ANNALS OF COMMUNICATIONS IN MATHEMATICS
Volume 5, Number 2 (2022), 127-137
ISSN: 2582-0818

NEUTROSOPHIC REGULAR SEMI COMPACTNESS AND CONNECTEDNESS

R. VIJAYALAKSHMI AND R. R. PRAVEENA*

ABSTRACT. In this paper, we introduce the concept of neutrosophic regular semi compactness, neutrosophic regular semi connectedness, neutrosophic regular semi strongly connectedness and neutrosophic regular semi-C_5-connectedness in neutrosophic topological spaces. Some interesting properties of these notions are studied. In this connection, interrelations are discussed. Example are provided wherever necessary.

1. INTRODUCTION

Zadeh [16] introduced the notion of fuzzy sets in the year 1965. The concept of fuzzy topological spaces have been introduced and developed by Chang [3]. In 1983, Atanassov [1] introduced the concept of intuitionistic fuzzy set which was generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. Later, Coker [4] introduced the concept of intuitionistic fuzzy topological spaces, by using the notion of the intuitionitic fuzzy set. Smarandache [6, 7, 8] introduced the concept of Neutrosophic set. Neutrosophic set is classified into three independent functions namely, membership function, indeterminancy and non membership function that are independently related. In 2012, Salama and Alblowi [12, 13, 14] introduced the concept of Neutrosophic topology. Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allow more general functions to be members of fuzzy topology. In 2014, Salama et. al., [13] introduced the concept of Neutrosophic closed sets and Neutrosophic continuous functions. In general topology, the concept of regular semiopen set was introduced by Cameron [2] in 1978. Elavarasan [5] introduced the concept of fuzzy regular semi compactness and connectedness in the sense of Sostak's. Recently Vijayalakshmi and Praveena [9, 10] introduced the concept of neutrosophic regular semiopen, neutrosophic regular semiclosed, neutrosophic regular semi continuous, neutrosophic regular semi irresolute, neutrosophic regular semi homeomorphisms and neutrosophic regular semi C-homeomorphisms in neutrosophic topological spaces. In this paper, we introduce the concepts of neutrosophic regular semi compactness, neutrosophic regular semi connectedness, neutrosophic regular semi strongly connectedness and neutrosophic regular

2010 *Mathematics Subject Classification.* 54A10, 54A40, 03E72.
Key words and phrases. NRS-compactness, NRS-connectedness, NRS-strongly connectedness and NRS-C_5-connectedness.
Received: August 09, 2022. Accepted: October 25, 2022. Published: November 30, 2022.
*Corresponding author.

semi-C_5-connectedness in neutrosophic topological spaces. Some interesting properties of these notions are studied. In this connection, interrelations are discussed. Example are provided wherever necessary.

2. Preliminaries

Definition 2.1. [12] Let X be a non-empty fixed set. A Neutrosophic set [for short, Ns] A is an object having the form $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $B_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminancy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark. [12] *A Ns* $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ *can be identified to an ordered triple* $A = \langle B_A(x), \sigma_A(x), \gamma_A(x)\rangle$ *in* $]^-0, 1^+[$ *on* X.

Remark. [12] *For the sake of simplicity, we shall use the symbol* $A = \langle B_A, \sigma_A, \gamma_A\rangle$ *for the Ns* $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Example 2.2. [12] Every intuitionsistic fuzzy set A is a non-empty set in X is obviously on Ns having the form $A = \{\langle x, B_A(x), 1 - B_A(x) + \gamma_A(x)\rangle : x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic sets 0_N and 1_N in X as follows:
$0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 2.3. [12] Let $A = \langle (B_A, \sigma_A, \gamma_A)\rangle$ be a Ns on X, then the complement of the set A(A^c or $C(A)$ for short) may be defined as $C(A) = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), B_A(x)\rangle : x \in X\}$.

Definition 2.4. [12] Let X be a non-empty set and Ns's A and B in the form $A = \{\langle x, B_A, \sigma_A, \gamma_A\rangle : x \in X\}$ and $B = \{\langle x, B_B, \sigma_B, \gamma_B\rangle : x \in X\}$. Then $(A \subseteq B)$ may defined as: $(A \subseteq B) \Leftrightarrow B_A(x) \leq B_B(x), \sigma_A(x) \leq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x) \forall x \in X$.

Definition 2.5. [12] Let X be a non-empty set and $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, B_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$ are Ns's. Then $A \cap B$ and $A \cup B$ may defined as:

(i) $A \cap B = \langle x, B_A(x) \wedge B_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle$
(ii) $A \cup B = \langle x, B_A(x) \vee B_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle$

Definition 2.6. [12] A Neutrosophic topology (for short, NT or nt) is a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:

(i) $0_N, 1_N \in \tau_N$,
(ii) $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
(iii) $\cup G_i \in \tau_N$ for every $\{G_i : i \in J\} \subseteq \tau_N$.

Throughout this paper, the pair of (X, τ_N) is called a neutrosophic topological space (for short, nts). The elements of τ_N or τ are called neutrosophic open set (for short, nos). A neutrosophic set F is neutrosophic closed set (for short, ncs)if and only if F^c is nos.

