$= \begin{cases} \nu_i^{-2}\gamma(a) \\ \nu_i^{s-2}\gamma(a) \\ \nu_i^{4\rho-2}\gamma(a) \end{cases}$$

for all $a \in E$. Hence the inequality (1.2) holds for
$L = 5^{-2}$ if $i = 0$ $\;and\;$ $L = \frac{1}{5^{-2}}$ if $i = 1$.
$L = 5^{\rho-2}$ $\;for\;$ $\rho < 2$ if $i = 0$ $\;and\;$ $L = \frac{1}{5^{\rho-2}}$ $\;for\;$ $\rho > 2$ if $i = 1$.
$L = 5^{4\rho-2}$ $\;for\;$ $\rho < \frac{2}{4}$ if $i = 0$ $\;and\;$ $L = \frac{1}{5^{4\rho-2}}$ $\;for\;$ $\rho > \frac{2}{4}$ if $i = 1$.

Now, from (3.5) we prove the following cases:
Case1: $L = 5^{-1}$ if $i = 0$.

$$||\phi(a) - Q_1(a)| \leq \frac{L^{1-i}}{1-L}\gamma(a) = \frac{(5^{-2})}{1-5^{-2}}\delta = \frac{\delta}{24}.$$

Case2: $L = \frac{1}{5^{-2}}$ if $i = 1$.

$$||\phi(a) - Q_1(a)|| \leq \frac{L^{1-i}}{1-L}\gamma(a) = \frac{1}{1-5^2}\delta = \frac{-\delta}{24}.$$

Case3: $L = 5^2$ for $\rho < 2$ if $i = 0$.

$$||\phi(a) - Q_1(a)|| \leq \frac{L^{1-i}}{1-L}\gamma(a) = \frac{5^{\rho-2}}{1-5^{\rho-2}}\frac{4\delta||a||^\rho}{5^\rho} = \frac{4\delta||a||^\rho}{5^2-5^\rho}.$$

Case4: $L = \frac{1}{5^{s-2}}$ for $\rho > 2$ if $i = 1$.

$$||\phi(a) - Q_1(a)|| \leq \frac{L^{1-i}}{1-L}\gamma(a) = \frac{1}{1-\frac{1}{5^{\rho-2}}}\frac{4\delta||a||^\rho}{5^\rho} = \frac{4\delta||a||^\rho}{5^\rho-5^2}.$$

Case5: $L = 5^{4\rho-2}$ for $\rho < \frac{2}{4}$ if $i = 0$.

$$||\phi(a) - Q_1(a)|| \leq \frac{L^{1-i}}{1-L}\gamma(a) = \frac{5^{4\rho-2}}{1-5^{4\rho-2}}\frac{5\delta||a||^{4\rho}}{5^{4\rho}} = \frac{5\delta||a||^{4\rho}}{5^2-5^{4\rho}}.$$

Case6: $L = \frac{1}{5^{4\rho-2}}$ for $\rho > \frac{2}{4}$ if $i = 1$.

$$||\phi(a) - Q_1(a)|| \leq \frac{L^{1-i}}{1-L}\gamma(a) = \frac{1}{1-\frac{1}{5^{4\rho-2}}}\frac{5\delta||a||^{4\rho}}{5^{4\rho}} = \frac{5\delta||a||^{4\rho}}{5^{4\rho}-5^2}.$$

Hence the proof. □

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