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SOMEWHAT PAIRWISE FUZZY e-IRRESOLUTE MAPPINGS

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ABSTRACT. The concepts of somewhat pairwise fuzzy e-irresolute mapping and somewhat pairwise fuzzy irresolute e-open mapping is introduced. Also some properties and comparisions of those mappings are given.

1. INTRODUCTION

The concepts of fuzzy sets introduced by Zadeh[7]. Chang [2] studied the notion of fuzzy topology in 1968. Seenivasan and Kamala [3] introduced the concept of fuzzy e-continous functions in fuzzy topological spaces. The concepts of somewhat fuzzy e-continuous functions and somewhat fuzzy e-open functions are introduced and studied by Swaminathan in [6]. The purpose of this paper is to introduce and study the concepts of somewhat pairwise fuzzy e-irresolute mappings and somewhat pairwise fuzzy irresolute e-open mappings on a fuzzy bitopological spaces and also we discuss some of their properties.

A fuzzy subset A of a space X is called fuzzy regular open [1] (resp. fuzzy regular closed) if $A = \text{Int}(\text{Cl}(A))$ (resp.$A = \text{Cl}(\text{Int}(A))$). Now $\text{Cl}(A)$ and $\text{Int}(A)$ are defined as follows:$\text{Cl}(A) = \bigwedge \{U : U \geq A,\ U \text{ is fuzzy closed in } X\}$ and
$\text{Int}(A) = \bigvee \{U \leq A,\ U \text{ is fuzzy open in } X\}$. The fuzzy δ-interior of a fuzzy subset A of X is the union of all fuzzy regular open sets contained in A. A fuzzy subset A is called fuzzy δ-open [4] if $A = \text{Int}_\delta(A)$. The complement of fuzzy δ-open set is called fuzzy δ-closed (i.e, $A = \text{Cl}_\delta(A)$).

A fuzzy subset A of a space X is called fuzzy e-open[3] if $A \leq \text{cl}(\text{int}_\delta A) \vee \text{int}(\text{cl}_\delta A)$ and fuzzy e-closed set if $A \geq \text{cl}(\text{int}_\delta A) \wedge \text{int}(\text{cl}_\delta A)$. Throughout this paper X and Y stand for $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(Y, \mathcal{F}_1, \mathcal{F}_2)$ respectively.

Definition 1.1. *A mapping $f : X \to Y$ is called fuzzy e-continuous* [3] *if $f^{-1}(V)$ is a fuzzy e-open set on X for any fuzzy open set V on Y.*

Definition 1.2. *A mapping $f : X \to Y$ is called somewhat fuzzy e-continuous*[6] *if there exists a fuzzy e-open set $U \neq 0_X$ on X such that $U \leq f^{-1}(V) \neq 0_X$ for any fuzzy open set V on Y.*

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Definition 1.3. *A mapping $f : X \to Y$ is called somewhat fuzzy e-open[6] if there exists a fuzzy e-open set $V \neq 0_Y$ on Y such that $V \leq f(U) \neq 0_Y$ for any fuzzy open set U on X.*

Definition 1.4. *A fuzzy set U on a fuzzy topological space X is called fuzzy e-dense[6] if there exists no fuzzy e-closed set V in X such that $U < V < 1$.*

2. SOMEWHAT PAIRWISE FUZZY e-IRRESOLUTE MAPPINGS

In this section we introduce a somewhat pairwise fuzzy e-irresolute mapping and compared few results.

