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SOME IMPROVISED SETS IN GRILL TOPOLOGICAL SPACES

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ABSTRACT. Aim of this paper, the new grill notions are studied using grill topological spaces and by using some defined sets where the sets $\mathcal{G}_t$-set and $\mathcal{G}_{\mathcal{R}}$-set are defined. Properties of this set and some relationships are investigate and deal with a grill topological spaces.

1. INTRODUCTION

The idea of grill on a topological space by Choquet [2], goes as follows : A non-null collection of subsets of a topological spaces X is said to be a $grill$ on X if

(1) $\phi \notin \mathcal{G}$.
(2) $D \in \mathcal{G}$ and $D \subseteq E$ imply that $E \in \mathcal{G}$ and
(3) $D, E \subseteq X$ and $D \cup E \in \mathcal{G}$ imply that $D \in \mathcal{G}$ or $E \in \mathcal{G}$.

For a topological space X, the operator $\Phi : \wp(X) \longrightarrow \wp(X)$ from the power set $\wp(X)$ of X to P(X) was first defined in [4] in terms of grill; the latter concept being defined by Choquet [2] several decades back. Interestingly, it is found from subsequent investigations that the notion of grills as an appliance like nets and filters. For a grill G on a topological space X, an operator from the power set $\wp(X)$ of X to $\wp(X)$ was defined in [8] in the following manner :

For any $D \in \wp(X)$,

$\Phi(D) = \{x \in X : U \cap D \in G, \text{for each open neighborhood } U \text{ of } x\}$

Then the operator $\psi : \wp(X) \longrightarrow \wp(X)$, given by $\psi(D) = D \cup \Phi(D)$, for $D \in \wp(X)$, was also shown in [8] to be a Kuratowski closure operator, defining a unique topology $\tau_{\mathcal{G}}$ on X such that $\tau \subseteq \tau_{\mathcal{G}}$. If (X, τ) is a topological space and $\mathcal{G}$ is a grill on X then the triple $(X, \tau, \mathcal{G})$ will be called a *grill topological space*. In this paper, the new grill notions are studied using grill topological spaces and by using some defined sets where the sets $\mathcal{G}_t$-set and $\mathcal{G}_{\mathcal{R}}$-set are defined. Properties of this set and some relationships are investigate and deal with a grill topological spaces.

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2. Preliminaries

There are some types of a grill topological space as like a cofinite topology and discrete topology [8].

For a topological space (X, τ) and $D \subseteq X$, throughout this paper, we mean $C(D)$ and $I(D)$ the closure set and the interior set of D, respectively.

Theorem 2.1. [8]

Let $(X, \tau, \mathcal{G})$ be a grill topological space. Then for $D, E \subseteq X$, the following properties hold:

(1) $D \subseteq E$ implies that $\Phi(D) \subseteq \Phi(E)$.
(2) $\Phi(D \cup E) = \Phi(D) \cup \Phi(E)$.
(3) $\Phi(\Phi(D)) \subseteq \Phi(D) = C(\Phi(D)) \subseteq C(D)$.
(4) If $U \in \tau$ then $U \cap \Phi(D) \subseteq \Phi(U \cap D)$.

Theorem 2.2. [8] *In a space $(X, \tau, \mathcal{G})$, if D and E are subsets of X, then the following results are true for the set operator ψ.*

(1) $D \subseteq \psi(D)$,
(2) $\psi(\phi) = \phi$ and $\psi(X) = X$,
(3) $If D \subseteq E$, then $\psi(D) \subseteq \psi(E)$,
(4) $\psi(D) \cup \psi(E) = \psi(D \cup E)$.
(5) $\psi(\psi(D)) = \psi(D)$.

Definition 2.1. [3] A subset D of space $(X, \tau, \mathcal{G})$ is said to be

(1) grill α-open (resp. $\mathcal{G}_\alpha$-open) if $D \subseteq I(\psi(I(D)))$,
(2) grill pre-open (resp. $\mathcal{G}_p$-open) if $D \subseteq I(\psi(D))$.