Definition 2.7. [12] Let (X, τ_N) be nts and $A = \langle x, B_A, \sigma_A, \gamma_A\rangle$ be a Ns in X. Then the neutrosophic closure and neutrosophic interior of A are defined by $NCl(A) = \cap\{K : K$ is a ncs in X and $A \subseteq K\}$, $NInt(A) = \{G : G$ is a nos in X and $G \subseteq A\}$. It can be also shown that $NCl(A)$ is ncs and $NInt(A)$ is a nos in X. A is nos if and only if $A = NInt(A)$, A is ncs if and only if $A = NCl(A)$.

Definition 2.8. [15] Let $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ be a Ns on a nts (X, τ_N) then A is called:

(i) neutrosophic regular open (for short, nro) iff $A = NInt(NCl(A))$.
(ii) neutrosophic regular closed (for short, nrc) iff $A = NCl(NInt(A))$.

Definition 2.9. [15] Let $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ be a Ns and $B = \{\langle x, B_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$ be a Ns on a nts (X, τ_N) then A is called neutrosophic semi-open (for short, nso) iff $A \subseteq NInt(NCl(A))$.

Definition 2.10. [9] Let $(X,\ \tau)$ be a nts. Then A is called
(i) neutrosophic regular semiopen (for short, nrso) if there exists an nro set B in X such that $B \subseteq A \subseteq NCl(B)$.
(ii) neutrosophic regular semiclosed (for short, nrsc) if there exists an nrc set B in X and $NInt(B) \subseteq A \subseteq B$.

Definition 2.11. [9] Let $(X,\ \tau)$ be a nts. Then
(i) the neutrosophic regular semiclosure of A defined by $nrscl(A) = \bigcap\{B \mid A \subseteq B$ and $B \in NRSCS(X,\tau)\}$ is a neutrosophic set.
(ii) the neutrosophic regular semiinterior of A defined by $nrsint(A) = \bigcup\{B \mid B \subseteq A$ and $B \in NRSOS(X,\tau)\}$ is a neutrosophic set.

Definition 2.12. [14] Let (X,τ) and (Y,σ) be any two nts's. A map $f : (X,\tau) \to (Y,\sigma)$ is neutrosophic continuous (for short, NC) if the inverse image of every neutrosophic closed set in (Y,σ) is neutrosophic closed set in (X,τ).

Definition 2.13. [10]Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be two nts's. A Neutrosophic function $f : X \to Y$ is said to be
(i) neutrosophic regular semi continuous (for short, $NRSC$) if for each nos A of Y, the inverse image $f^{-1}(A)$ is a nrso set of X.
(ii) neutrosophic regular semi irresolute (for short, $NRSI$) if for each nrso set A of Y, the inverse image $f^{-1}(A)$ is a nrso set of X.
(iii) neutrosophic regular semiopen function (for short, NRS-O) if for each nos B of X, the image $f(B)$ is a nrso set of Y.
(iv) neutrosophic regular semiclosed function (for short, NRS-C) if for each ncs set B of X, the image $f(B)$ is a nrsc set of Y.

Proposition 2.1. [9] *If R is nrso set in (X,τ), then R^c is also nrso set.*

Theorem 2.2. [11] *Let (X,τ) and (Y,σ) be two nts's and let $f : (X,\tau) \to (Y,\sigma)$ is neutrosophic weakly regular open and neutrosophic weakly regular continuous function, then $f^{-1}(A)$ is nro (resp. nrc) set for every nro set A in Y.*

3. Neutrosophic Regular Semi Compactness

Definition 3.1. A nts (X,τ) is called
(1) neutrosophic regular semi compact (for short, NRS-compact) if for every $NRSO$-cover $\{A_i : i \in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} A_i = 1_N$.
(2) Neutrosophic weakly regular semi compact (for short, $NWRS$-compact) if for every $NRSO$-cover $\{A_i : i \in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} NInt(A_i) = 1_N$.
(3) neutrosophic almost regular semi compact (for short, $NARS$-compact) if for every $NRSO$-cover $\{A_i : i \in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} NCl(A_i) = 1_N$.

Remark. *(1) Every $NWRS$-compact is NRS-compact.*
(2) Every NRS-compact is $NARS$-compact.

Theorem 3.1. *A nts (X,τ) is NRS-compact if and only if for each family $\{A_i|i \in J\}$ of nrso sets of X such that $\bigcap_{i\in J} A_i = 0_N$, there exists a finite subset J_0 of J such that $\bigcap_{i\in J_0} A_i = 0_N$.*

Theorem 3.2. *A nts (X,τ) is $NWRS$-compact if and only if for each family $\{A_i|i \in J\}$ of nrso sets of X such that $\bigcap_{i\in J} A_i = 0_N$, there exists a finite subset J_0 of J such that $\bigcap_{i\in J_0} NCl(A_i) = 0_N$.*

Proof. Suppose that (X,τ) is $NWRS$-compact. Let $\{A_i|i \in J\}$ be a family of nrso sets of X such that $\bigcap_{i\in J} A_i = 0_N$. Then by Theorem 2.1, $\{1 - A_i|i \in J\}$ is a family of nrso sets of X such that $\bigcup_{i\in J} 1 - A_i = 1 - \bigcap_{i\in J} A_i = 1_N$. Since (X,τ) is $NWRS$-compact, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} NInt(1 - A_i) = 1_N$. Hence $\bigcap_{i\in J_0} NCl(A_i) = 1 - (\bigcup_{i\in J_0} NInt(1 - A_i) = 0_N$. □

Converse follows by reversing the previous arguments.

