ANNALS OF COMMUNICATIONS IN MATHEMATICS
Volume 4, Number 3 (2021), 261-277
ISSN: 2582-0818
© http://www.technoskypub.com

GENERALIZED SPHERICAL FUZZY SOFT SETS IN MEDICAL DIAGNOSIS FOR A DECISION MAKING

M. PALANIKUMAR* AND K. ARULMOZHI

ABSTRACT. In the present communication, we introduce the theory of generalized spherical fuzzy soft set and define some operations such as complement, union, intersection, AND and OR. Notably, we tend to showed De Morgan's laws, associate laws and distributive laws that are holds in generalized spherical fuzzy soft set. Also, we advocate an algorithm to solve the decision making problem based on generalized soft set model. We introduce a similarity measure of two generalized spherical fuzzy soft sets and discuss its application in a medical diagnosis problem. Suppose that there are five patients $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4$ and $\mathcal{P}_5$ in a hospital with certain symptoms of dengue hemorrhagic fever. Let the universal set contain only three elements. That is $X = \{x_1$: severe, x_2: mild, x_3 : no$\}$, the set of parameters E is the set of certain symptoms of dengue hemorrhagic fever is represented by $E = \{e_1$: severe abdominal pain, e_2: persistent vomiting, e_3 : rapid breathing, e_4 : bleeding gums, e_5: restlessness and blood in vomit$\}$. An illustrative examples are mentioned to show that they can be successfully used to solve problems with uncertainties.

1. INTRODUCTION

After the demonstration of ordinary fuzzy sets by Zadeh, they have been very popular in almost all branches of science [28] and suggests that decision makers are to be solving uncertain problems by considering membership degree. The concept of intuitionistic fuzzy set is introduced by Atanassov and is characterized by a degree of membership and non-membership satisfying the condition that sum of its membership degree and non membership degree is not exceeding one [2]. However, we may interact a problem in decision making events where the sum of the degree of membership and non-membership of a particular attribute is exceeding one. So Yager [26] was introduced by the concept of Pythagorean fuzzy sets. It has been to extended the intuitionistic fuzzy sets and characterized by the condition that square sum of its degree of membership and non membership is not exceeding one. The concept of picture fuzzy sets [3], which are direct extensions of the fuzzy sets and the intuitionistic fuzzy sets. In 2015, picture fuzzy sets were developed by Cuong, picture fuzzy sets based models may be adequate in situations when we face

2010 *Mathematics Subject Classification.* 03E05, 06D72.
Key words and phrases. generalized spherical fuzzy soft set; spherical fuzzy soft set; decision making problem.
Received: July 25, 2021. Accepted: December 22, 2021. Published: December 31, 2021.
*Corresponding author.

human opinions involving more answers of types: yes, abstain, no, and refusal. Voting can be a good example of such a situation as the human voters may be divided into four groups of those who: vote for, abstain, vote against, refusal of the voting. These sets let decision makers use a larger area for assigning membership, non-membership, and hesitancy degrees. In 2018, Spherical fuzzy sets were introduced by Kahraman and Gundogdu as an extension of Pythagorean, neutrosophic and picture fuzzy sets. The idea behind spherical fuzzy set is to let decision makers to generalize other extensions of fuzzy sets by defining a membership function on a spherical surface and independently assign the parameters of that membership function with a larger domain. Shahzaib Ashraf et al. was discussed by spherical fuzzy sets which is an advanced tool of the fuzzy sets, intuitionistic fuzzy sets and picture fuzzy sets.

In 2018, Garg et al. algorithm for T-spherical fuzzy multi attribute decision making based on improved interactive aggregation operators. Ashraf and Abdullah proposed spherical aggregation operators and applied in multi attribute group decision making. Liu et al. extended the generalized Maclaurin symmetric mean (GMSM) operator to T-spherical fuzzy environment and proposed the T-spherical fuzzy GMSM operator (T-SFGMSM) and the T-spherical fuzzy weighted GMSM operator(T-SFWGMSM). In 2019, Quek at al. developed some new operational laws for T-spherical fuzzy sets, and based on these new operations, proposed two types of Einstein aggregation operators, namely the Einstein interactive averaging aggregation operators and the Einstein interactive geometric aggregation operators under T-spherical fuzzy environment. In 2019, Liu et al. proposed Muirhead mean(MM) operator and power average operator, the spherical fuzzy power Muirhead mean(SFPMM) operator, weighted SFPMM operator, spherical fuzzy power dual Muirhead mean(SFPDMM) operator, weighted SFPDMM operator and discussed their anticipated properties under T-spherical fuzzy environment. In 2019, Gundogdu and Kahraman introduced spherical fuzzy sets, their operational laws, and spherical fuzzy TOPSIS method. An extension of WASPAS with spherical fuzzy sets, VIKOR method using spherical fuzzy sets and correlation coefficients were presented by Gundogdu and Kahraman.

