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ULAM STABILITY FOR FINITE VARIABLE QUARTIC FUNCTIONAL EQUATION IN BANACH ALGEBRA

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ABSTRACT. In this paper, we determine some stability results concerning the quartic functional equation as of the form

$$\sum_{b=1}^{s}\phi\left(-v_b+\sum_{a=1;a\neq b}^{s}v_a\right)-4\sum_{1\leq a<b<c\leq s}\phi\left(v_a+v_b+v_c\right)-\sum_{b=1}^{s}\phi\left(2v_b\right)$$
$$=(-4s+14)\sum_{a=1;a\neq b}^{s}\phi\left(v_a+v_b\right)+(s-8)\phi\left(\sum_{a=1}^{s}v_a\right)$$
$$+2\left[\sum_{a=1;a\neq b}^{s}\phi\left(v_a-v_b\right)+(s^2-7s+7)\sum_{a=1}^{s}\phi(v_a)\right]$$

where any positive integer $s\geq 3$, in Banach algebra via direct and fixed point approaches.

1. INTRODUCTION

The stability theory of functional equations started with the talk of S. M. Ulam held at the Wisconsin university in 1940 as follows: under what condition does there exist an additive mapping near an approximately additive mapping? (See [16]).

The first partial solution to Ulams question was provided by D. H. Hyers [4]. Let X and Y are Banach spaces with norms $\|\cdot\|$ and $\|\cdot\|$, respectively. Hyers showed that if a function $f: X\to Y$ satisfies the following inequality

$$\|f(x+y)-f(x)-f(y)\|\leq\epsilon$$

for all $\epsilon\geq 0$ and for all $x,y\in X$, then the limit

$$a(x)=\lim_{n\to\infty}2^{-n}f(2^nx)$$

exists for each $x\in X$ and $a: X\to Y$ is the unique additive function such that

$$\|f(x)-a(x)\|\leq\epsilon$$

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for any $x \in X$. Won-Gil Park and Jae-Hyeong Bae [11], introduced the following functional equation

$$f(x+2y)+f(x-2y)=4f(x+y)+4f(x-y)+24f(y)-6f(x) \tag{1.1}$$

and they established the general solution of the functional equation (1.1). It is easy to see that the function $f(x)=x^4$ is a solution of the functional equation (1.1). Thus, it is natural that (1.1) is called a quartic functional equation and every solution of the quartic functional equation is said to be quartic mapping.Thirty seven years after Hyers Theorem, Th. M. Rassias in his paper [10], provided a remarkable generalization of Hyers result by allowing for the first time in the subject of functional equations and inequalities the Cauchy difference to be unbounded. C.Park[8, 9] investigates linear mappings of functional equations in banach algebras. Tamilvanan et al. [1, 7, 13, 14] have been investigated various functional equation in various spaces which are motivated by doing this work. This fact rekindled interest of several mathematicians worldwide in the study of several important functional equations of several variables. (see[2, 3, 12, 15]).

S. Murthy et al.,[6] introduced the following quartic functional equation

$$\sum_{b=1}^{s}\phi\left(-v_b+\sum_{a=1;a\neq b}^{s}v_a\right)-4\sum_{1\leq a<b<c\leq s}\phi\left(v_a+v_b+v_c\right)-\sum_{b=1}^{s}\phi\left(2v_b\right) \tag{1.2}$$

$$=(-4s+14)\sum_{a=1;a\neq b}^{s}\phi\left(v_a+v_b\right)+(s-8)\phi\left(\sum_{a=1}^{s}v_a\right) \tag{1.3}$$

$$+2\left[\sum_{a=1;a\neq b}^{s}\phi\left(v_a-v_b\right)+(s^2-7s+7)\sum_{a=1}^{s}\phi(v_a)\right] \tag{1.4}$$

where any positive integer $s\geq 3$ and obtained its general solution and moreover they examined its Hyers-Ulam stability in generalized 2-normed spaces. Now, we determine Hyers-Ulam stability results concerning the quartic functional equation (1.2), in Banach algebra via direct and fixed point approaches.