Definition 2.1. *A mapping $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ is called pairwise fuzzy e-continuous if $f^{-1}(V)$ is a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set on $(X, \mathcal{T}_1, \mathcal{T}_2)$ for any $\mathcal{F}_1$-fuzzy open or $\mathcal{F}_2$-fuzzy open set V on $(Y, \mathcal{F}_1, \mathcal{F}_2)$.*

Definition 2.2. *A mapping $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ is called pairwise fuzzy e-irresolute if $f^{-1}(V)$ is a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set on $(X, \mathcal{T}_1, \mathcal{T}_2)$ for any $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set V on $(Y, \mathcal{F}_1, \mathcal{F}_2)$.*

Definition 2.3. *A mapping $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ is called somewhat pairwise fuzzy e-continuous if there exists a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set $U \neq 0_X$ on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $U \leq f^{-1}(V) \neq 0_X$ for any $\mathcal{F}_1$-fuzzy open or $\mathcal{F}_2$-fuzzy open set $V \neq 0_Y$ on $(Y, \mathcal{F}_1, \mathcal{F}_2)$.*

Definition 2.4. *A mapping $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ is called somewhat pairwise fuzzy e-irresolute if there exists a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set $U \neq 0_X$ on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $U \leq f^{-1}(V) \neq 0_X$ for any $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set $V \neq 0_Y$ on $(Y, \mathcal{F}_1, \mathcal{F}_2)$.*

Remark. *From the above definitions, it is observed that the following reverse implications are false:*
(i)Every pairwise fuzzy e-continuous mapping is a somewhat pairwise fuzzy e-continuous mapping.
(ii)Every somewhat pairwise fuzzy e-irresolute mapping is a somewhat pairwise fuzzy e-continuous mapping.
(iii)Every pairwise fuzzy e-irresolute mapping is a somewhat pairwise fuzzy e-irresolute mapping.

Example 2.5. Let M_1,M_2,M_3,M_4 and M_5 be fuzzy sets on $X = Y = \{x, y, z\}$. Then $M_1 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.5}{c}$, $M_2 = \frac{0.6}{a} + \frac{0.5}{b} + \frac{0.5}{c}$, $M_3 = \frac{0.7}{a} + \frac{0.6}{b} + \frac{0.4}{c}$, $M_4 = \frac{0.3}{x} + \frac{0.4}{y} + \frac{0.4}{z}$ and $M_5 = \frac{0.6}{x} + \frac{0.5}{y} + \frac{0.4}{z}$ are defined as follows:Consider $\mathcal{T}_1 = \{0_X, M_1, M_2, M_4, M_5, 1_X\}$,$\mathcal{T}_2 = \{0_X, M_1, M_2, M_4, 1_X\}$,$\mathcal{F}_1 = \{0_X, M_1, M_1^c, M_2, M_3, M_4, M_5, 1_X\}$ and $\mathcal{F}_2 = \{0_X, M_1, M_5, 1_X\}$. Then $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(Y, \mathcal{F}_1, \mathcal{F}_2)$ are fuzzy bitopologies and $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ be an identity mapping. Then we have $M_1 \leq f^{-1}(M_1) = M_1$, $M_1 \leq f^{-1}(M_1^c) = M_1^c$,$M_1 \leq f^{-1}(M_2) = M_2$,$M_4 \leq f^{-1}(M_3) = M_3$,$M_4 \leq f^{-1}(M_4) = M_4$ and $M_4 \leq f^{-1}(M_5) = M_5$. Since M_1, M_2 and M_4 are $\mathcal{T}_1$-fuzzy e-open set on $(X, \mathcal{T}_1, \mathcal{T}_2)$, f is somewhat pairwise fuzzy e-continuous. But $f^{-1}(M_3) = M_3$ is not a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set on $(X, \mathcal{T}_1, \mathcal{T}_2)$. Hence f is not a pairwise fuzzy e-continuous mapping.