Molodtsov [11] proposed the theory of soft sets. In comparison with other uncertain theories, soft sets more accurately reflect the objectivity and complexity of decision making during actual situations. Moreover, the combination of soft sets with other mathematical models is also a critical research area. Maji et al. proposed by the concept of fuzzy soft set [8] and intuitionistic fuzzy soft set [9]. These two theories are applied to solve various decision making problems. Yong Yang et al. was discussed by picture fuzzy soft set [27]. In recent years, Peng et al [19] has extended fuzzy soft set to Pythagorean fuzzy soft set. This model solved a class of multi attribute decision making consists sum of the degree of membership and non membership value is exceeding one but the sum of the squares is equal or not exceeding one. Pinaki Majumdara et al. discussed generalized fuzzy soft sets [10]. In our generalization of spherical fuzzy soft set, a degree is attached with the parameterization of spherical fuzzy sets while defining an generalized spherical fuzzy soft set. The purpose of this paper is to extend the concept of generalized interval valued fuzzy soft set to parameterization of generalized spherical fuzzy set using generalized soft set model. We shall further establish a similarity measure based on this generalized soft set model. Relations on generalized spherical fuzzy soft sets are defined and their properties are studied and as an application a decision making problem is solved.

2. PRELIMINARIES

Definition 2.1. [7] Let X be a non-empty set, spherical set A in X is an object having the following form : $\widehat{A} = \{u, \vartheta_A(x), \varpi_A(x), \tau_A(x) | x \in X\}$, where $\vartheta_A(x)$, $\varpi_A(x)$ $\tau_A(x)$ represents the degree of positive membership, degree of neutral membership and degree of negative membership of A respectively. The mapping $\vartheta_A, \varpi_A, \tau_A : X \to [0,1]$ and $0 \leq (\vartheta_A(x))^2 + (\varpi_A(x))^2 + (\tau_A(x))^2 \leq 1$. The degree of refusal is determined as $r_A(x) = \left[\sqrt{1-(\vartheta_A(x))^2-(\varpi_A(x))^2-(\tau_A(x))^2}\right]$. Since $\widehat{A} = \langle \vartheta_A, \varpi_A, \tau_A\rangle$ is called a spherical fuzzy number(SFN).

Definition 2.2. Given that $\widehat{\beta_1} = \langle \vartheta_{\beta_1}, \varpi_{\beta_1}, \tau_{\beta_1}\rangle$, $\widehat{\beta_2} = \langle \vartheta_{\beta_2}, \varpi_{\beta_2}, \tau_{\beta_2}\rangle$ and $\widehat{\beta_3} = \langle \vartheta_{\beta_3}, \varpi_{\beta_3}, \tau_{\beta_3}\rangle$ are any three SFNs over (X,E), then the following properties are holds:

(i) $\widehat{\beta_1^c} = \langle \tau_{\beta_1}, \varpi_{\beta_1}, \vartheta_{\beta_1}\rangle$

(ii) $\widehat{\beta_1} \sqcup \widehat{\beta_2} = \Big\langle \max(\vartheta_{\beta_1}, \vartheta_{\beta_2}), \min(\varpi_{\beta_1}, \varpi_{\beta_2}), \min(\tau_{\beta_1}, \tau_{\beta_2})\Big\rangle$

(iii) $\widehat{\beta_1} \sqcap \widehat{\beta_2} = \Big\langle \min(\vartheta_{\beta_1}, \vartheta_{\beta_2}), \min(\varpi_{\beta_1}, \varpi_{\beta_2}), \max(\tau_{\beta_1}, \tau_{\beta_2})\Big\rangle$