Throughout this paper we consider A and B as normed algebra and Banach algebra respectively. **Theorem (Banach Contraction Principle)**: Let (A,d)be a complete metric space and consider a mapping $T:A\to A$ which is strictly contractive mapping, that is
(A1) $d\left(Tu,Tv\right)\leq Ld\left(u,v\right)$ for some (Lipschitz constant) $L<1$, then

(1) The mapping T has one and only fixed point $u^*=T\left(u^*\right);$
(2) The fixed point for each given element u^* is globally attractive that is

(A2) $\lim_{s\to\infty}T^su=u^*$, for any starting point $u\in A$;

(1) One has the following estimation inequalities:

(A3) $d\left(T^su,u^*\right)\leq\frac{1}{1-L}d\left(T^su,T^{s+1}u\right)$, for all $s\geq 0,\ \ u\in A$.
(A4) $d\left(u,u^*\right)\leq\frac{1}{1-L}d\left(u,u^*\right),\quad \forall\ u\in A$.
Theorem (The Alternative of fixed point): Suppose that for a complete generalized metric space (A,d) and a strictly contractive mapping $T:A\longrightarrow A$ with Lipschitz constant L. Then, for each given element $u\in A$ either
(B1) $d(T^su,T^{s+1}u)=+\infty$, for all $s\geq 0$, or
(B2) There exists natural number s_0 such that

i) $d(T^s u, T^{s+1}u) < \infty$ for all $s \geq s_0$;
ii) The sequence $(T^s u)$ is convergent to a fixed point v^* of T;
iii) v^* is the unique fixed point of T in the set $B = \{v \in A; d(T^{s_0}u, v) < \infty\}$;
iv) $d(v^*, v) \leq \frac{1}{1-L} d(v, Tv)$ for all $v \in B$.

Define a mapping $\phi : A \to B$ by

$$D\phi(v_1, v_2, \cdots, v_s) = \sum_{b=1}^{s} \phi\left(-v_b + \sum_{a=1; a\neq b}^{s} v_a\right) - 4 \sum_{1\leq a<b<c\leq s} \phi\left(v_a + v_b + v_c\right) - \sum_{b=1}^{s} \phi\left(2v_b\right) - (-4s+14) \sum_{a=1; a\neq b}^{s} \phi\left(v_a + v_b\right) - (s-8)\phi\left(\sum_{a=1}^{s} v_a\right) - 2\left[\sum_{a=1; a\neq b}^{s} \phi\left(v_a - v_b\right) + (s^2 - 7s + 7)\sum_{a=1}^{s} \phi(v_a)\right]$$

for all$v_1, v_2, \cdots, v_s \in A$.

Definition 1.1. Let A be Banach Algebra. A mapping $\phi : A \to A$ is said to be quartic derivation if the quartic function ϕ satisfies,

$$\phi(v_1 v_2) = \phi(v_1)v_2^4 + v_1^4 \phi(v_2)$$

for all $v_1, v_2 \in A$. Also the quartic derivation for s-variables satisfies

$$\phi(v_1 v_2 \cdots v_s) = \phi(v_1)v_2^4 \cdots v_s^4 + s_1^4 \phi(v_2)v_3^4 \cdots v_s^4 + \cdots + v_1^4 v_2^4 \cdots v_{s-1}^4 \phi(v_s) \quad (1.5)$$

for all $v_1, v_2, \cdots, v_s \in A$.

2. Stability Result for (1.2) : Direct Method

Theorem 2.1. *Let $\phi : A \to B$ be a mapping for which there exists a function $\vartheta, \eta : A^s \to [0, \infty)$ with the condition*
$\sum_{\gamma=0}^{\infty} \frac{\vartheta(2^\gamma v_1, 2^\gamma v_2, \cdots, 2^\gamma v_s)}{2^{4\gamma}}$ *converges in $\mathbb{R}$ and*

$$\lim_{\gamma\to\infty} \frac{\vartheta(2^\gamma v_1, 2^\gamma v_2, \cdots, 2^\gamma v_s)}{2^{4\gamma}} = 0 \quad (2.1)$$

and also
$\sum_{\gamma=0}^{\infty} \frac{\eta(2^\gamma v_1, 2^\gamma v_2, \cdots, 2^\gamma v_s)}{2^{4s\gamma}}$ *converges in $\mathbb{R}$ and*

$$\lim_{\gamma\to\infty} \frac{\eta(2^\gamma v_1, 2^\gamma v_2, \cdots, 2^\gamma v_s)}{2^{4s\gamma}} = 0 \quad (2.2)$$

