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GENERALIZED SYMMETRIC BI-DERIVATIONS OF LATTICES

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ABSTRACT. In this article, the notion of a new kind of derivation is introduced for a lattice L called symmetric bi-(T, F)-derivations on L as a generalization of derivation of lattices and characterized some of its related properties. Some equivalent conditions provided for a lattice L with greatest element 1 by the notion of isotone symmetric bi-(T, F)-derivation on L. By using the concept of isotone derivation, we characterized the modular and distributive lattices by the notion of isotone symmetric bi-(T, F)-derivation.

1. INTRODUCTION

The notion of lattice theory first introduced by Birkhof [7]. After the initiation of lattices many researchers studied lattice theory in different point of view such as, Balbes and Dwinger [3] gave the concept on distributive lattices and Hoffmann gave the notion of partially ordered set (Poset). The application of lattice theory plays an important role in different areas such as information theory [4], information retrieval [10], information access controls [39] and cryptanalysis [14]. Recently, the properties of lattices were studied by some authors [13, 16, 26] in analytic and algebraic point of view.

Derivations is a very interesting research area in the theory of algebraic structure in mathematics. Posner [37] provided the concept of derivation on rings. Based on this concept Bell and Kappe [5] studied that rings in which derivations satisfy certain algebraic conditions and Kaya [28] applied the notions of derivations on prime rings. The notion of generalized derivation in ring introduced by Braser [8, 9] and Hvala [17]. This concept of derivation further carried out by many authors [2, 15] in prime rings and lie ideal in prime rings. Jun and Xin [25] applied the notion of derivation in ring and near ring theory to BCI-algebra. Later on, Muhiuddin et al. studied the theory of derivations in BCI-algebras on different aspects (see for e.g., [31],[32], [33], [34]). Jana et al. [21-30] and, Bej and Pal [6] and Senapati et al. [40] has done lot of works on BCK/BCI-algebra and $B/BG/G$-algebras which is related to these algebras. Zhan and Liu [45] studied the notion of left-right (respectively, right-left) f-derivation of BCI-algebras and investigated its properties. The study of derivation in lattice theory is an important topic in application of different mode. Xin et al. [43] introduced the notion of derivation in lattices and discussed its properties. Thereafter, many authors generalized this idea in lattices. For example Yilmaz and Öztürk [44] introduced the notion of f-derivation on lattices and its

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some related properties discussed, Çeven [12] studied symmetric bi-derivation on lattice, Kim [27] further investigated symmetric bi-f-derivations on lattices, Alshehri [1] studied generalized derivation on lattices and Chaudhry and Ullah [11] introduced the notion of (α, β)-generalized derivations on lattices and some of its related properties investigated. After symmetric bi-derivation studied by Maksa [29, 30], many researchers introduced this concept to study symmetric bi-derivation on rings and near-rings [35, 36, 38, 41, 42]. Recently, Çven [12] studied symmetric bi-derivation on lattices and investigated some properties on it. Motivated by the above works and best of our knowledge there is no work available on symmetric bi-(T, F)-derivations on lattices. For this reason we developed theoretical study of symmetric bi-(T, F)-derivation on lattices.

In this paper, the notion of symmetric bi-(T, F)-derivation which is a generalization of derivation in lattices is introduced and studied some properties of it. We gave some equivalent condition for which a derivation to be an isotone symmetric bi-(T, F)-derivation for a lattices with greatest element. We characterized modular lattices and distributive lattices by the concept of isotone symmetric bi-(T, F)-derivation.

2. Preliminaries

Definition 2.1. [7] Let L be a non-empty set endowed with operations $\wedge$ and $\vee$. Then the set $(L, \wedge, \vee)$ is called lattices if for all $x, y, z \in L$ satisfies the following conditions:
$(L1)$ $x \wedge x = x$, $x \vee x = x$
$(L2)$ $x \wedge y = y \wedge x$, $x \vee y = y \vee x$
$(L3)$ $(x \wedge y) \wedge z = x \wedge (y \wedge z)$, $(x \vee y) \vee z = x \vee (y \vee z)$
$(L4)$ $(x \wedge y) \vee x = x$, $(x \vee y) \wedge x = x$.

Definition 2.2. [7] A Lattice $(L, \wedge, \vee)$ is called distributive lattice if for all $x, y, z \in L$ satisfies the following conditions:
$(L5)$ $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
$(L6)$ $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.

It is notified that in a Lattice the conditions $(L5)$ and $(L6)$ are equivalent.

Definition 2.3. [7] Let $(L, \wedge, \vee)$ be a lattice. A binary relations $(\leq)$ on L defined by $x \leq y$ is holds if and only if $x \wedge y = x$ and $x \vee y = y$.

Definition 2.4. [3] A lattice $(L, \wedge, \vee)$ is called a modular lattice if for all $x, y, z \in L$ satisfies the following conditions:
$(L7)$ If $x \leq y$ implies $x \vee (y \wedge z) = (x \vee y) \wedge z$.

Definition 2.5. [43] Let $(L, \wedge, \vee)$ be a lattice. Then $(L, \leq)$ is a poset, i.e. it is a partially ordered set and for any $x, y \in L$, $x \wedge y$ is the g.l.b of $\{x, y\}$, and $x \vee y$ is the l.u.b of $\{x, y\}$.