(iv) $\widehat{\beta_1} \leq \widehat{\beta_2}$ iff $\vartheta_{\beta_1} \leq \vartheta_{\beta_2}$ and $\varpi_{\beta_1} \leq \varpi_{\beta_2}$ and $\tau_{\beta_1} \geq \tau_{\beta_2}$

(v) $\widehat{\beta_1} = \widehat{\beta_2}$ iff $\vartheta_{\beta_1} = \vartheta_{\beta_2}$ and $\varpi_{\beta_1} = \varpi_{\beta_2}$ and $\tau_{\beta_1} = \tau_{\beta_2}$.

Definition 2.3. Let X be a non-empty set of the universe and E be a set of parameter. The pair $(\widehat{\mathcal{U}}, A)$ is called a spherical soft set on X if $A \sqsubseteq E$ and $\widehat{\mathcal{U}} : A \to \widehat{S\mathcal{U}(X)}$, where $\widehat{S\mathcal{U}(X)}$ is a parameterized family of subsets of the universe X.

Definition 2.4. [10] Let $X = \{x_1, x_2, ..., x_n\}$ be a non-empty set of the universe and $E = \{e_1, e_2, ..., e_m\}$ be a set of parameter. The pair (X, E) is a soft universe. Consider the mapping $\mathcal{U} : E \to I^X$ and ξ be a fuzzy subset of E, ie. $\xi : E \to I = [0,1]$, where I^X is the collection of all fuzzy subsets of X. Let $\mathcal{U}_\xi : E \to I^X \times I$ be a function defined as $\mathcal{U}_\xi(e) = (\mathcal{U}(e)(x), \xi(e)), \forall x \in X$. Then $\mathcal{U}_\xi$ is called a generalized fuzzy soft set (GFSS) on (X, E). Here for each parameter e_i, $\mathcal{U}_\xi(e_i) = (\mathcal{U}(e_i)(x), \xi(e_i))$ indicates not only the degree of belongingness of the elements of X in $\mathcal{U}(e_i)$ but also the degree of possibility of such belongingness which is represented by $\xi(e_i)$. So we can write $\mathcal{U}_\xi(e_i)$ as follows: $\mathcal{U}_\xi(e_i) = \left(\left\{\frac{x_1}{\mathcal{U}(e_i)(x_1)}, \frac{x_2}{\mathcal{U}(e_i)(x_2)}, ..., \frac{x_n}{\mathcal{U}(e_i)(x_n)}\right\}, \xi(e_i)\right)$, where $\mathcal{U}(e_i)(x_1)$, $\mathcal{U}(e_i)(x_2)$,...,$\mathcal{U}(e_i)(x_n)$ are the degrees of belongingness and $\xi(e_i)$ is the degree of possibility of such belongingness.

Definition 2.5. [1] Let $X = \{x_1, x_2, ..., x_n\}$ be a non-empty set of the universe and $E = \{e_1, e_2, ..., e_m\}$ be a set of parameter. The pair (X, E) is a soft universe. Consider the mapping $\mathcal{U} : E \to \mathcal{U}(X)$ and ξ be a fuzzy subset of E, ie. $\xi : E \to \mathcal{U}(X)$. Let $\mathcal{U}_\xi : E \to \mathcal{U}(X) \times \mathcal{U}(X)$ be a function defined as $\mathcal{U}_\xi(e) = (\mathcal{U}(e)(x), \xi(e)(x)), \forall x \in X$. Then $\mathcal{U}_\xi$ is called a possibility fuzzy soft set (PFSS) on (X, E). Here for each parameter e_i, $\mathcal{U}_\xi(e_i) = (\mathcal{U}(e_i)(x), \xi(e_i)(x))$ indicates not only the degree of belongingness of the elements of X in $\mathcal{U}(e_i)$ but also the degree of possibility of such belongingness which is represented by $\xi(e_i)$. So we can write $\mathcal{U}_\xi(e_i)$ as follows:
$\mathcal{U}_\xi(e_i) = \left\{ \left(\frac{x_1}{\mathcal{U}(e_i)(x_1)}, \xi(e_i)(x_1)\right), \left(\frac{x_2}{\mathcal{U}(e_i)(x_2)}, \xi(e_i)(x_2)\right), ..., \left(\frac{x_n}{\mathcal{U}(e_i)(x_n)}, \xi(e_i)(x_n)\right) \right\}$.