such that

$$||D\phi(v_1, v_2, \cdots, v_s)|| \leq \vartheta(v_1, v_2, v_3, \cdots, v_s) \quad (2.3)$$

and

$$\| D\phi(v_1 v_2 \cdots, v_s) - D\phi(v_1)v_2^4 \cdots v_s^4 - v_1^4 D\phi(v_2)v_3^4 \cdots v_s^4 - \cdots - v_1^4 v_2^4 \cdots v_{s-1}^4 D\phi(v_s) \| \leq \eta(v_1, v_2, \cdots, v_s) \quad (2.4)$$

for all $v_1, v_2, \cdots, v_s \in A$. Then there exist a unique quartic derivation $Q_4 : A \to B$ fulfils (1.2) and

$$||\phi(v) - Q_4(v)|| \leq \frac{1}{16} \sum_{\gamma=0}^{\infty} \frac{\vartheta(2^\gamma v, 0, \cdots, 0)}{2^{4\gamma}} \tag{2.5}$$

for all $v \in A$. The function Q_4 is given by

$$Q_4(v) = \lim_{\gamma \to \infty} \frac{\phi(2^\gamma v)}{2^{4\gamma}} \tag{2.6}$$

for all $v \in A$.

Proof. Switching $(v_1, v_2, \cdots, v_s)$ by $(v, 0, \cdots, 0)$ in (2.3), we obtain

$$\|16\phi(v) - \phi(2v)\| \leq \vartheta(v, 0, \cdots, 0), \quad \forall\, v \in A. \tag{2.7}$$

From (2.7) that

$$\|\frac{\phi(2v)}{2^4} - \phi(v)\| \leq \frac{1}{16}\vartheta(v, 0, \cdots, 0), \quad \forall\, v \in A. \tag{2.8}$$

Now, changing v by $2v$ and dividing by 2^4 in (2.8), we reach

$$\|\frac{\phi(2^2 v)}{2^8} - \frac{\phi(2v)}{2^4}\| \leq \frac{1}{2^8}\vartheta(2v, 0, \cdots, 0), \quad \forall\, v \in A. \tag{2.9}$$

Utilizing (2.8) and (2.9), we obtain

$$\|\frac{\phi(2^2 v)}{2^8} - \phi(v)\| \leq \frac{1}{2^4}\left(\vartheta(v, 0, \cdots, 0) + \frac{\vartheta(2v, 0, \cdots, 0)}{2^4}\right)$$

for all $v \in A$. In general for any non-negative integer p , one can easy to prove that

$$\|\frac{\phi(2^p v)}{2^{4p}} - \phi(v)\| \leq \frac{1}{2^4} \sum_{\gamma=0}^{\infty} \frac{\vartheta(2^\gamma v, 0, \cdots, 0)}{2^{4\gamma}}, \quad \forall\, v \in A. \tag{2.10}$$

In order to prove the convergence of the sequence $\{\frac{\phi(2^p v)}{2^{4p}}\}$, switching v by $2^q v$ and dividing 2^{4q} in (2.10), for $p, q > 0$, we arrive

$$\|\frac{\phi(2^{p+q} v)}{2^{4(p+q)}} - \frac{\phi(2^q v)}{2^{4q}}\| \leq \frac{1}{2^4} \sum_{\gamma=0}^{p-1} \frac{\vartheta(2^{\gamma+q} v, 0, \cdots, 0)}{2^{4(\gamma+q)}} \to 0 \ \ as \ \ q \to \infty \tag{2.11}$$

for all $v \in A$. Hence the sequence $\{\frac{\phi(2^p v)}{2^{4p}}\}$ is a Cauchy sequence. As B is complete, there exists a mapping $Q_4 : A \to B$ such that

$$Q_4(v) = \lim_{p \to \infty} \frac{\phi(2^p v)}{2^{4p}}$$

for all $v \in A$. Taking $p \to \infty$ in (2.10) we see that (2.5) holds for $v \in A$. To prove that Q_4 fulfils (1.2), replacing $(v_1, v_2, \cdots, v_s)$ by $(2^q v, 2^q v, \cdots, 2^q v)$ and dividing 2^{4q} in (2.3), we reach

$$\frac{1}{2^{4q}}\|Q_4(2^q v, 2^q v, \cdots, 2^q v)\| \leq \frac{1}{2^{4q}}\vartheta(2^q v, 2^q v, \cdots, 2^q v)$$

for all $v_1, v_2, \cdots, v_s \in A$. Passing $q \to \infty$ in above inequality and utilizing the definition of $Q_4(v)$, we obtain that $DQ_4(v_1, v_2, \cdots, v_s) = 0$. Hence Q_4 fulfils (1.2) for all $v \in A$. Next, to show that Q_4 fulfils (1.5), Interchanging $(v_1, v_2, \cdots, v_s)$ by