Definition 2.6. [7] Let $F : L \to M$ be a function from the lattice L to the lattice M and is called lattice homomorphism if it satisfies the conditions:
(L_{10}) $F(x \wedge y) = F(x) \wedge F(y)$, $F(x \vee y) = F(x) \vee F(y)$ for all $x, y \in L$.

Definition 2.7. [43] Let L be a lattice and d be a self-map on L. Then for all $x, y \in L$, d is called derivation on L if satisfying the following identity :

$$d(x \wedge y) = (d(x \wedge y) \vee (x \wedge d(y))$$

Proposition 2.1. [43] Let L be a lattice and d be a derivation on L. Then following conditions are hold:
(1) $d(x) \leq x$

(2) $d(x) \wedge d(y) \leq d(x \wedge y) \leq d(x) \vee d(y)$
(3) If L has a least element 0 and a greatest element 1, then $d(0) = 0$ and $d(1) \leq 1$.

Definition 2.8. [43] Let L be a lattice and d be a derivation on L
(5) If $x \leq y$ implies $d(x) \leq d(y)$, then d is called an isotone derivation
(6) If d is one-to-one, then d is called a monomorphic derivation
(7) If d is onto, then d is called an epimorphic derivation.

Definition 2.9. [12, 41] Let $(L, \wedge, \vee)$ be a lattice. A function $\mathcal{D} : L \times L \to L$ is called symmetric if it satisfies the condition $\mathcal{D}(x, y) = \mathcal{D}(y, x)$ for all $x, y \in L$.

Definition 2.10. [35] Let L be a lattice. A function $d : L \times L \to L$ defined by $d(x) = D(x, x)$ is called trace of D, where D is a symmetric function.

Definition 2.11. [12] Let L be a lattice and Let $\mathcal{D} : L \times L \to L$ be a symmetric function on L. Then D is called symmetric bi-derivation on L if it satisfies the following identity:

$$D(x \wedge y, z) = (D(x, z) \wedge y) \vee (x \wedge D(y, z))$$

for all $x, y, z \in L$. Also, a symmetric bi-derivation D satisfies the following relation

$$D(x, y \wedge z) = (D(x, y) \wedge z) \vee (y \wedge D(x, z))$$

for all $x, y, z \in L$.

3. Symmetric bi-(T, F)-derivations on lattices

In this section, symmetric bi-(T, F)-derivation on a lattices is introduced.

Definition 3.1. Let L be a lattice. Then for any $T \in L$, we define a self-map $\mathcal{D}_T : L \times L \to L$ by $\mathcal{D}_T(x, y) = (x \wedge y) \wedge T$ for all $x, y \in L$.

Definition 3.2. Let L be a lattice. Then for any $T \in L$, a self-map $\mathcal{D}_T : L \times L \to L$ is defined as for any $T \in L$, $\mathcal{D}_T(x, y) = (x \wedge y) \wedge T$ for all $x \in L$. Then then function $\mathcal{D}_T : L \times L \to L$ is called symmetric bi-(T, F)-derivation of L if there exist a function $F : L \to L$ satisfies the condition:

$$\mathcal{D}_T(x \wedge y, z) = (\mathcal{D}_T(x, z) \wedge F(y)) \vee (F(x) \wedge \mathcal{D}_T(y, z))$$

for all $x, y, z \in L$. Also, a symmetric bi-(T, F)-derivation $\mathcal{D}_T$ satisfies the following relation

$$\mathcal{D}_T(x, y \wedge z) = (\mathcal{D}_T(x, y) \wedge F(z)) \vee (F(y) \wedge \mathcal{D}_T(x, z))$$

for all $x, y, z \in L$.

It is notified in the Definition 3.2 that if F is an identity function then D_T is a symmetric bi-T-derivation on L. Therefore, according to Definition 3.2, D_T is a symmetric bi-(T, F)-derivation on L if F must satisfied the identity of the Definition 3.2.

Example 3.1. Let $L = \{0, a, b, 1\}$ be a lattice shown by the Hasse diagram of Figure 1
For any $T \in L$, define a self-map $\mathcal{D}_T : L \times L \to L$ of a lattice L given in figure 2
Define the mapping $\mathcal{D}_T$ as follows:
for $T = 0$, $\mathcal{D}_T(x, y) = 0$ for all $(x, y) \in L \times L$
for $T = a$, $\mathcal{D}_T(x, y) = 0$ for all $(x, y) \in \{(0, 0), (0, a), (a, 0), (b, 0), (0, b), (1, 0), (0, 1)\}$