3. GENERALIZED SPHERICAL FUZZY SOFT SETS (GSFSS)

Definition 3.1. Let $X = \{x_1, x_2, ..., x_n\}$ be a non-empty set of the universe and $E = \{e_1, e_2, ..., e_m\}$ be a set of parameter. The pair (X, E) is called a soft universe. Suppose that $\overbrace{\mathcal{U}} : E \to \overbrace{S\mathcal{U}(X)}$ and p is a spherical fuzzy subset of E. That is $p : E \to \overbrace{[0,1]}$, where $\overbrace{S\mathcal{U}(X)}$ denotes the collection of all spherical subsets of X. If $\overbrace{\mathcal{U}_p} : E \to \overbrace{S\mathcal{U}(X)} \times \overbrace{[0,1]}$ is a function defined as $\overbrace{\mathcal{U}_p(e)} = \left(\overbrace{\mathcal{U}(e)(x)}, \overbrace{p(e)}\right), x \in X$, then $\overbrace{\mathcal{U}_p}$ is a GSFSS on (X, E). For each parameter e, $\overbrace{\mathcal{U}_p(e_i)} = (\Big\{\frac{x_1}{(\vartheta_{\mathcal{U}(e)}(x_1), \varpi_{\mathcal{U}(e)}(x_1), \tau_{\mathcal{U}(e)}(x_1))}, ..., \frac{x_n}{(\vartheta_{\mathcal{U}(e)}(x_n), \varpi_{\mathcal{U}(e)}(x_n), \tau_{\mathcal{U}(e)}(x_n))}\Big\}, (p_1(e_i), p_2(e_i), p_3(e_i)))$.

To demonstrate the Definition 3.1, we provide a numerical example as follows:

Example 3.2. Let $X = \{x_1, x_2, x_3\}$ be a set of three heart patients with symptoms, $E = \{e_1\text{: hyper tension}, \ e_2\text{: highly blood pressure}, \ e_3\text{: weight loss}\}$ is a set of parameters. Suppose that $\overbrace{\mathcal{U}_p} : E \to \overbrace{S\mathcal{U}(X)} \times \overbrace{[0,1]}$ is given by

$$\overbrace{\mathcal{U}_p(e_1)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.50,0.15,0.65)} \\ \frac{x_2}{(0.60,0.20,0.50)} \\ \frac{x_3}{(0.40,0.30,0.55)} \end{array} \right\}, (0.55, 0.50, 0.20) \right);$$

$$\overbrace{\mathcal{U}_p(e_2)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.40,0.20,0.65)} \\ \frac{x_2}{(0.50,0.35,0.55)} \\ \frac{x_3}{(0.45,0.25,0.65)} \end{array} \right\}, (0.35, 0.25, 0.45) \right);$$

$$\overbrace{\mathcal{U}_p(e_3)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.15,0.40,0.65)} \\ \frac{x_2}{(0.25,0.50,0.45)} \\ \frac{x_3}{(0.15,0.30,0.55)} \end{array} \right\}, (0.45, 0.35, 0.45) \right)$$

Definition 3.3. Let X be a non-empty set of the universe and E be a set of parameter. Suppose that $\overbrace{\mathcal{U}_p}$ and $\overbrace{\mathcal{V}_q}$ are two GSFSSs on (X, E). Now $\overbrace{\mathcal{U}_p}$ is a generalized spherical fuzzy soft subset of $\overbrace{\mathcal{V}_q}$ (denoted by $\overbrace{\mathcal{U}_p} \sqsubseteq \overbrace{\mathcal{V}_q}$) if and only if

(i) $\overbrace{\mathcal{U}(e)(x)} \sqsubseteq \overbrace{\mathcal{V}(e)(x)}$ if $\vartheta_{\mathcal{U}(e)}(x) \leq \vartheta_{\mathcal{V}(e)}(x)$, $\varpi_{\mathcal{U}(e)}(x) \leq \varpi_{\mathcal{V}(e)}(x)$, $\tau_{\mathcal{U}(e)}(x) \geq \tau_{\mathcal{V}(e)}(x)$,

(ii) $p(e)(x) \leq q(e)(x), \forall\, e \in E$ and $\forall\, x \in X$.