$(2^r v_1, 2^r v_2, \cdots, 2^r v_s)$ and dividing by 2^{4rs} in (2.4) that

$$\begin{aligned}&\| Q_4(v_1 v_2 \cdots v_s) - Q_4(v_1) v_2^4 \cdots v_s^4 - v_1^4 Q_4(v_2) v_3^2 \cdots v_s^4 - \cdots - v_1^4 v_2^4 \cdots v_{s-1}^4 Q_4(v_s) \| \\ &\leq \frac{1}{2^{4sr}} \| \phi(2^r(v_1 v_2 \cdots v_s)) - Q_4(2^r v_1)(2^r v_2)^4 \cdots (2^r v_s)^4 \\ &-(2^r v_1)^4 Q_4(2^r v_2)(2^r v_3)^2 \cdots (2^r v_s)^4 - \cdots - (2^r v_1)^4 (2^r v_2)^4 \cdots (2^r v_{s-1})^4 Q_4(2^r v_s) \| \\ &\leq \frac{1}{2^{4rs}} \eta(2^r(v_1, v_2, \cdots, v_s)) \\ &\to 0 \text{ as } r \to \infty\end{aligned}$$

for all $v_1, v_2, \cdots, v_s \in A$. Hence Q_4 satisfies (1.5). To show that Q_4 is unique. Consider R_4 be the another quartic mapping fulfilling (1.2) and (2.5), then

$$\begin{aligned}\|Q_4(v) - R_4(v)\| &\leq \frac{1}{2^{4q}} \{ \|Q_4(2^q v) - \phi(2^q)\| + \|\phi(2^q v) - R_4(2^q v))\| \} \\ &\leq \frac{1}{2^4} \sum_{\gamma=0}^{\infty} \frac{\vartheta(2^{\gamma+q} v, 0, \cdots, 0)}{2^{4(\gamma+q)}} \to 0 \ \ as \ \ q \to \infty\end{aligned}$$

for all $v \in A$. Hence Q_4 is unique.Hence the proof of the theorem. □

Theorem 2.2. *Let $\phi : A \to B$ be a mapping for which there exists a function $\vartheta, \eta : A^s \to [0, \infty)$ with the condition*
$\sum_{\gamma=0}^{\infty} 2^{4\gamma} \vartheta\left(\frac{v_1}{2^\gamma}, \frac{v_2}{2^\gamma}, \cdots, \frac{v_s}{2^\gamma}\right)$ *converges in* $\mathbb{R}$ *and*

$$\lim_{\gamma \to \infty} 2^{4\gamma} \vartheta\left(\frac{v_1}{2^\gamma}, \frac{v_2}{2^\gamma}, \cdots, \frac{v_s}{2^\gamma}\right) = 0 \tag{2.12}$$

and also
$\sum_{\gamma=0}^{\infty} 2^{4s\gamma} \eta\left(\frac{v_1}{2^\gamma}, \frac{v_2}{2^\gamma}, \cdots, \frac{v_s}{2^\gamma}\right)$ *converges in* $\mathbb{R}$ *and*

$$\lim_{\gamma \to \infty} 2^{4s\gamma} \eta\left(\frac{v_1}{2^\gamma}, \frac{v_2}{2^\gamma}, \cdots, \frac{v_s}{2^\gamma}\right) = 0 \tag{2.13}$$

such that

$$||D\phi(v_1, v_2, \cdots, v_s)|| \leq \vartheta(v_1, v_2, v_3, \cdots, v_s) \tag{2.14}$$

and

$$\begin{aligned}&\| D\phi(v_1 v_2 \cdots, v_s) - D\phi(v_1) v_2^4 \cdots v_s^4 - v_1^4 D\phi(v_2) v_3^4 \cdots v_s^4 - \cdots - v_1^4 v_2^4 \cdots v_{s-1}^4 D\phi(v_s) \| \\ &\leq \eta(v_1, v_2, \cdots, v_s)\end{aligned} \tag{2.15}$$