Theorem 3.3. *Let (X,τ) be a nts. Then the following are equivalent:*

(1) (X,τ) is $NWRS$-compact.
(2) For each family $\{A_i|i \in J\}$ of nrso sets of X such that $\bigcap_{i\in J} A_i = 0_N$, there exists a finite subset J_0 of J such that $\bigcap_{i\in J_0} NCl(A_i) = 0_N$.
(3) For each neutrosophic regular closed cover $\{A_i|i \in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} NInt(A_i) = 1_N$.

Proof. (1)⇒(2): Trivial.

(2)⇒(1): Let $\{A_i|i \in J\}$ be a family of nrso sets of X such that $\bigcap_{i\in J} A_i = 0_N$. Since A_i is an nrso set for each $i \in J$, $NCl(A_i) = NCl(NInt(A_i))$ for each $i \in J$. Since $\{NInt(A_i)|i \in J\}$ is a family of nro sets of X such that $\bigcap_{i\in J} NInt(A_i) = 0_N$, by (2) there exists a finite subset J_0 of J such that $\bigcap_{i\in J_0} NCl(A_i) = \bigcap_{i\in J_0} NCl(NInt(A_i)) = 0_N$. Thus (X,τ) is $NWRS$-compact.

(2)⇔(3): It is obvious. □

Definition 3.2. Let (X,τ) and (Y,σ) be a nts's. Let $f : (X,\tau) \to (Y,\sigma)$ be a function. Then f is called

(1) Neutrosophic weakly continuous if for each nos B of Y, $f^{-1}(B) \subseteq NInt(f^{-1}(NCl(B)))$.
(2) Neutrosophic weakly open if for each nos B of X, $f(B) \subseteq NInt(f(NCl(B)))$.

Theorem 3.4. *Let (X,τ) and (Y,σ) be two nts's and let $f : (X,\tau) \to (Y,\sigma)$ be surjective, neutrosophic weakly open and neutrosophic weakly continuous function. If (X,τ) is $NWRS$-compact, then so is (Y,σ).*

Proof. Let $\{B_i|i \in J\}$ be an neutrosophic regular closed cover of Y. By Theorem 2.2, $\{f^{-1}(B_i)|i \in J\}$ is an neutrosophic regular closed cover of X. Since X is $NWRS$-compact, by Theorem 3.2, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} NInt(f^{-1}(B_i)) = 1_N$. From the surjectivity and neutrosophic weakly openness of f, we have

$1_N = f(\bigcap_{i\in J_0}(NInt(f^{-1}(B_i))))$
$= \bigcup_{i\in J_0} f(NInt(f^{-1}(B_i)))$
$\leq \bigcup_{i\in J_0} NInt(f(NCl(NInt(f^{-1}(B_i)))))$
$= \bigcup_{i\in J_0}(NInt(f(f^{-1}(B_i)))$
$= \bigcup_{i\in J_0} NInt(B_i)$.

Hence $\bigcup_{i\in J_0} NInt(B_i) = 1_N$, and thus (Y,σ) is $NWRS$-compact. □

Theorem 3.5. *A nts (X,τ) is $NARS$-compact if and only if for each family $\{A_i|i \in J\}$ of nrso sets of X such that $\bigcap_{i\in J} A_i = 0_N$, there exists a finite subset J_0 of J such that $\bigcap_{i\in J_0} NInt(A_i) = 0_N$.*

Proof. Let (X,τ) be $NARS$-compact and let $\{A_i|i \in J\}$ be a family of nrso sets of X such that $\bigcap_{i\in J} A_i = 0_N$. Then $\{1 - A_i|i \in J\}$ is a family of nrso sets of X such that $\bigcup_{i\in J} 1 - A_i = 1 - (\bigcap_{i\in J} A_i) = 1_N$. Since (X,τ) is $NARS$-compact, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} NCl(1 - A_i) = 1_N$. Hence $\bigcap_{i\in J_0} NInt(A_i) = 1 - \bigcup_{i\in J_0} NCl(1 - A_i) = 0_N$.

The converse can be proved similarly. □

Theorem 3.6. *Let (X,τ) be a nts. Then the following statements are equivalent:*

(1) (X,τ) is $NARS$-compact.
(2) For each family $\{A_i|i \in J\}$ of nro sets of X such that $\bigcap_{i\in J} A_i = 0_N$, there exists a finite subset J_0 of J such that $\bigcap_{i\in J_0} A_i = 0_N$.
(3) For each neutrosophic regular closed cover $\{A_i|i \in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} A_i = 1_N$.

Proof. Straightforward. □

Definition 3.3. A nts (X,τ) is called an neutrosophic S-closed if and only if for every neutrosophic semiopen cover $\{A_i|i \in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} NCl(A_i) = 1_N$.

Theorem 3.7. *A nts (X,τ) is $NARS$-compact if and only if (X,τ) is neutrosophic S-closed.*

Proof. Let (X,τ) be neutrosophic S-closed. Since every nrso set is neutrosophic semiopen, (X,τ) is $NARS$-compact.