To illustrate the above Definition, we provide a numerical example as follows:

Example 3.4. Consider the GSFSS $\overbrace{\mathcal{U}_p}$ in Example 3.2. Let $\overbrace{\mathcal{V}_q}$ be another GSFSS defined as:

$$\overbrace{\mathcal{V}_q(e_1)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.60,0.35,0.35)} \\ \frac{x_2}{(0.75,0.35,0.15)} \\ \frac{x_3}{(0.65,0.55,0.25)} \end{array} \right\}, (0.70, 0.60, 0.50) \right);$$

$$\overbrace{\mathcal{V}_q(e_2)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.55,0.45,0.20)} \\ \frac{x_2}{(0.65,0.65,0.10)} \\ \frac{x_3}{(0.75,0.45,0.25)} \end{array} \right\}, (0.70, 0.55, 0.60) \right);$$

$$\widehat{\mathcal{V}_q(e_3)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.45,0.60,0.15)} \\ \frac{x_2}{(0.55,0.65,0.15)} \\ \frac{x_3}{(0.55,0.50,0.20)} \end{array} \right\}, (0.65, 0.55, 0.60) \right)$$

Definition 3.5. Let X be a non-empty set of the universe and E be a set of parameter. Suppose that $\widehat{\mathcal{U}_p}$ and $\widehat{\mathcal{V}_q}$ are two GSFSSs on (X, E). These two GSFSSs are equal (denoted by $\widehat{\mathcal{U}_p} = \widehat{\mathcal{V}_q}$) if and only if $\widehat{\mathcal{U}_p} \sqsubseteq \widehat{\mathcal{V}_q}$ and $\widehat{\mathcal{U}_p} \sqsupseteq \widehat{\mathcal{V}_q}$.

Definition 3.6. Let X be a non-empty set of the universe and E be a set of parameter. Let $\widehat{\mathcal{V}_q}$ be a GSFSS on (X, E). The complement of $\widehat{\mathcal{V}_q}$ is denoted by $\widehat{\mathcal{V}_q^c}$ and is defined by $\widehat{\mathcal{V}_q^c} = \left(\widehat{\mathcal{V}^c(e)(x)}, \widehat{q^c(e)}\right)$, where $\widehat{\mathcal{V}^c(e)(x)} = \left\{\frac{x}{\left(\tau_{\mathcal{V}(e)}(x), \varpi_{\mathcal{V}(e)}(x), \vartheta_{\mathcal{V}(e)}(x)\right)}\right\}$ and $\widehat{q^c(e)} = (q_3(e), q_2(e), q_1(e))$. Also true that $\left(\widehat{\mathcal{V}_q^c}\right)^c = \widehat{\mathcal{V}_q}$

Example 3.7. By the Example 3.4,

$$\widehat{\mathcal{V}_q^c(e_1)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.35,0.35,0.60)} \\ \frac{x_2}{(0.15,0.35,0.75)} \\ \frac{x_3}{(0.25,0.55,0.65)} \end{array} \right\}, (0.50, 0.60, 0.70) \right);$$

$$\widehat{\mathcal{V}_q^c(e_2)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.20,0.45,0.55)} \\ \frac{x_2}{(0.10,0.65,0.65)} \\ \frac{x_3}{(0.25,0.45,0.75)} \end{array} \right\}, (0.60, 0.55, 0.70) \right);$$

$$\widehat{\mathcal{V}_q^c(e_3)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.15,0.60,0.45)} \\ \frac{x_2}{(0.15,0.65,0.55)} \\ \frac{x_3}{(0.20,0.50,0.55)} \end{array} \right\}, (0.60, 0.55, 0.65) \right)$$