for all $v_1, v_2, \cdots, v_s \in A$. Then there exist a unique quartic derivation $Q_4 : A \to B$ fulfils (1.2) and

$$||\phi(v) - Q_4(v)|| \leq \frac{1}{16} \sum_{\gamma=0}^{\infty} 2^{4\gamma} \vartheta\left(\frac{v}{2^\gamma}, 0, \cdots, 0\right) \tag{2.16}$$

for all $v \in A$. The function Q_4 is given by

$$Q_4(v) = \lim_{\gamma \to \infty} 2^{4\gamma} \phi\left(\frac{v}{2^\gamma}\right) \tag{2.17}$$

for all $v \in A$.

Proof. Replacing $(v_1, v_2, \cdots, v_s)$ by $(v, 0, \cdots, 0)$ in (2.14), we have

$$\|16\phi(v) - \phi(2v)\| \leq \vartheta(v, 0, \cdots, 0), \quad \forall\, v \in A. \tag{2.18}$$

Now, changing v by $\frac{v}{2}$ in (2.18), we arrive

$$\|2^4 \phi\left(\frac{v}{2}\right) - \phi(v)\| \leq \vartheta\left(\frac{v}{2}, 0, \cdots, 0\right), \quad \forall\, v \in A. \tag{2.19}$$

The remaining proof is derived by similar manner of the Proof of Theorem 2.1. □

Corollary 2.3. *Let δ and s be a non-negative real numbers. Let $\phi : A \to B$ be a function satisfying the inequality*

$$||D\phi(v_1, v_2, \cdots, v_s)|| \leq \begin{cases} \delta \\ \delta(\sum_{r=1}^{s} ||v_r||^w) \\ \delta(\prod_{r=1}^{s} ||v_r||^w + \sum_{r=1}^{s} ||v_r||^{sw}) \end{cases}$$

and

$$\| D\phi(v_1 v_2 \cdots v_s) - D\phi(v_1)v_2^4 \cdots v_s^4 - v_1^4 D\phi(v_2)v_3^4 \cdots v_s^4 - \cdots - v_1^4 v_2^4 \cdots v_{s-1}^4 D\phi(v_s) \|$$

$$\leq \begin{cases} \delta \\ \delta(\sum_{r=1}^{s} ||v_r||^w) \\ \delta(\prod_{r=1}^{s} ||v_r||^w + \sum_{a=1}^{s} ||v_r||^{sw}) \end{cases}$$

for all $v_1, v_2, \cdots, v_s \in A$. Then there exists a unique quartic function $Q_4 : A \to B$ such that

$$||\phi(v) - Q_4(v)|| \leq \begin{cases} \frac{\delta}{|15|} \\ \frac{\delta||v||^w}{|2^4 - 2^w|} & ; \quad w \neq 4 \\ \frac{\delta||v||^{sw}}{|2^4 - 2^{sw}|} & ; \quad w \neq \frac{4}{s} \end{cases}$$

for all $v \in A$.

Proof. Let us define $\vartheta(2^\gamma v, 0, \cdots, 0) = \vartheta\left(\frac{v}{2^\gamma}, 0, \cdots, 0\right) = \begin{cases} \delta \\ \delta \sum_{i=1}^{s} \|v_i\|^w \\ \delta\left(\prod_{i=1}^{s} \|v_i\|^w + \sum_{i=1}^{s} \|v_i\|^{sw}\right) \end{cases}$.

Then, from Theorem 2.1 and Theorem 2.2, the conclusion follows. □

3. Stability Result for (1.2): Fixed Point Method

Theorem 3.1. *Let $Q_4 : A \to B$ be a mapping for which there exists a function $\vartheta, \eta : A^s \to [0, \infty)$ with the condition $\sum_{\gamma=0}^{\infty} \frac{\vartheta(\beta_r^{\gamma\varsigma} v_1, \beta_r^{\gamma\varsigma} v_2, \cdots, \beta_r^{\gamma\varsigma} v_s)}{\beta_r^{4\gamma\varsigma}}$ converges in $\mathbb{R}$ and*

$$\lim_{\gamma\to\infty} \frac{\vartheta(\beta_r^{\gamma\varsigma} v_1, \beta_r^{\gamma\varsigma} v_2, \cdots, \beta_r^{\gamma\varsigma} v_s)}{\beta_r^{4\gamma\varsigma}} = 0 \qquad (3.1)$$