Example 2.6. *Let $\mu_1(x)$, $\mu_2(x)$ and $\mu_3(x)$ be fuzzy sets on $I = [0, 1]$ defined as follows:*

$$\mu_1(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq 1 \end{cases}$$

$$\mu_2(x) = \begin{cases} 1, & 0 \le x \le \frac{1}{2} \\ -4x+2, & \frac{1}{2} \le x \le \frac{3}{4} \\ 0, & \frac{3}{4} \le x \le 1 \end{cases}$$

$$\mu_3(x) = \begin{cases} 0, & 0 \le x \le \frac{1}{4} \\ \frac{1}{3}(4x-1), & \frac{1}{4} \le x \le 1 \end{cases}$$

Let $\mathcal{T}_1 = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ and $\mathcal{T}_2 = \{0, \mu_1 \vee \mu_2, 1\}$, $\mathcal{T}_3 = \{0, \mu_2^{'}, 1\}$ and $\mathcal{T}_4 = \{0, \mu_1^{'}, 1\}$ be a fuzzy topologies on I. Let $f : (I, \mathcal{T}_1, \mathcal{T}_2) \to (I, \mathcal{F}_1, \mathcal{F}_2)$ be a function defined by $f(x) = \frac{x}{2}$ for each $x \in I$. We can see that for fuzzy e-open sets $\mu_1^{'}$ and $\mu_2^{'}$ on $(I, \mathcal{F}_1, \mathcal{F}_2)$, $f^{-1}(\mu_1^{'}) = 1_X$,$\mu_1 \le f^{-1}(\mu_2^{'}) = \mu_1$. Since μ_1 is a fuzzy e-open set on $(I, \mathcal{T}_1, \mathcal{T}_2)$. Therefore f is somewhat pairwise fuzzy e-continuous mapping. Consider a fuzzy open set μ_3 which is fuzzy e-open set on $(I, \mathcal{T}_1, \mathcal{T}_2)$, $f^{-1}(\mu_3) = \mu_3 f(\frac{x}{2}) = \alpha(x) = \begin{cases} 0, & 0 \le x \le \frac{1}{2} \\ \frac{1}{3}(2x-1), & \frac{1}{2} \le x \le 1 \end{cases}$ for each $x \in I$. But there is no non-zero fuzzy e-open set smaller than $f^{-1}(\mu_3(x)) = \alpha(x)$ on $(I, \mathcal{T}_1, \mathcal{T}_2)$. Hence f is not somewhat pairwise fuzzy e-irresolute mapping.

Example 2.7. *In Example 2.5, for an $\mathcal{F}_1$-fuzzy e-open sets on $(Y, \mathcal{F}_1, \mathcal{F}_2)$, $M_4 \le f^{-1}(M_1) = M_1$, $M_4 \le f^{-1}(M_1^c) = M_1^c$,$M_4 \le f^{-1}(M_2) = M_2$,$M_4 \le f^{-1}(M_3) = M_3$,$M_4 \le f^{-1}(M_4) = M_4$ and $M_4 \le f^{-1}(M_5) = M_5$. Since M_4 is a $\mathcal{T}_1$-fuzzy e-open set on $(X, \mathcal{T}_1, \mathcal{T}_2)$, f is somewhat pairwise fuzzy e-continuous But $f^{-1}(M_3) = M_3$ is not a $\mathcal{T}_1$-fuzzy or $\mathcal{T}_2$-fuzzy e-open set on $(X, \mathcal{T}_1, \mathcal{T}_2)$. Hence f is not a pairwise fuzzy e-irresolute mapping.*

Definition 2.8. *A fuzzy set U on a fuzzy bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ is called pairwise e-dense fuzzy set if there exists no $\mathcal{T}_1$-fuzzy e-closed or $\mathcal{T}_2$-fuzzy e-closed set V in $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $U < V < 1$.*

Theorem 2.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ be a mapping. Then the following are equivalent:
(1) f is somewhat pairwise fuzzy e-irresolute .
(2) If V is an $\mathcal{F}_1$-fuzzy e-closed or $\mathcal{F}_2$-fuzzy e-closed set of $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $f^{-1}(V) \ne 1_X$, then there exists a $\mathcal{T}_1$-fuzzy e-closed or $\mathcal{T}_2$-fuzzy e-closed set $U \ne 1_X$ of $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $f^{-1}(V) \le U$.
(3) If U is a pairwise e-dense fuzzy set on $(X, \mathcal{T}_1, \mathcal{T}_2)$, then $f(U)$ is a pairwise e-dense fuzzy set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$.