$\mathcal{D}_T(x,y) = a$ for all $(x,y) \in \{(a,a),(a,b),(b,a),(a,1),(1,a),(b,b),(b,1),(1,b),(1,1)\}$ for $T = b$, $\mathcal{D}_T(x,y) = 0$ for all $(x,y) \in \{(0,0),(a,0),(0,a),(0,b),(b,0),(1,0),(0,1)\}$, $\mathcal{D}_T(x,y) = a$ for all $(x,y) \in \{(a,a),(a,b),(b,a),(a,1),(1,a)\}$ and $\mathcal{D}_T(x,y) = b$ for all $(x,y) \in \{(b,b),(b,1),(1,b),(1,1)\}$
For $T = 1$, $\mathcal{D}_T(x,y) = 0$ for all $(x,y) \in \{(0,0),(0,a),(a,0),(b,0),(0,b),(1,0),(0,1)\}$, $\mathcal{D}_T(x,y) = a$ for all $(x,y) \in \{(a,a),(a,b),(b,a),(a,1),(1,a)$, $\mathcal{D}_T(x,y) = b$ for all $(x,y) \in \{(b,b),(b,1),(1,b)\}$ and $\mathcal{D}_T(x,y) = 1$ for $(x,y) = (1,1)$. If we defined the function F by $F(0) = 0$, $F(a) = a$, $F(b) = 1$ and $F(1) = b$, then it is verified that for each $T \in L$, $\mathcal{D}_T$ is a symmetric bi-(T,F)-derivation on L.

Where as, if we defined the function F by $F(0) = 0$, $F(a) = b$, $F(b) = a$ and $F(1) = 1$, then it is justified that for each $T \in L$, $\mathcal{D}_T$ is not a symmetric bi-(T,F)-derivation of L, because
for $T = b$, we have $\mathcal{D}_T(a \wedge b, 1) = \mathcal{D}_T(a,1) = (a \wedge 1) \wedge b = a \wedge b = a$, but
$(\mathcal{D}_T(a,1) \wedge F(b)) \vee (F(a) \wedge \mathcal{D}_T(b,1))$
$= ((a \wedge 1) \wedge a) \vee (b \wedge (b \wedge 1))$
$= (a \wedge a) \vee (b \wedge b) = a \vee b = b$. Therefore,

$$\mathcal{D}_T(a \wedge b, 1) = a \neq b = (\mathcal{D}_T(a,1) \wedge F(b)) \vee (F(a) \wedge \mathcal{D}_T(b,1)).$$

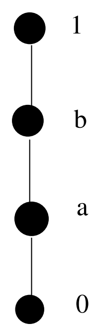

FIGURE 1. The lattice in example 3.3

Example 3.2. Let L be a lattice and $T \in L$. Defining a function $\mathcal{D}_T : L \to L$ by $\mathcal{D}_T(x,y) = (F(x) \wedge F(y)) \wedge T$ for all $x, y \in L$ where $F : L \to L$ satisfying $F(x \wedge y) = F(x) \wedge F(y)$ for all $x, y \in L$. Then $\mathcal{D}_T$ is a symmetric bi-(T,F)-derivation of L. In addition, if F is an increasing function then $\mathcal{D}_T$ is an isotone symmetric bi-(T,F) derivation on L.

Theorem 3.3. Let L be a lattice and $\mathcal{D}_T$ be a trace of symmetric bi-(T,F)-derivation $\mathcal{D}_T$. Then following conditions are hold for all $x, y \in L$.
(1) $\mathcal{D}_T(x,y) \leq F(x)$ and $\mathcal{D}_T(x,y) \leq F(y)$
(2) $\mathcal{D}_T(x,y) \wedge \mathcal{D}_T(w,y) \leq \mathcal{D}_T(x \wedge w, y) \leq \mathcal{D}_T(x,y) \vee \mathcal{D}_T(w,y)$
(3) $\mathcal{D}_T(x \wedge w, y) \leq F(x) \vee F(w)$
(4) $\mathcal{D}_T(x,y) \leq F(x) \wedge F(y)$
(5) $\mathcal{D}_T(x) \leq F(x)$
(6) $d_T^2(x) = \mathcal{D}_T(x)$.

Proof. (1) Since $\mathcal{D}_T(x,y) = \mathcal{D}_T(x \wedge x, y) = (\mathcal{D}_T(x,y) \wedge F(x)) \vee (F(x) \wedge \mathcal{D}_T(x,y)) = F(x) \wedge \mathcal{D}_T(x,y)$ from which we get $\mathcal{D}_T(x,y) \leq F(x)$. Similarly, $\mathcal{D}_T(x,y) \leq F(y)$ for all $x, y \in L$.
(2) Since $\mathcal{D}_T(x,y) \leq F(x)$ and $\mathcal{D}_T(w,y) \leq F(w)$. Then, we have $\mathcal{D}_T(x,y) \wedge \mathcal{D}_T(w,y) \leq F(x) \wedge \mathcal{D}_T(w,y)$, and from (1) $\mathcal{D}_T(x,y) \wedge \mathcal{D}_T(w,y) \leq F(w) \wedge \mathcal{D}_T(x,y)$ for all $x, y, w \in L$. Hence, $\mathcal{D}_T(x,y) \wedge \mathcal{D}_T(w,y) \leq (F(x) \wedge \mathcal{D}_T(w,y)) \vee (F(w) \wedge \mathcal{D}_T(x,y)) = \mathcal{D}_T(x \wedge w, y)$. Also, since $F(x) \wedge \mathcal{D}_T(w,y) \leq \mathcal{D}_T(w,y)$ and $F(w) \wedge \mathcal{D}_T(x,y) \leq \mathcal{D}_T(x,y)$, and hence obtained $(F(x) \wedge \mathcal{D}_T(w,y)) \vee (F(w) \wedge \mathcal{D}_T(x,y) \leq \mathcal{D}_T(x,y) \vee \mathcal{D}_T(w,y)$. Thus, $\mathcal{D}_T(x \wedge w, y) \leq \mathcal{D}_T(x,y) \vee \mathcal{D}_T(w,y)$.
(3) Since $\mathcal{D}_T(x,y) \wedge F(w) \leq F(w)$ and $F(x) \wedge \mathcal{D}_T(w,y) \leq F(x)$. Therefore, $(\mathcal{D}_T(x,y) \wedge F(w)) \vee (F(x) \wedge \mathcal{D}_T(w,y)) \leq F(x) \vee F(w)$. Hence, $\mathcal{D}_T(x \wedge w, y) \leq F(x) \vee F(w)$.
(4) From (1) it is clear that $\mathcal{D}_T(x,y) \leq F(x) \wedge F(y)$ for all $x, y \in L$.
(5) Since $\mathcal{D}_T(x) = \mathcal{D}_T(x \wedge x, x) = (\mathcal{D}_T(x,x) \wedge F(x)) \vee (F(x) \wedge \mathcal{D}_T(x,x)) = F(x) \wedge \mathcal{D}_T(x,y)$ from which we obtained $\mathcal{D}_T(x) \leq F(x)$ for all $x \in L$.
(6) From (5) it is seen that $d_T^2(x) = \mathcal{D}_T(\mathcal{D}_T(x)) \leq \mathcal{D}_T(x) \leq F(x)$ and also from (A) gives $\mathcal{D}_T(x, \mathcal{D}_T(x)) \leq \mathcal{D}_T(x)$. Then, we have