3. On some new sets in grill topological spaces

Definition 3.1. A subset D of a space $(X, \tau, \mathcal{G})$ is called

(1) grill t-set (resp. $\mathcal{G}_t$-set) if $I(D) = I(\psi(D))$,
(2) grill t_α-set (resp. $\mathcal{G}_{t_\alpha}$-set) if $I(D) = I(\psi(I(D)))$,
(3) grill $\mathcal{R}$-set (resp. $\mathcal{G}_\mathcal{R}$-set) if $D = D_1 \cap D_2$, where D_1 is open and D_2 is $\mathcal{G}_t$-set,
(4) grill $\mathcal{R}_\alpha$-I-set (resp, $\mathcal{G}_{\mathcal{R}_\alpha}$-set) if $D = D_1 \cap D_2$, where D_1 is open and D_2 is $\mathcal{G}_{t_\alpha}$-set.

Example 3.1. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$.*

If $\mathcal{G} = \{\{a\}, \{a, b\}, \{a, c\}, X\}$. Then $\{\phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$ is $\mathcal{G}_t$-sets and $\mathcal{G}_{t_\alpha}$-sets, $\wp(X)$ is $\mathcal{G}_\mathcal{R}$-sets and $\mathcal{G}_{\mathcal{R}_\alpha}$-sets.

Remark. *Let $(X, \tau, \mathcal{G})$ be a grill topological space,*

(1) every open set is $\mathcal{G}_\mathcal{R}$-set.
(2) every $\mathcal{G}_t$-set is $\mathcal{G}_\mathcal{R}$-set.

Remark. *The converses of Remark 3 are not true as shown in the next Examples.*

Example 3.2. *Here the Example 3.1,*

(1) $\{c\}$ is $\mathcal{G}_\mathcal{R}$-set but not open set.
(2) $\{a\}$ is $\mathcal{G}_\mathcal{R}$-set but not $\mathcal{G}_t$-set.

Proposition 3.3. *Let D and E be subsets of a space $(X, \tau, \mathcal{G})$. If D and E are $\mathcal{G}_t$-sets, then $D \cap E$ is $\mathcal{G}_t$-set.*

Proof.
Let D and E be $\mathcal{G}_t$-sets. Then we have $I(D\cap E) \subset I(\psi(D\cap E)) \subset I(\psi(D)\cap\psi(E)) = I(\psi(D)) \cap I(\psi(E)) = I(D) \cap I(E) = I(D \cap E)$. Then $I(D \cap E) = I(\psi(D \cap E))$ and hence $D \cap E$ is a $\mathcal{G}_t$-set. □

Example 3.4. *In Example 3.1, $H = \{a, c\}$ and $K = \{b, c\}$ is $\mathcal{G}_t$-set. But $D \cap E = \{c\}$ is $\mathcal{G}_t$-set.*

Proposition 3.5. *For a subset D of a space $(X, \tau, \mathcal{G})$, the next properties are equivalent:*

(1) D is open,
(2) D is $\mathcal{G}_p$-open and $\mathcal{G}_{\mathcal{R}}$-set.

Proof.
(1) ⇒ (2) : Let D be open. Then $D = I(D) \subset I(\psi(D))$ and D is $\mathcal{G}_p$-open. Also by Remark 3, D is $\mathcal{G}_{\mathcal{R}}$-set.
(2) ⇒ (1) : Given D is $\mathcal{G}_{\mathcal{R}}$-set. So $D = D_1 \cap D_2$ where D_1 is open and $I(Q) = I(C(Q))$. Then $D \subseteq D_1 = I(D_1)$. Also, D is $\mathcal{G}_p$-open implies $D \subseteq I(C(D)) \subset I(\psi(D_2)) = I(D_2)$ by assumption. Thus $D \subseteq I(D_1) \cap I(D_2) = I(D_1 \cap D_2) = I(D)$ and hence D is open. □

Remark. *Let $(X, \tau, \mathcal{G})$, be a grill topological space, the concept of $\mathcal{G}_p$-open sets and $\mathcal{G}_{\mathcal{R}}$-sets are independent.*

Example 3.6. *Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a, b\}, \{c, d\}, X\}$.*
If $\mathcal{G} = \{\{c\}, \{c, d\}, X\}$.

(1) $\{c\}$ is $\mathcal{G}_p$-open but not $\mathcal{G}_{\mathcal{R}}$-set.
(2) In Example 3.1, $\{c\}$ is $\mathcal{G}_{\mathcal{R}}$-set but not $\mathcal{G}_p$-open.