Proof. (1)$\Rightarrow$ (2): Let V be an $\mathcal{F}_1$-fuzzy e-closed or $\mathcal{F}_2$-fuzzy e-closed set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $f^{-1}(V) \ne 1_X$. Then V^c is an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ and $f^{-1}(V^c) = (f^{-1}(V))^c \ne 0_X$. Since f is somewhat pairwise e-irresolute , there exists a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set $U^c \ne 0_X$ on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $U^c \le f^{-1}(V^c)$. Hence there exists $\mathcal{T}_1$-fuzzy e-closed or $\mathcal{T}_2$-fuzzy e-closed set $U \ne 0_X$ on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $f^{-1}(V) = 1 - f^{-1}(V^c) \le 1 - U^c = U$.

(2) $\Rightarrow$ (3): Let U be a pairwise e-dense fuzzy set on $(X, \mathcal{T}_1, \mathcal{T}_2)$ and suppose $f(U)$ is not pairwise e-dense fuzzy set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$. Then there exists an $\mathcal{F}_1$-fuzzy e-closed or $\mathcal{F}_2$-fuzzy e-closed set V on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $f(U) < V < 1$. Since $V < 1$ and $f^{-1}(V) \ne 1_X$, there exists a $\mathcal{T}_1$-fuzzy e-closed or $\mathcal{T}_2$-fuzzy e-closed set $D \ne 1_X$ such that $U \le f^{-1}(f(U)) < f^{-1}(V) \le D$. This contradicts to the assumption that U is a

pairwise e-dense fuzzy set on $(X, \mathcal{T}_1, \mathcal{T}_2)$. Hence $f(U)$ is a pairwise e-dense fuzzy set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$.

(3) $\Rightarrow$ (1): Let $V \neq 0_Y$ be an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ and let $f^{-1}(V) \neq 0_X$. Suppose that there exists no $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set $U \neq 0_X$ on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $U \leq f^{-1}(V)$. Then $(f^{-1}(V))^c$ is a $\mathcal{T}_1$-fuzzy set or $\mathcal{T}_2$-fuzzy set on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that there is no $\mathcal{T}_1$-fuzzy e-closed or $\mathcal{T}_2$-fuzzy e-closed set D on $(X, \mathcal{T}_1, \mathcal{T}_2)$ with $(f^{-1}(V))^c < D < 1$. In fact, if there exists a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set D^c such that $D^c \leq f^{-1}(V)$, then it is a contradiction. So $(f^{-1}(V))^c$ is a pairwise e-dense fuzzy set on $(X, \mathcal{T}_1, \mathcal{T}_2)$. Then $f((f^{-1}(V))^c)$ is a pairwise e-dense fuzzy set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$. But $f((f^{-1}(V))^c) = f(f^{-1}(V^c)) \neq V^c < 1$. This is a contradiction to the fact that $f((f^{-1}(V))^c)$ is pairwise e-dense fuzzy set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$. Hence there exists a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set $U \neq 0_X$ on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $U \leq f^{-1}(V)$. Consequently, f is somewhat pairwise fuzzy e-irresolute . □