$$\begin{aligned} d_T^2(x) &= \mathcal{D}_T(\mathcal{D}_T(x)) = \mathcal{D}_T(F(x) \wedge \mathcal{D}_T(x)) \\ &= \mathcal{D}_T(F(x), \mathcal{D}_T(x)) \vee (F(x) \wedge d_T^2(x)) \vee (\mathcal{D}_T(x) \wedge F(x)) \\ &= \mathcal{D}_T(F(x), \mathcal{D}_T(x)) \vee d_T^2(x) \vee \mathcal{D}_T(x) \\ &= \mathcal{D}_T(F(x), \mathcal{D}_T(x)) \vee \mathcal{D}_T(x). \end{aligned}$$

□

Corollary 3.1. *Let L be a lattice and $\mathcal{D}_T$ be a symmetric bi-(T,F)-derivation on L with least element 0 and greatest element 1, then $F(0) = 0$ and $F(1) = 1$ implies $\mathcal{D}_T(0,x) = 0$ and $\mathcal{D}_T(1,x) \leq F(x)$ for all $x \in L$.*

Proof: The proof of the corollary is trivial by Theorem 3.3(1). □

Theorem 3.4. Let L be a lattice and $\mathcal{D}_T$ be symmetric bi-(T,F)-derivation of L and $\mathcal{D}_T$ be the trace of symmetric bi-(T,F)-derivation $\mathcal{D}_T$. Then,

$$\mathcal{D}_T(x \wedge y) = \mathcal{D}_T(x,y) \vee (F(x) \wedge \mathcal{D}_T(y)) \vee (F(y) \wedge \mathcal{D}_T(x))$$

for all $x, y \in L$.

Proof. By using the Theorem 3.3 (1) and (5), we have

$$\begin{aligned} \mathcal{D}_T(x \wedge y) &= \mathcal{D}_T(x \wedge y, x \wedge y) \\ &= (\mathcal{D}_T(x \wedge y, x) \wedge F(y)) \vee (\mathcal{D}_T(x \wedge y, y) \wedge F(x)) \\ &= \mathcal{D}_T(x \wedge y, x) \vee \mathcal{D}_T(x \wedge y, y) \\ &= ((\mathcal{D}_T(x) \wedge F(y)) \vee (F(x) \wedge \mathcal{D}_T(x,y))) \vee ((\mathcal{D}_T(x,y) \wedge F(y)) \vee (F(x) \wedge \mathcal{D}_T(y))) \\ &= ((\mathcal{D}_T(x) \wedge F(y)) \vee \mathcal{D}_T(x,y)) \vee (\mathcal{D}_T(x,y) \vee (F(x) \wedge \mathcal{D}_T(y))) \\ &= \mathcal{D}_T(x,y) \vee (F(x) \wedge \mathcal{D}_T(y)) \vee (F(y) \wedge \mathcal{D}_T(x)). \end{aligned}$$

□

Corollary 3.2. *Let L be a lattice and $\mathcal{D}_T$ be symmetric bi-(T,F)-derivation of L and $\mathcal{D}_T$ be the trace of symmetric bi-(T,F)-derivation $\mathcal{D}_T$. Then the followings inequalities hold: for all $x, y \in L$*
(1) $\mathcal{D}_T(x,y) \leq \mathcal{D}_T(x \wedge y)$

(2) $F(x) \wedge \mathcal{D}_T(y) \leq \mathcal{D}_T(x \wedge y)$
(3) $F(y) \wedge \mathcal{D}_T(x) \leq \mathcal{D}_T(x \wedge y)$
(4) $\mathcal{D}_T(x) \wedge \mathcal{D}_T(y) \leq \mathcal{D}_T(x \wedge y)$.