Remark. *Let $(X, \tau, \mathcal{G})$, be a grill topological space,*

(1) every open set is $\mathcal{G}_{\mathcal{R}_\alpha}$-set.
(2) every $\mathcal{G}_{t_\alpha}$-set is $\mathcal{G}_{\mathcal{R}_\alpha}$-set.

Example 3.7. *In Example 3.1,*

(1) $\{c\}$ is $\mathcal{G}_{\mathcal{R}_\alpha}$-set but not open set.
(2) $\{a, b\}$ is $\mathcal{G}_{\mathcal{R}_\alpha}$-set but not $\mathcal{G}_{t_\alpha}$-set.

Proposition 3.8. *If D_1 and D_2 are $\mathcal{G}_{t_\alpha}$-sets, then the intersection $D_1 \cap D_2$ is a $\mathcal{G}_{t_\alpha}$-set.*

Proof.
Let D_1 and D_2 be $\mathcal{G}_{t_\alpha}$-sets. Then we have $I(D_1 \cap D_2) \subset I(\psi(I(D_1 \cap D_2))) \subseteq I[\psi(I(D_1))\cap\psi(I(D_2))] = I(\psi(I(D_1)))\cap I(\psi(I(D_2))) = I(D_1)\cap I(D_2) = I(D_1\cap D_2)$. Then $I(D_1 \cap D_2) = I(\psi(I(D_1 \cap D_2)))$ and hence $D_1 \cap D_2$ is a $\mathcal{G}_{t_\alpha}$-set. □

Example 3.9. *In Example 3.1, $H = \{a, c\}$ and $K = \{b, c\}$ is $\mathcal{G}_{t_\alpha}$-set. But $H \cap K = \{c\}$ is $\mathcal{G}_{t_\alpha}$-set.*

Proposition 3.10. *For a subset D of a space $(X, \tau, \mathcal{G})$ be a grill topological space, the next properties are equivalent:*

(1) D is open,
(2) D is $\mathcal{G}_\alpha$-open and $\mathcal{G}_{\mathcal{R}_\alpha}$-set.

Proof.
(1) ⇒ (2) : Let D be a open. Then $D = I(D) \subseteq \psi(I(D))$ and $D = I(D) \subseteq I(\psi(I(D)))$. Therefore D is $\mathcal{G}_\alpha$-open. Also by Remark 3(1), D is a $\mathcal{G}_{\mathcal{R}_\alpha}$-set.

$(2) \Rightarrow (1)$: Given D is a $\mathcal{G}_{\mathcal{R}_\alpha}$-set. So $D = D_1 \cap D_2$ where D_1 is open and $I(D_2) = I(\psi(I(D_2)))$. Then $D \subseteq D_1 = I(D_1)$. Also D is $\mathcal{G}_\alpha$-open implies $D \subseteq I(\psi(I(D))) \subseteq I(\psi(I(D_2))) = I(D_2)$ by assumption. Thus $D \subseteq I(D_1) \cap I(D_2) = I(D_1 \cap D_2) = I(D)$ and D is open. □

Remark. *These relations are shown in the diagram.*

$$
\begin{array}{ccccc}
\mathcal{G}_{t_\alpha}\textbf{-set} & & & & \\
\uparrow & & & & \\
\textbf{open set} & \longrightarrow & \mathcal{G}_{\mathcal{R}}\textbf{-set} & \not\leftrightarrow & \mathcal{G}_p\textbf{-set} \\
\downarrow & & \downarrow & & \\
\mathcal{G}_{\mathcal{R}_\alpha}\textbf{-set} & & \mathcal{G}_t\textbf{-set} & &
\end{array}
$$

The converse of above diagram is not true.

4. Conclusion

Because of the topological space is stripped of the geometric form and it is used to measure things that are difficult to measure, such as intelligence, beauty and goodness. So, we used the concept of grill to expand this space to help us measure the things that are difficult to measure. In this paper, the new grill notions are studied using grill topological spaces and by using some defined sets where the sets $\mathcal{G}_t$-set and $\mathcal{G}_{\mathcal{R}}$-set are defined. Properties of this set and some relationships are investigate and deal with a grill topological spaces.

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