Conversely, suppose that (X,τ) is $NARS$-compact and let $\{A_i|i \in J\}$ be an neutrosophic semiopen cover of X. Then there exists B_i in X with B_i is nos, such that $B_i \le A_i \le NCl(B_i)$, for each $i \in J$. We can easily show that $NCl(B_i)$ is an nrc for each $i \in J$. Since $B_i \le A_i \le NCl(A_i)$, for each $i \in J$, $NCl(B_i) \le NCl(A_i) \le NCl(NCl(B_i))$ for each $i \in J$. Thus $NCl(A_i) = NCl(B_i)$ for each $i \in J$. Thus $\{NCl(A_i)|i \in J\}$ is an neutrosophic regular closed cover of X. Since (X,τ) is $NARS$-compact, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} NCl(A_i) = 1_N$. Hence (X,τ) is neutrosophic S-closed. □

Theorem 3.8. *A nts (X,τ) is an $NWRS$-compact if and only if for every an neutrosophic semiopen cover $\{A_i|i \in J\}$ of X, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} NInt(NCl(A_i)) = 1_N$.*

Proof. Similar to Theorem 3.7. □

Theorem 3.9. *Let (X,τ) and (Y,σ) be two nts's and let $f : (X,\tau) \to (Y,\sigma)$ be a surjective, neutrosophic weakly open and neutrosophic weakly continuous function. If (X,τ) is $NARS$-compact, then so is (Y,σ).*

Proof. Let $\{B_i|i \in J\}$ be an neutrosophic regular closed cover of Y. By Theorem 2.2, $\{f^{-1}(B_i)|i \in J\}$ is an neutrosophic regular closed cover of X. Since (X,τ) is $NARS$-compact, by Theorem 2.2, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0} f^{-1}(B_i) = 1_N$. From the surjectivity of f we have

$1_N = f(\bigcup_{i\in J_0} f^{-1}(B_i)) = \bigcup_{i\in J_0} f(f^{-1}(B_i)) = \bigcup_{i\in J_0} B_i$.

Hence $\bigcup_{i\in J_0} B_i = 1_N$. Thus (Y,σ) is $NARS$-compact. □

Definition 3.4. A nts (X,τ) is called neutrosophic extremally disconnected (for short, NED) if $NCl(A)$ is nos and A is nos.

Theorem 3.10. *Let (X,τ) and (Y,σ) be two nts, and let $f:(X,\tau)\to(Y,\sigma)$ be a surjective, neutrosophic weakly open and neutrosophic weakly continuous function. If (X,τ) is NED, then so is (Y,σ).*

Proof. Let A be nro (nos) set in Y. Then $A=NInt(A)$. Hence $NCl(A)$ is nrc set. By Theorem 2.2, $f^{-1}(NCl(A))$ is nrc, i.e., $f^{-1}(NCl(A))=NCl(NInt(f^{-1}(NCl(A))))$. Since (X,τ) is NED and $NInt(f^{-1}(NCl(A)))$ is nos and $NCl(NInt(f^{-1}(NCl(A))))$ is nos. From the surjectivity and neutrosophic weakly openness of f we have

$NCl(A)=f(f^{-1}(NCl(A)))$
$=f(NCl(NInt(f^{-1}(NCl(A)))))$
$\leq NInt(f(NCl(NInt(f^{-1}(NCl(A))))))$
$=NInt(f(NCl(f^{-1}(NCl(A)))))$
$=NInt(f(f^{-1}(NCl(A))))$
$=NInt(NCl(A))$.

Hence $NCl(A)=NInt(NCl(A))$ and so $NCl(A)$ is nos in Y. Thus (Y,σ) is NED. □

Theorem 3.11. *Let a nts (X,τ) be NED. If A of X is nrso, then $NInt(A)=A=NCl(A)$.*

Proof. Let A be an nrso set. Then there exists an nro set B such that $B\leq A\leq NCl(B)$. Since X is NED, $B=NCl(B)$. And we get $B=NInt(B)$, since B is an nro set. Thus we have the following, $B=NInt(B)\leq NInt(A)\leq A\leq NCl(A)\leq NCl(B)=B$. Hence $NInt(A)=A=NCl(A)$. □

From the above theorem, we get the following:

Theorem 3.12. *Let a nts (X,τ) be NED. Then the following are equivalent:*

(1) (X,τ) is $NWRS$-compact.
(2) (X,τ) is NRS-compact.
(3) (X,τ) is $NARS$-compact.

Theorem 3.13. *For an NED nts (X,τ), the following are true:*

(1) neutrosophic compactness implies $NWRS$-compactness.
(2) neutrosophic nearly compactness implies NRS-compactness.
(3) neutrosophic almost compactness implies $NARS$-compactness.

Proof. (2) Let (X,τ) be an NED and neutrosophic nearly compact space, let $\{A_i|i\in J\}$ be an $NRSO$ cover of X. Then there exists an nro set B_i such that $B_i\leq A_i\leq NCl(B_i)$ for each $i\in J$. Since (X,τ) is NED and $B_i=NInt(NCl(B_i))$ for each $i\in J$, $A_i=NInt(A_i)$ for each $i\in J$. Thus we get $A_i=NInt(NCl(A_i))$ for each $i\in J$ from Proposition 2.1. Hence (X,τ) is NRS-compact since X is neutrosophic nearly compact.