Proof. The proof of (1), (2) and (3) are trivial by Theorem 3.4. (4) can be proved by using (2), (3) and Theorem 3.3(5). □

Proposition 3.5. Let L be a lattice with least element 0 and greatest element 1, and $\mathcal{D}_T$ be symmetric bi-(T, F)-derivation of L and d_T be the trace of symmetric bi-(T, F)-derivation $\mathcal{D}_T$, then following results hold:
(1) If $F(x) \geq \mathcal{D}_T(1, y)$, then $\mathcal{D}_T(x, y) \geq \mathcal{D}_T(1, y)$
(2) If $F(x) \leq \mathcal{D}_T(1, y)$, then $\mathcal{D}_T(x) = F(x)$

Proof. (1) Let $F(1) = 1$, then

$$\begin{aligned}\mathcal{D}_T(x, y) &= \mathcal{D}_T(x \wedge 1, y)\\ &= (\mathcal{D}_T(x, y) \wedge F(1)) \vee (F(x) \wedge \mathcal{D}_T(1, y))\\ &= \mathcal{D}_T(x, y) \vee \mathcal{D}_T(1, y).\end{aligned}$$

Hence, $\mathcal{D}_T(x, y) \geq \mathcal{D}_T(1, y)$ for all $x, y \in L$
(2)

$$\begin{aligned}\mathcal{D}_T(x, y) &= \mathcal{D}_T(x \wedge 1, y)\\ &= (\mathcal{D}_T(x, y) \wedge F(1)) \vee (F(x) \wedge \mathcal{D}_T(1, y))\\ &= \mathcal{D}_T(x, y) \vee F(x).\end{aligned}$$

Then, $F(x) \leq \mathcal{D}_T(x, y)$. Hence by Theorem 3.3(1), $\mathcal{D}_T(x, y) = F(x)$ for all $x, y \in L$. □

Theorem 3.6. Let L be a lattice with greatest element 1 and let $\mathcal{D}_T$ be a trace of a symmetric bi-(T, F)-derivation $\mathcal{D}_T$. Then following conditions are equivalent:
(1) $\mathcal{D}_T$ is an isotone mapping
(2) $\mathcal{D}_T(x) = F(x) \wedge \mathcal{D}_T(1)$
(3) $\mathcal{D}_T(x \wedge y) = \mathcal{D}_T(x) \wedge \mathcal{D}_T(y)$
(4) $\mathcal{D}_T(x) \vee \mathcal{D}_T(y) \leq \mathcal{D}_T(x \vee y)$.

Proof. $(1) \Rightarrow (2)$. Since $\mathcal{D}_T$ is isotone and $x \leq 1$, then $F(x) \leq \mathcal{D}_T(1)$. Also, $\mathcal{D}_T(x) \leq F(x) \wedge \mathcal{D}_T(1)$ by Theorem 3.3(E). By Corollary 3.2 (B), we have $F(x) \wedge \mathcal{D}_T(1) \leq \mathcal{D}_T(x)$ for all $x \in L$. Hence, $\mathcal{D}_T(x) = F(x) \wedge \mathcal{D}_T(1)$ for all $x \in L$.

$(2) \Rightarrow (3)$. Let $F(x) \wedge \mathcal{D}_T(1) = \mathcal{D}_T(x)$ for all $x \in L$. Then, $\mathcal{D}_T(x \wedge y) = F(x \wedge y) \wedge \mathcal{D}_T(1) = (F(x) \wedge \mathcal{D}_T(1)) \wedge (F(y) \wedge \mathcal{D}_T(1)) = \mathcal{D}_T(x) \wedge \mathcal{D}_T(y)$ for all $x, y \in L$.

$(3) \Rightarrow (1)$. Let $\mathcal{D}_T(x \wedge y) = \mathcal{D}_T(x) \wedge \mathcal{D}_T(y)$ for all $x, y \in L$ and $x \leq y$. Then, $\mathcal{D}_T(x) = \mathcal{D}_T(x \wedge y) = \mathcal{D}_T(x) \wedge \mathcal{D}_T(y)$, and hence $\mathcal{D}_T(x) \leq \mathcal{D}_T(y)$.

$(1) \Rightarrow (4)$. Let $\mathcal{D}_T$ be isotone. Since $x \leq x \vee y$ and $y \leq x \vee y$, then $\mathcal{D}_T(x) \leq \mathcal{D}_T(x \vee y)$ and $\mathcal{D}_T(y) \leq \mathcal{D}_T(x \vee y)$. Thus, $\mathcal{D}_T(x) \vee \mathcal{D}_T(y) \leq \mathcal{D}_T(x \vee y)$ for all $x, y \in L$.

$(4) \Rightarrow (1)$. Let $x \leq y$, then $\mathcal{D}_T(x) \leq \mathcal{D}_T(x \vee y) = \mathcal{D}_T(y)$. Hence, $\mathcal{D}_T$ is isotone. □

Definition 3.3. Let L be a lattice and $\mathcal{D}_T$ be a symmetric bi-(T, F)-derivation of L
(1) If $x \leq w$ implies $D_T(x, y) \leq \mathcal{D}_T(w, y)$, then D_T is called an isotone symmetric bi-(T, F)-derivation
(2) If D_T is one-to-one, then D_T is called a monomorphic symmetric bi-(T, F)-derivation
(3) If D_T is onto, then D_T is called an epic symmetric bi-(T, F)-derivation.