Definition 3.8. Let X be a non-empty set of the universe and E be a set of parameter. Let $\widehat{\mathcal{U}_p}$ and $\widehat{\mathcal{V}_q}$ be two GSFSSs on (X, E). The union and intersection of $\widehat{\mathcal{U}_p}$ and $\widehat{\mathcal{V}_q}$ over (X, E) are denoted by $\widehat{\mathcal{U}_p} \sqcup \widehat{\mathcal{V}_q}$ and $\widehat{\mathcal{U}_p} \sqcap \widehat{\mathcal{V}_q}$ respectively and is defined by $\widehat{J_j} : E \to \widehat{S\mathcal{U}(X)} \times \widehat{[0,1]}$, $\widehat{I_i} : E \to \widehat{S\mathcal{U}(X)} \times \widehat{[0,1]}$ such that $\widehat{J_j(e)(x)} = \left(\widehat{J(e)(x)}, \widehat{j(e)}\right)$, $\widehat{I_i(e)(x)} = \left(\widehat{I(e)(x)}, \widehat{i(e)}\right)$, where $\widehat{J(e)(x)} = \widehat{\mathcal{U}(e)(x)} \sqcup \widehat{\mathcal{V}(e)(x)}$, $\widehat{j(e)} = \widehat{p(e)} \sqcup \widehat{q(e)}$, $\widehat{I(e)(x)} = \widehat{\mathcal{U}(e)(x)} \sqcap \widehat{\mathcal{V}(e)(x)}$ and $\widehat{i(e)} = \widehat{p(e)} \sqcap \widehat{q(e)}$, for all $x \in X$.

Example 3.9. Let $\widehat{\mathcal{U}_p}$ and $\widehat{\mathcal{V}_q}$ be the two GSFSSs on (X, E) is defined by

$$\widehat{\mathcal{U}_p(e_1)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.5,0.4,0.6)} \\ \frac{x_2}{(0.5,0.6,0.4)} \\ \frac{x_3}{(0.7,0.5,0.3)} \end{array} \right\}, (0.4, 0.6, 0.8) \right);$$

$$\widehat{\mathcal{U}_p(e_2)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.6,0.1,0.7)} \\ \frac{x_2}{(0.5,0.2,0.6)} \\ \frac{x_3}{(0.7,0.3,0.4)} \end{array} \right\}, (0.4, 0.6, 0.5) \right);$$

and

$$\overbrace{\mathcal{V}_q(e_1)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.4,0.3,0.5)} \\ \frac{x_2}{(0.3,0.1,0.8)} \\ \frac{x_3}{(0.6,0.4,0.3)} \end{array} \right\}, (0.2, 0.3, 0.4) \right);$$

$$\overbrace{\mathcal{V}_q(e_2)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.2,0.3,0.6)} \\ \frac{x_2}{(0.3,0.1,0.6)} \\ \frac{x_3}{(0.5,0.4,0.7)} \end{array} \right\}, (0.4, 0.5, 0.7) \right);$$

Thus, $\overbrace{\mathcal{U}_p} \sqcup \overbrace{\mathcal{V}_q}$ is obtained as:

$$\overbrace{\mathcal{U}_p(e_1)} \sqcup \overbrace{\mathcal{V}_q(e_1)} = \left(\left\{ \begin{array}{c} \frac{x_1}{0.5,0.3,0.5} \\ \frac{x_2}{(0.5,0.1,0.4)} \\ \frac{x_3}{(0.7,0.4,0.3)} \end{array} \right\}, (0.4, 0.6, 0.8) \right);$$

$$\overbrace{\mathcal{U}_p(e_2)} \sqcup \overbrace{\mathcal{V}_q(e_2)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.6,0.1,0.6)} \\ \frac{x_2}{(0.5,0.1,0.6)} \\ \frac{x_3}{(0.7,0.3,0.4)} \end{array} \right\}, (0.4, 0.6, 0.7) \right);$$

Thus, $\overbrace{\mathcal{U}_p} \sqcap \overbrace{\mathcal{V}_q}$ is obtained as:

$$\overbrace{\mathcal{U}_p(e_1)} \sqcap \overbrace{\mathcal{V}_q(e_1)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.4,0.3,0.6)} \\ \frac{x_2}{(0.3,0.1,0.8)} \\ \frac{x_3}{(0.6,0.4,0.3)} \end{array} \right\}, (0.2, 0.3, 0.4) \right);$$

$$\overbrace{\mathcal{U}_p(e_2)} \sqcap \overbrace{\mathcal{V}_q(e_2)} = \left(\left\{ \begin{array}{c} \frac{x_1}{(0.2,0.1,0.7)} \\ \frac{x_2}{(0.3,0.1,0.6)} \\ \frac{x_3}{(0.5,0.3,0.7)} \end{array} \right\}, (0.4, 0.5, 0.5) \right);$$