Theorem 2.2. Let $(X_1, \mathcal{T}_1, \mathcal{T}_2), (X_2, \mathcal{G}_1, \mathcal{G}_2), (Y_1, \mathcal{F}_1, \mathcal{F}_2), (Y_2, \mathcal{K}_1, \mathcal{K}_2)$ be fuzzy bitopological spaces. Let $(X_1, \mathcal{T}_1, \mathcal{T}_2)$ be product related to $(X_2, \mathcal{G}_1, \mathcal{G}_2)$ and let $(Y_1, \mathcal{F}_1, \mathcal{F}_2)$ be product related to $(Y_2, \mathcal{K}_1, \mathcal{K}_2)$. If $f_1 : (X_1, \mathcal{T}_1, \mathcal{T}_2) \to (Y_1, \mathcal{F}_1, \mathcal{F}_2)$ and $f_2 : (X_2, \mathcal{G}_1, \mathcal{G}_2) \to (Y_2, \mathcal{K}_1, \mathcal{K}_2)$ is a somewhat pairwise fuzzy e-irresolute mappings, then the product $f_1 \times f_2 : (X_1, \mathcal{T}_1, \mathcal{T}_2) \times (X_2, \mathcal{G}_1, \mathcal{G}_2) \to (Y_1, \mathcal{F}_1, \mathcal{F}_2) \times (Y_2, \mathcal{K}_1, \mathcal{K}_2)$ is also somewhat pairwise fuzzy e-irresolute.

Proof. Let $C = \bigvee_{i,j}(U_i \times V_j)$ be $\mathcal{F}_i$-fuzzy e-open or $\mathcal{K}_j$-fuzzy e-open set on $(Y_1, \mathcal{F}_1, \mathcal{F}_2) \times (Y_2, \mathcal{K}_1, \mathcal{K}_2)$ where $U_i \neq 0_{Y_1}$ is $\mathcal{F}_i$-fuzzy e-open set and $V_j \neq 0_{Y_2}$ is $\mathcal{K}_j$-fuzzy e-open set on $(Y_1, \mathcal{F}_1, \mathcal{F}_2)$ and $(Y_2, \mathcal{K}_1, \mathcal{K}_2)$ respectively. Then $(f_1 \times f_2)^{-1}(C) = \bigvee_{i,j}(f_1^{-1}(U_i) \times f_2^{-1}(V_j))$. Since f_1 is somewhat pairwise fuzzy e-irresolute , there exists a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set $D_i \neq 0_{X_1}$ such that $D_i \leq f_1^{-1}(U_i) \neq 0_{X_1}$. And, since f_2 is somewhat pairwise fuzzy e-irresolute , there exists a $\mathcal{G}_1$-fuzzy e-open or $\mathcal{G}_2$-fuzzy e-open set $A_j \neq 0_{X_2}$ such that $A_j \leq f_2^{-1}(V_j) \neq 0_{X_2}$. Now $D_i \times A_j \leq f_1^{-1}(U_i) \times f_2^{-1}(V_j) = (f_1 \times f_2)^{-1}(U_i \times V_j)$ and $D_i \times A_j \neq 0_{X_1 \times X_2}$ is a D_i-fuzzy e-open or V_j-fuzzy e-open set on $(X_1, \mathcal{T}_1, \mathcal{T}_2) \times (X_2, \mathcal{G}_1, \mathcal{G}_2)$. Hence $D_i \times A_j$ is a $\mathcal{T}_i$-fuzzy e-open or $\mathcal{G}_j$-fuzzy e-open set on $(X_1, \mathcal{T}_1, \mathcal{T}_2) \times (X_2, \mathcal{G}_1, \mathcal{G}_2)$ such that $\bigvee_{i,j}(D_i \times A_j) \leq \bigvee_{i,j}(f_1^{-1}(U_i) \times f_2^{-1}(V_j)) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(U_i \times V_j)) = (f_1 \times f_2)^{-1}(C) \neq 0_{X_1 \times X_2}$. Therefore, $f_1 \times f_2$ is somewhat pairwise fuzzy e-irresolute . □

Theorem 2.3. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ be a mapping. If the graph $g : (X, \mathcal{T}_1, \mathcal{T}_2) \to (X, \mathcal{T}_1, \mathcal{T}_2) \times (Y, \mathcal{F}_1, \mathcal{F}_2)$ of f is a somewhat pairwise fuzzy e-irresolute mapping, then f is also somewhat pairwise fuzzy e-irresolute .