Lemma 3.7. Let $\mathcal{D}_T$ be a symmetric bi-(T, F)-derivation on lattice L. Then followings hold:
(1) $\mathcal{D}_T(x \wedge w, y) = \mathcal{D}_T(x, y) \wedge \mathcal{D}_T(w, y)$ for all $x, y, w \in L$
(2) $\mathcal{D}_T(x \vee w, y) \geq \mathcal{D}_T(x, y) \vee \mathcal{D}_T(w, y)$ for all $x, y, w \in L$.

Proof. (1) Since $x \wedge w \leq x$ and $x \wedge w \leq w$, then we have $\mathcal{D}_T(x \wedge w, y) \leq \mathcal{D}_T(x, y)$ and $\mathcal{D}_T(x \wedge w, y) \leq \mathcal{D}_T(w, y)$. Thus $\mathcal{D}_T(x \wedge w, y) \leq \mathcal{D}_T(x, y) \wedge \mathcal{D}_T(w, y)$. Hence, by Theorem 3.3(2), we get $\mathcal{D}_T(x \wedge w, y) = \mathcal{D}_T(x, y) \wedge \mathcal{D}_T(w, y)$ for all $x, y, w \in L$.
(2) Since $x \leq x \vee y$ and $y \leq x \vee y$, so we have $\mathcal{D}_T(x, y) \leq \mathcal{D}_T(x \vee w, y)$ and $\mathcal{D}_T(w, y) \leq \mathcal{D}_T(x \vee w, y)$. Therefore, we obtained $\mathcal{D}_T(x \vee w, y) \geq \mathcal{D}_T(x, y) \vee \mathcal{D}_T(w, y)$ for all $x, y, w \in L$. □

Proposition 3.8. Let L be a lattice and $\mathcal{D}_T$ be a symmetric bi-(T, F)-derivation on L. Then following conditions hold:
(1) $\mathcal{D}_T(x, y) = \mathcal{D}_T(x, y) \vee (\mathcal{D}_T(x \vee s, y) \wedge x)$, when $\mathcal{D}_T$ is an symmetric bi-(T, F)-derivation on L
(2) $\mathcal{D}_T(x, y) = \mathcal{D}_T(x, y) \vee (\mathcal{D}_T(x \vee s, y) \wedge F(x))$, when F is a join-homomorphism on L
(3) Then $\mathcal{D}_T(x, y) = \mathcal{D}_T(x, y) \vee (F(x) \wedge \mathcal{D}_T(x \vee s, y)$, when F is an increasing function on L.

Proof. (1) Let $\mathcal{D}_T$ be an isotone symmetric bi-(T, F)-derivation. Then,

$$\begin{aligned}\mathcal{D}_T(x, y) &= \mathcal{D}_T((x \vee s) \wedge x, y)\\ &= (\mathcal{D}_T(x \vee s, y) \wedge F(x)) \vee (F(x \vee s) \wedge \mathcal{D}_T(x, y))\\ &= (\mathcal{D}_T(x \vee s, y) \wedge F(x)) \vee \mathcal{D}_T(x, y).\end{aligned}$$

As, $\mathcal{D}_T(x, y) \leq \mathcal{D}_T(x \vee s, y) \leq F(x \vee s)$.
(2) Since $\mathcal{D}_T(x, y) \leq F(x) \leq F(x) \vee F(s)$ and $F(x \vee s) = F(x) \vee F(s)$, so obtained

$$\begin{aligned}\mathcal{D}_T(x, y) &= \mathcal{D}_T((x \vee s) \wedge x, y)\\ &= (\mathcal{D}_T(x \vee s, y) \wedge F(x)) \vee (F(x \vee s) \wedge \mathcal{D}_T(x, y))\\ &= (\mathcal{D}_T(x \vee s, y) \wedge F(x)) \vee \mathcal{D}_T(x, y).\end{aligned}$$

(3) Since F is an increasing function and $x \leq x \vee y$, so $F(x) \leq F(x \vee y)$. Therefore,

$$\begin{aligned}\mathcal{D}_T(x, y) &= \mathcal{D}_T((x \vee s) \wedge x, y)\\ &= (\mathcal{D}_T(x \vee s, y) \wedge F(x)) \vee (F(x \vee s) \wedge \mathcal{D}_T(x, y))\\ &= (\mathcal{D}_T(x \vee s, y) \wedge F(x)) \vee \mathcal{D}_T(x, y).\end{aligned}$$

□

Theorem 3.9. Let L be a lattice with greatest element 1 and $\mathcal{D}_T$ be a symmetric bi-(T, F)-derivation on L and $F(x \wedge y) = F(x) \wedge F(y)$. Then followings equivalent:
(1) $\mathcal{D}_T$ is isotone symmetric bi-(T, F)-derivation
(2) $\mathcal{D}_T(x, y) \vee \mathcal{D}_T(s, y) \leq \mathcal{D}_T(x \vee s, y)$ for all $x, y \in L$
(3) $\mathcal{D}_T(x, y) = F(x) \wedge \mathcal{D}_T(1, y)$ for all $x, y \in L$
(4) $\mathcal{D}_T(x \wedge s, y) = \mathcal{D}_T(x, y) \wedge \mathcal{D}_T(s, y)$ for all $x, y, s \in L$.