(1) and (3) are similar to (2). □

Corollary 3.14. *If a nts (X,τ) is NED, then the following are equivalent:*

(1) neutrosophic nearly compactness.
(2) neutrosophic almost compactness.
(3) neutrosophic S-closeness.

Proof. We get the results from Theorems 3.7, 3.12 and 3.13. □

4. Neutrosophic Regular Semi Connectedness

Definition 4.1. Let (X, τ) be a nts and $A,\ B \in X$. A NRS-separation on 1_N is a pair of non null proper nrso sets A and B such that $A \cap B = 0_N$ and $A \cup B = 1_N$.

Definition 4.2. A nts (X, τ) is said to be neutrosophic regular semi connected (for short, NRS-connected) if and only if there is no NRS-separation of 1_N. Otherwise, (X, τ) is said to be neutrosophic regular semi disconnected space.

Example 4.3. Let $X = \{a, b, c\}$ and $\tau = \{0_N, 1_N, A\}$ where A, B, C are Ns's of X defined as follows:
$A = \left\langle (\frac{\mu_a}{0.2}, \frac{\mu_b}{0.3}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\gamma_a}{0.5}, \frac{\gamma_b}{0.5}, \frac{\gamma_c}{0.5}) \right\rangle$,
$B = \left\langle (\frac{\mu_a}{0.6}, \frac{\mu_b}{0.3}, \frac{\mu_c}{0.4}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\gamma_a}{0.5}, \frac{\gamma_b}{0.5}, \frac{\gamma_c}{0.5}) \right\rangle$,
$C = \left\langle (\frac{\mu_a}{0.7}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}), (\frac{\gamma_a}{0.5}, \frac{\gamma_b}{0.5}, \frac{\gamma_c}{0.5}) \right\rangle$.
Clearly B and C are nrso sets in (X, τ), $B \neq 0_N$, $C \neq 0_N$, $B \cup C \neq 1_N$ and $B \cap C \neq 0_N$. Hence (X, τ) is NRS-connected.

Proposition 4.1. *A nts (X, τ) is a NRS-connected if and only if there exists no non-null nrso sets $A,\ B \in X$ such that $A = 1 - B$.*

Proof. Necessity: Let A and B be two nrso sets in (X, τ) such that $A \neq 0_N, 1 - B \neq 0_N$ and $A = 1 - B$. Therefore $1 - B$ is a nrsc set. Since $A \neq 0_N,\ B \neq 1_N$. This implies that B is a proper neutrosophic set which is both nrso and nrsc in (X, τ). Hence (X, τ) is not a NRS-connected space. But this is a contradiction to our hypothesis. Thus there exists no non-null nrso sets A and B in (X, τ) such that $A = 1 - B$.

Sufficiency: Let A be both nrso and nrsc in (X, τ) such that $A \neq 0_N,\ A \neq 1_N$. Let $1 - A = B$. Then B is a nrso set and $1 - B \neq 1_N$. This implies that $B = 1 - A \neq 0_N$, which is a contradiction to our hypothesis. Hence (X, τ) is a NRS-connected space. $\square$

Proposition 4.2. *A nts (X, τ) is a NRS-connected space if and only if there exists no non-null nrso sets $A,\ B \in X$ such that $A = 1 - B$, $B = 1 - NRSCl(A)$ and $A = 1 - NRSCl(B)$.*

Proof. Necessity: Assume that there exists a Ns's A and B such that $A \neq 0_N,\ 1 - B \neq 0_N$, $A = 1 - B$, $B = 1 - NRSCl(A)$ and $A = 1 - NRSCl(B)$. Since $1 - NRSCl(A)$ and $1 - NRSCl(B)$ are nrso sets in (X, τ), A and B are nrso sets in (X, τ). This implies (X, τ) is not a NRS-connected space, which is a contradiction. Thus there exists no non-null nrso sets A and B in (X, τ) such that $A = 1 - B$, $B = 1 - NRSCl(A)$ and $A = 1 - NRSCl(B)$.

Sufficiency: Let A be both nrso and nrsc in (X, τ) such that $A \neq 0_N,\ A \neq 1_N$. Now by taking $1 - A = B$, we obtain a contradiction to our hypothesis. Hence (X, τ) is a NRS-connected space. $\square$

Definition 4.4. A nts (X, τ) is said to be neutrosophic C_5-disconnected if there exists Ns $A \in X$, which is both nos and ncs such that $A \neq 0_N$ and $A \neq 1_N$. If (X, τ) is not neutrosophic C_5-disconnected then it is said to be neutrosophic C_5-connected.

Proposition 4.3. *Let (X, τ) and (Y, σ) be two nts's. Let $f : (X, \tau) \to (Y, \sigma)$ is a $NRSC$ and surjective function. If (X, τ) is NRS-connected, then (Y, σ) is a neutrosophic C_5-connected.*

Proof. Let (X, τ) is NRS-connected. Suppose (Y, σ) is not a neutrosophic C_5-connected space, then there exists a proper Ns $A \in Y$, which is both nos and ncs. Since f is a $NRSC$

function, $f^{-1}(A)$ is both nrso and nrsc in (X,τ). But this is a contradiction to hypothesis. Hence (Y,σ) is a neutrosophic C_5-connected space. □

Definition 4.5. A Ns in a nts (X,τ) is said to be nrsco set, which is both nrso and nrsc set.