Proof. Let V be an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$. Then $f^{-1}(V) = 1 \wedge f^{-1}(V) = g^{-1}(1 \times V)$. Since g is somewhat pairwise fuzzy e-irresolute and $1 \times V$ is a $\mathcal{T}_i$-fuzzy e-open or $\mathcal{F}_j$-fuzzy e-open set on $(X, \mathcal{T}_1, \mathcal{T}_2) \times (Y, \mathcal{F}_1, \mathcal{F}_2)$, there exists a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set $U \neq 0_X$ on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $U \leq g^{-1}(1 \times V) = f^{-1}(V) \neq 0_X$. Therefore, f is somewhat pairwise fuzzy e-irresolute . □

3. Somewhat pairwise fuzzy irresolute e-open mappings

In this section, we introduce a somewhat pairwise fuzzy irresolute e-open mapping and we characterize a somewhat pairwise fuzzy irresolute e-open mapping.

Definition 3.1. *A mapping $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ is called pairwise fuzzy e-open if $f(U)$ is an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ for any $\mathcal{T}_1$-fuzzy open or $\mathcal{T}_2$-fuzzy open set U on $(X, \mathcal{T}_1, \mathcal{T}_2)$.*

Definition 3.2. *A mapping $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ is called pairwise fuzzy irresolute e-open if $f(U)$ is an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ for any $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set U on $(X, \mathcal{T}_1, \mathcal{T}_2)$.*

Definition 3.3. *A mapping $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ is called somewhat pairwise fuzzy e-open if there exists an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set $V \neq 0_Y$ on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $V \leq f(U) \neq 0_Y$ for any $\mathcal{T}_1$-fuzzy open or $\mathcal{T}_2$-fuzzy open set U on $(X, \mathcal{T}_1, \mathcal{T}_2)$.*

Definition 3.4. *A mapping $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ is called somewhat pairwise fuzzy irresolute e-open if there exists an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set $V \neq 0_Y$ on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $V \leq f(U) \neq 0_Y$ for any $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set $U \neq 0_X$ on $(X, \mathcal{T}_1, \mathcal{T}_2)$.*

Theorem 3.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ be a bijection. Then the following are equivalent:
(1) f is somewhat pairwise fuzzy irresolute e-open.
(2) If U is a $\mathcal{T}_1$-fuzzy e-closed or $\mathcal{T}_2$-fuzzy e-closed set on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $f(U) \neq 1_Y$, then there exists an $\mathcal{F}_1$-fuzzy e-closed or $\mathcal{F}_2$-fuzzy e-closed set $V \neq 1_Y$ on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $f(U) < V$.

Proof. (1) $\Rightarrow$ (2): Let U be a $\mathcal{T}_1$-fuzzy e-closed or $\mathcal{T}_2$-fuzzy e-closed set on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $f(U) \neq 1_Y$. Since f is bijective and U^c is a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set on $(X, \mathcal{T}_1, \mathcal{T}_2)$, $f(U^c) = (f(U))^c \neq 0_Y$. And, since f is somewhat pairwise fuzzy irresolute e-open mapping, there exists an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set $D \neq 0_Y$ on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $D < f(U^c) = (f(U))^c$. Consequently, $f(U) < D^c = V \neq 1_Y$ and V is an $\mathcal{F}_1$-fuzzy e-closed or $\mathcal{F}_2$-fuzzy e-closed set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$.

(2) $\Rightarrow$ (1): Let U be a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $f(U) \neq 0_Y$. Then U^c is a $\mathcal{T}_1$-fuzzy e-closed or $\mathcal{T}_2$-fuzzy e-closed set on $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $f(U^c) \neq 1_Y$. Hence there exists an $\mathcal{F}_1$-fuzzy e-closed or $\mathcal{F}_2$-fuzzy e-closed set $V \neq 1_Y$ on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $f(U^c) < V$. Since f is bijective, $f(U^c) = (f(U))^c < V$. Hence $V^c < f(U)$ and $V^c \neq 0_X$ is an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$. Therefore, f is somewhat pairwise fuzzy irresolute e-open.