Proof. $(1) \Rightarrow (2)$. We assume that $\mathcal{D}_T$ is an isotone symmetric bi-(T, F)-derivation on L. Since $x \leq x \vee s$ and $s \leq x \vee s$, and so $\mathcal{D}_T(x, y) \leq \mathcal{D}_T(x \vee s, y)$ and $\mathcal{D}_T(s, y) \leq \mathcal{D}_T(x \vee s, y)$. Hence, $\mathcal{D}_T(x, y) \vee \mathcal{D}_T(s, y) \leq \mathcal{D}_T(x \vee s, y)$ for all $x, y, s \in L$.

$(2) \Rightarrow (3)$. Suppose that $\mathcal{D}_T(x,y) \vee \mathcal{D}_T(s,y) \leq \mathcal{D}_T(x \vee s, y)$ and $x \leq s$. Then, we get $\mathcal{D}_T(x,y) \leq \mathcal{D}_T(x,y) \vee \mathcal{D}_T(s,y) \leq \mathcal{D}_T(x \vee s, y) = \mathcal{D}_T(s,y)$. Therefore, $\mathcal{D}_T$ is an isotone symmetric bi-(T,F)-derivation on L.

$(1) \Rightarrow (3)$. Suppose $\mathcal{D}_T$ is an isotone symmetric bi-(T,F)-derivation on L.
Since, $\mathcal{D}_T(x,y) \leq \mathcal{D}_T(1,y)$, we have $\mathcal{D}_T(x,y) \leq F(x) \wedge \mathcal{D}_T(1,y)$ by Theorem 3.3 (A). Using Proposition 3.8 and by taking $s = 1$, we get

$$\begin{aligned} \mathcal{D}_T(x,y) &= (\mathcal{D}_T(1,y) \wedge F(x)) \vee \mathcal{D}_T(x,y) \\ &= \mathcal{D}_T(1,y) \wedge F(x). \end{aligned}$$

$(3) \Rightarrow (4)$. Assume that $\mathcal{D}_T(x,y) = F(x) \wedge \mathcal{D}_T(1,y)$, then $\mathcal{D}_T(x \wedge s, y) = F(x \wedge s) \wedge \mathcal{D}_T(1,y) = F(x) \wedge F(s) \wedge \mathcal{D}_T(1,y) = (F(x) \wedge \mathcal{D}_T(1,y)) \vee (F(s) \wedge \mathcal{D}_T(1,y)) = \mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y)$ for all $x, y, 1 \in L$
$(4) \Rightarrow (1)$. Let $\mathcal{D}_T(x \wedge s, y) = \mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y)$ and $x \leq s$. Then, $\mathcal{D}_T(x,y) = \mathcal{D}_T(x \wedge s, y) = \mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y)$. Hence, $\mathcal{D}_T(x,y) \leq \mathcal{D}_T(s,y)$. □

Theorem 3.10. Let L be a modular lattice and $\mathcal{D}_T$ be a symmetric bi-(T,F)-derivation on L. Then, followings hold.
(1) If $\mathcal{D}_T$ is an isotone symmetric bi-(T,F)-derivation on L if and only if $\mathcal{D}_T(x \wedge s, y) = \mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y)$
(2) If $\mathcal{D}_T$ is an isotone symmetric bi-(T,F)-derivation on L and $F(x \vee s) = F(x) \vee F(s)$, $\mathcal{D}_T(x,y) = F(x)$, then $\mathcal{D}_T(x \vee s, y) = \mathcal{D}_T(x,y) \vee \mathcal{D}_T(s,y)$.

Proof. (1) Let $\mathcal{D}_T$ be a symmetric bi-(T,F)-derivation on L. Since $x \wedge s \leq x$ and $x \wedge s \leq s$, then $\mathcal{D}_T(x \wedge s, y) \leq \mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y)$. Therefore,

$$\begin{aligned} \mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y) &= (\mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y)) \wedge (F(x) \wedge F(s)) \\ &= (\mathcal{D}_T(x,y) \wedge F(s)) \wedge (F(x) \wedge \mathcal{D}_T(s,y)) \\ &\leq (\mathcal{D}_T(x,y) \wedge F(s)) \vee (\mathcal{D}_T(s,y) \wedge F(x)) \\ &= \mathcal{D}_T(x \wedge s, y). \end{aligned}$$