Definition 4.6. A nts (X,τ) is said to be NRS-C_5-disconnected if there exists nrsco set $A \in X$, such that $A \neq 0_N$ and $A \neq 1_N$. If (X,τ) is not NRS-C_5- disconnected then it is said to be NRS-C_5-connected.

Proposition 4.4. *A nts (X,τ) is NRS-C_5 connected, then it is NRS-connected.*

Proof. Suppose that there exists non-null nrso sets A and B such that $A \cup B = 1_N$ and $A \cap B = 0_N$ (NRS-disconnected), then $A = A \cup B$ and $A = A \cap B$. In other words, $A = 1 - B$. Hence A is a nrsco set which implies that (X,τ) is NRS-C_5-disconnected. □

Remark. *The converse of the above Proposition need not be true as shown by the following example.*

Example 4.7. Let $X = \{a,b,c\}$ and $\tau = \{0_N, 1_N, A_1, A_2, A_3, A_4, A_5\}$ where $A_1, A_2, A_3, A_4, A_5, B, C$ are Ns's of X defined as follows:
$A_1 = \left\langle\left(\frac{\mu_a}{0.4},\frac{\mu_b}{0.5},\frac{\mu_c}{0.6}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\gamma_a}{0.5},\frac{\gamma_b}{0.5},\frac{\gamma_c}{0.5}\right)\right\rangle$,
$A_2 = \left\langle\left(\frac{\mu_a}{0.4},\frac{\mu_b}{0.5},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\gamma_a}{0.5},\frac{\gamma_b}{0.5},\frac{\gamma_c}{0.5}\right)\right\rangle$,
$A_3 = \left\langle\left(\frac{\mu_a}{0.5},\frac{\mu_b}{0.5},\frac{\mu_c}{0.5}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\gamma_a}{0.5},\frac{\gamma_b}{0.5},\frac{\gamma_c}{0.5}\right)\right\rangle$,
$A_4 = \left\langle\left(\frac{\mu_a}{0.5},\frac{\mu_b}{0.5},\frac{\mu_c}{0.6}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\gamma_a}{0.5},\frac{\gamma_b}{0.5},\frac{\gamma_c}{0.5}\right)\right\rangle$,
$A_5 = \left\langle\left(\frac{\mu_a}{0.4},\frac{\mu_b}{0.5},\frac{\mu_c}{0.5}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\gamma_a}{0.5},\frac{\gamma_b}{0.5},\frac{\gamma_c}{0.5}\right)\right\rangle$,
$B = \left\langle\left(\frac{\mu_a}{0.5},\frac{\mu_b}{0.5},\frac{\mu_c}{0.4}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\gamma_a}{0.5},\frac{\gamma_b}{0.5},\frac{\gamma_c}{0.5}\right)\right\rangle$,
$C = \left\langle\left(\frac{\mu_a}{0.6},\frac{\mu_b}{0.5},\frac{\mu_c}{0.6}\right),\left(\frac{\sigma_a}{0.5},\frac{\sigma_b}{0.5},\frac{\sigma_c}{0.5}\right),\left(\frac{\gamma_a}{0.5},\frac{\gamma_b}{0.5},\frac{\gamma_c}{0.5}\right)\right\rangle$.
Clearly B and C are nrso sets in (X,τ). Also, $B \neq 0_N$, $C \neq 0_N$, $B \cup C \neq 1_N$ and $B \cap C \neq 0_N$. Hence (X,τ) is NRS-connected, but it is NRS-C_5-disconnected, since A_3 is both nrso and nrsc set.

Proposition 4.5. *Let (X,τ) and (Y,σ) be nts's. Let $f : (X,\tau) \to (Y,\sigma)$ be a $NRSI$ and surjective function. If (X,τ) is NRS-connected, then (Y,σ) is NRS-connected.*

Proof. Assume that (Y,σ) is not NRS-connected. Thus there exists non-null nrso sets $A, B \in Y$, such that $A \cup B = 1_N$ and $A \cap B = 0_N$. Since f is $NRSI$ function, $C = f^{-1}(A)$, $D = f^{-1}(B)$ are nrso sets in (X,τ). From $A \neq 0_N$, we get $C = f^{-1}(A) \neq 0_N$. (If $f^{-1}(A) \neq 0_N$, then $A = f(f^{-1}(A)) = f(0_N) = 0_N$, which is a contradiction.) Similarly we obtain $D = 0_N$. Now, $A \cup B = 1_N$ $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(1_N)$, $C \cup D = 1_N$, $A \cap B = 0_N$ $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(0_N)$ $C \cap D = 0_N$. This implies that $C \cup D = 1_N$ and $C \cap D = 0_N$. Thus (X,τ) is NRS-connected, which is a contradiction to our hypothesis. Hence (Y,σ) is NRS-connected. □

Proposition 4.6. *A nts (X,τ) is NRS-C_5-connected if and only if there exists no non-null nrso sets $A, B \in X$ such that $A = 1_N - B$.*

Proof. Suppose that A and B are nrso sets in X such that $A \neq 0_N$, $B \neq 0_N$, $A = 1 - B$. Since $A = 1 - B$, $1 - B$ is a nrso set and B is a nrsc set. And $A \neq 0_N$ implies $B \neq 1_N$. But this is a contradiction to the fact that (X,τ) is NRS-C_5-connected.