and also $\sum_{\gamma=0}^{\infty} \frac{\vartheta(\beta_r^{\gamma\varsigma} v_1, \beta_r^{\gamma\varsigma} v_2, \cdots, \beta_r^{\gamma\varsigma} v_s)}{\beta_r^{4s\gamma\varsigma}}$ converges in $\mathbb{R}$ and

$$\lim_{\gamma\to\infty} \frac{\eta(\beta_r^{\gamma\varsigma} v_1, \beta_r^{\gamma\varsigma} v_2, \cdots, \beta_r^{\gamma\varsigma} v_s)}{\beta_r^{4s\gamma\varsigma}} = 0 \qquad (3.2)$$

where $\beta_r = \begin{cases} 2 \;\; if \;\; r = 0; \\ \frac{1}{2} \;\; if \;\; r = 1 \end{cases}$ satisfying the functional inequalities

$$||D\phi(v_1, v_2, \cdots, v_s)|| \leq \vartheta(v_1, v_2, \cdots, v_s)$$

and

$$\| D\phi(v_1 v_2 \cdots v_s) - D\phi(v_1)v_2^4 \cdots v_s^4 - v_1^4 D\phi(v_2)v_3^4 \cdots v_s^4 - \cdots - v_1^4 v_2^4 \cdots v_{s-1}^4 D\phi(v_s) \| \leq \eta(v_1, v_2, \cdots, v_s)$$

for all $v_1, v_2, \cdots, v_s \in A$. *Then there exists* $L = L(r) < 1$ *such that the function* $v \to \chi(v) = \vartheta\left(\frac{v}{2}, 0, \cdots, 0\right)$ *has the property* $\frac{1}{\beta_r^4}\chi(\beta_r v) = L\chi(v)$ *for all* $v \in A$. *Then there exist a unique quartic derivation* $Q_4 : A \to B$ *which satisfies the functional equation (1.2) and*

$$\| \phi(v) - Q_4(v) \| \leq \frac{L^{1-r}}{1-L}\eta(v).$$

for all $v \in A$.

Corollary 3.2. *Let* δ *and* s *be a non-negative real numbers. Let* $\phi : A \to B$ *be a function satisfying the inequality*

$$||D\phi(v_1, v_2, \cdots, v_s)|| \leq \begin{cases} \delta \\ \delta(\sum_{r=1}^{s} ||v_r||^w) \\ \delta(\prod_{r=1}^{s} ||v_r||^w + \sum_{r=1}^{s} ||v_r||^{sw}) \end{cases}$$

and

$$\| D\phi(v_1 v_2 \cdots v_s) - D\phi(v_1)v_2^4 \cdots v_s^4 - v_1^4 D\phi(v_2)v_3^4 \cdots v_s^4 - \cdots - v_1^4 v_2^4 \cdots v_{s-1}^4 D\phi(v_s) \|$$

$$\leq \begin{cases} \delta \\ \delta(\sum_{r=1}^{s} ||v_r||^w) \\ \delta(\prod_{r=1}^{s} ||v_r||^w + \sum_{a=1}^{s} ||v_r||^{sw}) \end{cases}$$

for all $v_1, v_2, \cdots, v_s \in A$. *Then there exists a unique quartic function* $Q_4 : A \to B$ *such that*

$$||\phi(v) - Q_4(v)|| \leq \begin{cases} \frac{\delta}{|15|} \\ \frac{\delta ||v||^w}{|2^4 - 2^w|} & ; \quad w \neq 4 \\ \frac{\delta ||v||^{sw}}{|2^4 - 2^{sw}|} & ; \quad w \neq \frac{4}{s} \end{cases}$$

for all $v \in A$.

Proof. Setting $\vartheta(v_1, v_2, \cdots, v_s) = \begin{cases} \delta \\ \delta(\sum_{r=1}^{s} ||v_r||^w) \\ \delta(\prod_{r=1}^{s} ||v_r||^w + \sum_{a=1}^{s} ||v_r||^{sw}) \end{cases}$.

By using Theorem 3.1, we obtain the results. □

4. Conclusion

In this work, we investigated the Hyers-Ulam stability for a finite variable quartic functional equation in Banach algebra by using the different technique of direct and fixed point methods.

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