Conversely, let $\mathcal{D}_T(x \wedge s, y) = \mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y)$ and $x \leq s$. Thus, $\mathcal{D}_T(x,y) = \mathcal{D}_T(x \wedge s, y) = \mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y)$, and hence $\mathcal{D}_T(x,y) \leq \mathcal{D}_T(s,y)$ for all $x, y, s \in L$.
(2) Let $\mathcal{D}_T$ be a symmetric bi-T-derivation on L and $\mathcal{D}_T(x,y) = x$. Then, by Proposition 3.8 and since L is a modular lattice, thus, $\mathcal{D}_T(s,y) = (\mathcal{D}_T(s,y) \vee \mathcal{D}_T(x \vee s, y)) \wedge F(s) = F(s) \wedge \mathcal{D}_T(x \vee s, y)$. Thus,

$$\begin{aligned} \mathcal{D}_T(x,y) \vee \mathcal{D}_T(s,y) &= \mathcal{D}_T(x,y) \vee (F(s) \wedge \mathcal{D}_T(x \vee s, y)) \\ &= (\mathcal{D}_T(x,y) \vee F(s)) \wedge \mathcal{D}_T(x \vee s, y) \\ &= (F(x) \vee F(s)) \wedge \mathcal{D}_T(x \vee s, y) \\ &= F(x \vee s) \wedge \mathcal{D}_T(x \vee s, y) \\ &= \mathcal{D}_T(x \vee s, y). \end{aligned}$$

□

Theorem 3.11. Let L be a distributive lattice and $\mathcal{D}_T$ be a symmetric bi-(T,F)-derivation on L, and $F(x \vee s) = F(x) \vee F(s)$. Then, following conditions are hold.
(A) If $\mathcal{D}_T$ is an isotone symmetric bi-(T,F)-derivation on L, then $\mathcal{D}_T(x \wedge s, y) = \mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y)$
(B) If $\mathcal{D}_T$ is an isotone symmetric bi-(T,F)-derivation on L if and only if $\mathcal{D}_T(x \vee s, y) = \mathcal{D}_T(x,y) \vee \mathcal{D}_T(s,y)$.

Proof. Since, $\mathcal{D}_T$ is an isotone symmetric bi-T-derivation and $\mathcal{D}_T(x \wedge s, y) = \mathcal{D}_T(x, y) \wedge \mathcal{D}_T(s, y)$. By Theorem 3.3 (A), we have

$$\begin{aligned}
\mathcal{D}_T(x,y) \wedge \mathcal{D}_T(s,y) &= ((\mathcal{D}_T(x,y) \wedge F(x)) \wedge ((F(s) \wedge \mathcal{D}_T(s,y)) \\
&= (\mathcal{D}_T(x,y) \vee F(s)) \wedge (F(x) \wedge \mathcal{D}_T(s,y) \\
&\leq (\mathcal{D}_T(x,y) \wedge F(s)) \vee (F(x) \wedge \mathcal{D}_T(s,y) \\
&= \mathcal{D}_T(x \wedge s, y).
\end{aligned}$$

Therefore, $\mathcal{D}_T(x \wedge s, y) = \mathcal{D}_T(x, y) \wedge \mathcal{D}_T(s, y)$ for all $x, y, s \in L$. (B) Let $\mathcal{D}_T$ be an isotone symmetric bi-(T, F)-derivation. Then, using Theorem 3.3(A) and Proposition 3.8, we have

$$\begin{aligned}
\mathcal{D}_T(s,y) &= (\mathcal{D}_T(s,y) \vee (F(s) \wedge \mathcal{D}_T(x \vee s, y)) \\
&= (\mathcal{D}_T(s,y) \wedge F(s)) \wedge (\mathcal{D}_T(s,y) \vee \mathcal{D}_T(x \vee s, y)) \\
&= F(s) \wedge \mathcal{D}_T(x \vee s, y).
\end{aligned}$$

In similar way, $\mathcal{D}_T(x, y) = F(x) \wedge \mathcal{D}_T(x \vee s, y)$. Thus,

$$\begin{aligned}
\mathcal{D}_T(x,y) \vee \mathcal{D}_T(s,y) &= (F(x) \wedge \mathcal{D}_T(x \vee s, y)) \vee (F(s) \wedge \mathcal{D}_T(x \vee s, y)) \\
&= (F(x) \vee F(s)) \wedge \mathcal{D}_T(x \vee s, y) \\
&= F(x \vee s) \wedge \mathcal{D}_T(x \vee s, y) \\
&= \mathcal{D}_T(x \vee s, y).
\end{aligned}$$

Conversely, let $\mathcal{D}_T(x \vee s, y) = \mathcal{D}_T(x, y) \vee \mathcal{D}_T(s, y)$ and $x \leq s$, then obtained $\mathcal{D}_T(s, y) = \mathcal{D}_T(x \vee s, y) = \mathcal{D}_T(x, y) \vee \mathcal{D}_T(s, y)$, which imply $\mathcal{D}_T(x, y) \leq \mathcal{D}_T(s, y)$ for all $x, y, s \in L$.

□

4. CONCLUSIONS AND FUTURE WORK

In this paper, we discussed the notion of symmetric bi-(T, F)-derivation on lattice and investigated some useful properties of it. In our opinion, these results can be similarly extended to the other algebraic structure such as BCI-algebras, B-algebras, BG-algebras, BF-algebras, MV-algebras, d-algebras, Q-algebras, Incline algebras and so forth. The study of symmetric bi-(T, F)-derivation on different algebraic structures may have a lot of applications in different branches of theoretical physics, engineering, information theory, information retrieval, information control access, cryptanalysis and computer science, etc.

We hope that this work will give a deep impact on the upcoming research in this field and other algebraic study to open up a new horizons of interest and innovations. It is our hope that this work would serve as a foundation for further study in the theory of derivations of lattice.

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