Conversely, let A be both nrso and nrsc in X such that $A \neq 0_N$, $A \neq 1_N$. Now take $B = 1 - A$. In this case B is a nrso set and $A \neq 1_N$. Which implies that $B = 1 - A = 0_N$, which is a contradiction. □

Proposition 4.7. *A nts (X, τ) is NRS-C_5-connected if and only if there exists no non-null Ns A, B in X such that $1_N - A = B$, $B = 1_N - NRSCl(A)$, $A = 1_N - NRSCl(B)$.*

Proof. Assume that there exists a Ns sets A and B such that $A \neq 0_N$, $B \neq 0_N$, $1_N - A = B$, $B = 1_N - NRSCl(A)$ and $A = 1_N - NRSCl(B)$. Since $1_N - NRSCl(A)$ and $1_N - NRSCl(B)$ are nrso sets over X, A and B are nrso sets in X, which is a contradiction.

Conversely, let A be both nrso and nrsc in X such that $A \neq 0_N$, $A \neq 1_N$. Taking $B = 1_N - A$, we obtain a contradiction. □

Definition 4.8. A nts (X, τ) is said to be NRS-strongly connected if there exists no non-null nrsc sets A, B in X such that $A + B \leq 1_N$.

In otherwords, a nts (X, τ) is said to be NRS-strongly connected if there exists no non-null nrsc sets A, B in X such that $A \cap B = 1_N$.

Proposition 4.8. *A nts (X, τ) is NRS-strongly connected if and only if there exists no non-null nrso sets A, B in X such that $A \neq 1_N$, $B \neq 1_N$ and $A + B \geq 1_N$.*

Proof. Necessity: Let A and B are nrso sets in (X, τ) such that $A \neq 1_N$, $B \neq 1_N$ and $A + B \geq 1_N$. If we take $C = 1_N - A$ and $D = 1_N - B$, then C and D become nrsc sets in X and $C \neq 0_N$, $D \neq 0_N$ and $C + D \leq 1_N$. Which is a contradiction. Hence (X, τ) is NRS-strongly connected.

Sufficiency: Let A and B be non-null nrsc sets in (X, τ) such that $A + B \leq 1_N$. If $C = 1_N - A$ and $D = 1_N - B$, then C and D become nrso sets in (X, τ) and $C \neq 1_N$, $D \neq 1_N$ and $C + D \geq 1_N$. Which is a contradiction. Thus there exists no non-null nrso sets A and B in (X, τ) such that $A \neq 1_N$, $B \neq 1_N$ and $A + B \geq 1_N$. □

Proposition 4.9. *Let (X, τ) and (Y, σ) be nts's. Let $f : (X, \tau) \to (Y, \sigma)$ be a $NRSI$ and surjective function. If (X, τ) is NRS-strongly connected, then (Y, σ) is NRS-strongly connected.*

Proof. Suppose that (Y, σ) is not NRS-strongly connected. Then there exists non-null nrsc sets C_1 and C_2 in (Y, σ) such that $C_1 \neq 0_N$, $C_2 \neq 0_N$, $C_1 + C_2 \leq 0_N$. Since f is $NRSI$ function, $f^{-1}(C_1)$, $f^{-1}(C_2)$ are nrsc sets in (X, τ) and $f^{-1}(C_1) \cap f^{-1}(C_2) = 0_N$, $f^{-1}(C_1) \neq 0_N$, $f^{-1}(C_2) \neq 0_N$. (If $f^{-1}(C_1) = 0_N$, then $f(f^{-1}(C_1)) = C_1$ which implies $f(0_N) = C_1$. So $0_N = C_1$ a contradiction.) Hence (X, τ) is NRS-strongly connected, a contradiction to our hypothesis. Thus (Y, σ) is NRS-strongly connected. □

Remark. *NRS-strongly connected does not imply NRS-C_5-connected.*

Example 4.9. In Example 4.7, (X, τ) is NRS-strongly connected, since there is no nrsc sets A_1, A_2, $A_1 + A_2 \leq 1_N$. But (X, τ) is NRS-C_5-disconnected.

Remark. *NRS-C_5-connected does not imply NRS-strongly connected.*

Example 4.10. In Example 4.3, (X, τ) is NRS-C_5-strongly connected, since there is no Ns set A is both nrso and nrsc set. But (X, τ) is not NRS-strongly connected, since there is the nrsc sets A and B, $A + B \leq 1_N$.

Definition 4.11. Let (X, τ) be nts, A, B in X. The non-null Ns sets A and B are said to be

(1) NRS-weakly separated if $NRSCl(A) \leq 1_N - B$ and $NRSCl(B) \leq 1_N - A$.
(2) NRS-q-separated if $NRSCl(A) \cap B = 0_N = A \cap NRSCl(B)$.

Definition 4.12. A nts (X, τ) is said to be NRS-C_W-disconnected if there exists NRS-weakly separated non-null Ns sets A and B in X such that $A \cup B = 1_N$.