□

Theorem 3.2. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ be a surjection. Then the following are equivalent:
(1) f is somewhat pairwise fuzzy irresolute e-open.
(2) If V is a pairwise e-dense fuzzy set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$, then $f^{-1}(V)$ is a pairwise e-dense fuzzy set on $(X, \mathcal{T}_1, \mathcal{T}_2)$.

Proof. (1) $\Rightarrow$ (2): Let V be a pairwise e-dense fuzzy set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$. Suppose $f^{-1}(V)$ is not pairwise e-dense fuzzy set on $(X, \mathcal{T}_1, \mathcal{T}_2)$. Then there exists a $\mathcal{T}_1$-fuzzy e-closed or $\mathcal{T}_2$-fuzzy e-closed set U on $(X, \mathcal{T}_1, \mathcal{T}_2)$ such that $f^{-1}(V) < U < 1$. Since f is somewhat pairwise fuzzy irreolute e-open and U^c is a $\mathcal{T}_1$-fuzzy e-open or $\mathcal{T}_2$-fuzzy e-open set on $(X, \mathcal{T}_1, \mathcal{T}_2)$, there exists an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set $D \neq 0_Y$ on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $D \leq f(IntU^c) \leq f(U^c)$. Since f is surjective, $D \leq f(U^c) < f(f^{-1}(V^c)) = V^c$. Thus there exists an $\mathcal{F}_1$-fuzzy e-closed or $\mathcal{F}_2$-fuzzy e-closed set D^c

on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $V < D^c < 1$. This is a contradiction. Hence $f^{-1}(V)$ is pairwise e-dense fuzzy set on $(X, \mathcal{T}_1, \mathcal{T}_2)$.

(2) $\Rightarrow$ (1): Let U be a $\mathcal{T}_1$-fuzzy open or $\mathcal{T}_2$-fuzzy open set on $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $f(U) \neq 0_Y$. Suppose there exists no $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_2$-fuzzy e-open set $V \neq 0_Y$ on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $V \leq f(U)$. Then $(f(U))^c$ is an $\mathcal{F}_1$-fuzzy set or $\mathcal{F}_2$-fuzzy set D on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that there exists no $\mathcal{F}_1$-fuzzy e-closed or $\mathcal{F}_2$-fuzzy e-closed set D on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ with $(f(U))^c < D < 1$. This means that $(f(U))^c$ is pairwise e-dense fuzzy set on $(Y, \mathcal{F}_1, \mathcal{F}_2)$. Thus $f^{-1}((f(U))^c)$ is pairwise e-dense fuzzy set on $(X, \mathcal{T}_1, \mathcal{T}_2)$. But $f^{-1}((f(U))^c) = (f^{-1}(f(U)))^c \leq U^c < 1$. This is a contradiction to the fact that $f^{-1}(f(V))^c$ is pairwise e-dense fuzzy set on $(X, \mathcal{T}_1, \mathcal{T}_2)$. Hence there exists an $\mathcal{F}_1$-fuzzy e-open or $\mathcal{F}_1$-fuzzy e-open set $V \neq 0_Y$ on $(Y, \mathcal{F}_1, \mathcal{F}_2)$ such that $V \leq f(U)$. Therefore, f is somewhat pairwise fuzzy irresolute e-open. $\square$

4. Conclusions and/or Discussions

Even though the concept of somewhat fuzzy continuous functions are not at all fuzzy continuous functions , it has some interseting stuff to develope further. In this aspect the somewhat fuzzy δ-irresolute continuous and somewhat fuzzy e-irresolute continuous mappings were investigated in [5] and [6] respectively. From these papers, we have developed and studied this research article in fuzzy bitopology as somewhat pairwise fuzzy e-irresolute mappings;this paper only concerned with somewhat pairwise fuzzy e-irresolute functions with some interesting properties.

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