Example 4.13. Let $X = \{a, b, c\}$ and $\tau = \{0_N, 1_N, A, B\}$ where A, B are Ns's of X defined as follows:
$A = \big\langle (\frac{\mu_a}{0.0}, \frac{\mu_b}{1.0}, \frac{\mu_c}{0.0}), (\frac{\sigma_a}{0.0}, \frac{\sigma_b}{1.0}, \frac{\sigma_c}{0.0}), (\frac{\gamma_a}{0.0}, \frac{\gamma_b}{1.0}, \frac{\gamma_c}{0.0}) \big\rangle$,
$B = \big\langle (\frac{\mu_a}{1.0}, \frac{\mu_b}{0.0}, \frac{\mu_c}{1.0}), (\frac{\sigma_a}{1.0}, \frac{\sigma_b}{0.0}, \frac{\sigma_c}{1.0}), (\frac{\gamma_a}{1.0}, \frac{\gamma_b}{0.0}, \frac{\gamma_c}{1.0}) \big\rangle$,
Clearly A and B are nrso sets in (X, τ), $NRSCl(A) \leq 1_N - B$, $NRSCl(B) \leq 1_N - A$. Hence A and B are NRS-weakly separated and $A \cup B = 1_N$. Hence (X, τ) is NRS-C_W-disconnected.

Definition 4.14. A nts (X, τ) is said to be NRS-C_Q-disconnected if there exists NRS-*q*-separated non-null Ns sets A and B in X such that $A \cup B = 1_N$.

Example 4.15. In Example 4.13, the Ns sets A and B are nrso sets, $NRSCl(A) = (1_N - B) \cap B = 0_N$ and $NRSCl(B) = (1_N - A) \cap A = 0_N$. Hence A and B are NRS-*q*-separated and $A \cup B = 1_N$. Thus (X, τ) is NRS-C_Q-disconnected.

Remark. *A nts (X, τ) is said to be NRS-C_W-connected if and only if (X, τ) is NRS-C_Q-connected.*

Definition 4.16. Let (X, τ) be a nts and $Y \subseteq X$. Let A^Y is defined as follows $A^Y(x) = \begin{cases} 1_N & \text{if } x \in Y \\ 0_N & \text{if } x \notin Y \end{cases}$. Let $\tau_Y = \{A^Y \cap B : B \text{ is nos}\}$, then the NT τ_Y on Y is called neutrosophic subspace topology and (Y, τ_Y) is called neutrosophic subspace of (X, τ).

Definition 4.17. A neutrosophic subspace (Y, τ_Y) of nts (X, τ) is said to be NRS-open (resp. NRS-closed, NRS-connected) subspace if $A^Y \in NRSO(X)$ (resp. $A^Y \in NRSC(X)$, A^Y is NRS-connected).

Theorem 4.10. *Let (Y, τ_Y) be a NRS-connected subspace of nts (X, τ) such that $C^Y \cap B \in NRSO(X)$. If 1_N has a NRS-separations A and B, then either $C^Y \leq A$ or $C^Y \leq B$.*

Proof. Let $A,\ B$ be NRS-separation on 1_N. By hypothesis, $A \cap C^Y \in NRSO(X)$, $B \cap C^Y \in NRSO(X)$ and $[A \cap C^Y] \cup [B \cap C^Y] = C^Y$. Since C^Y is NRS-connected. Then either $A \cap C^Y = 0_N$ or $B \cap C^Y = 0_N$. Therefore, either $C^Y \leq A$ or $C^Y \leq B$. □

Theorem 4.11. *If (X, τ_2) is a NRS-connected space and τ_1 is Neutrosophic coarser than τ_2, then (X, τ_1) is also a NRS-connected.*

Proof. Let $A,\ B$ in X be NRS-separation on (X, τ_1). Then $A,\ B$ are nrso sets. Since $\tau_1 \leq \tau_2$. Then $A,\ B$ in (X, τ_2) such that $A,\ B$ is NRS-separation on (X, τ_2), which is a contradiction with the NRS-connectedness of (X, τ_2). Hence, (X, τ_1) is NRS-connected. □

Theorem 4.12. *A neutrosophic subspace (Y, τ_Y) of a NRS-disconnected space (X, τ) is NRS-disconnected if $C^Y \cap B \in NRSO(X)$, $\forall B \in NRSO(X)$.*

Proof. Let (Y, τ_Y) be NRS-connected. Since (X, τ) is NRS-disconnected. Then there exists NRS-separation $A,\ B$ on (X, τ). By hypothesis, $A \cap C^Y \in NRSOS(X)$, $B \cap C^Y \in NRSOS(X)$ and $[A \cap C^Y] \cup [B \cap C^Y] = C^Y$, which is a contradiction with the NRS-connectedness of (Y, τ_Y). Therefore (Y, τ_Y) is NRS-disconnected. □

5. CONCLUSIONS

In this paper, we have introduced neutrosophic regular semi compactness and gave basic definition and theorems of the concept. Also, we introduce neutrosophic regular semi connectedness, neutrosophic regular semi strongly connectedness and neutrosophic regular semi-C_5-connectedness. Some interesting properties of these notions are studied.

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R. VIJAYALAKSHMI

DEPARTMENT OF MATHEMATICS, ARIGNAR ANNA GOVERNMENT ARTS COLLEGE, NAMMAKKAL, TAMIL NADU-637 002, INDIA.

Email address: viji.lakshmi80@rediffmail.com

R. R. PRAVEENA

RESEARCH SCHOLAR, DEPARTMENT OF MATHEMATICS, ANNAMALAI UNIVERSITY, ANNAMALAINAGAR, TAMIL NADU-608 002, INDIA.

Email address: praveenaphd24@gmail.com