Definition 3.10. A GSFSS $\overbrace{\emptyset_\theta(e)(x)} = \left(\overbrace{\emptyset(e)(x)}, \theta(e)(x)\right)$ is said to a generalized null spherical fuzzy soft set $\overbrace{\emptyset_\theta} : E \to \overbrace{S\mathcal{U}(X)} \times \overbrace{[0,1]}$, where $\overbrace{\emptyset(e)(x)} = (0,0,1)$ and $\overbrace{\theta(e)(x)} = \overline{0},\ \forall\ x \in X$.

Definition 3.11. A GSFSS $\overbrace{\Omega_\Lambda(e)(x)} = \left(\overbrace{\Omega(e)(x)}, \Lambda(e)(x)\right)$ is said to a generalized absolute spherical fuzzy soft set $\overbrace{\Omega_\Lambda} : E \to \overbrace{S\mathcal{U}(X)} \times \overbrace{[0,1]}$, where $\overbrace{\Omega(e)(x)} = (1,1,0)$, $\overbrace{\Lambda(e)(x)} = \overline{1},\ \forall\ x \in X$.

Theorem 3.1. *Let $\overbrace{\mathcal{U}_p}$ be a GSFSS on (X,E). Then the following properties are holds:*
(i) $\overbrace{\mathcal{U}_p} = \overbrace{\mathcal{U}_p} \sqcup \overbrace{\mathcal{U}_p}$, $\overbrace{\mathcal{U}_p} = \overbrace{\mathcal{U}_p} \sqcap \overbrace{\mathcal{U}_p}$
(ii) $\overbrace{\mathcal{U}_p} \sqsubseteq \overbrace{\mathcal{U}_p} \sqcup \overbrace{\mathcal{U}_p}$, $\overbrace{\mathcal{U}_p} \sqsubseteq \overbrace{\mathcal{U}_p} \sqcap \overbrace{\mathcal{U}_p}$
(iii) $\overbrace{\mathcal{U}_p} \sqcup \overbrace{\emptyset_\theta} = \overbrace{\mathcal{U}_p}$, $\overbrace{\mathcal{U}_p} \sqcap \overbrace{\emptyset_\theta} = \overbrace{\emptyset_\theta}$
(iv) $\overbrace{\mathcal{U}_p} \sqcup \overbrace{\Omega_\Lambda} = \overbrace{\Omega_\Lambda}$, $\overbrace{\mathcal{U}_p} \sqcap \overbrace{\Omega_\Lambda} = \overbrace{\mathcal{U}_p}$.

Remark. *Let $\overbrace{\mathcal{U}_p}$ be a GSFSS on (X,E). If $\overbrace{\mathcal{U}_p} \neq \overbrace{\Omega_\Lambda}$ or $\overbrace{\mathcal{U}_p} \neq \overbrace{\emptyset_\theta}$, then $\overbrace{\mathcal{U}_p} \sqcup \overbrace{\mathcal{U}_p^c} \neq \overbrace{\Omega_\Lambda}$ and $\overbrace{\mathcal{U}_p} \sqcap \overbrace{\mathcal{U}_p^c} \neq \overbrace{\emptyset_\theta}$.*

Theorem 3.2. *Let $\widehat{\mathcal{U}_p}$ and $\widehat{\mathcal{V}_q}$ are any two GSFSSs over (X, E). Then the commutative and De Morgan's laws of GSFSSs are holds:*
(1) $\widehat{\mathcal{U}_p} \sqcup \widehat{\mathcal{V}_q} = \widehat{\mathcal{V}_q} \sqcup \widehat{\mathcal{U}_p}$
(2) $\widehat{\mathcal{U}_p} \sqcap \widehat{\mathcal{V}_q} = \widehat{\mathcal{V}_q} \sqcap \widehat{\mathcal{U}_p}$
(3) $\left(\widehat{\mathcal{U}_p} \sqcup \widehat{\mathcal{V}_q}\right)^c = \widehat{\mathcal{U}_p^c} \sqcap \widehat{\mathcal{V}_q^c}$
(4) $\left(\widehat{\mathcal{U}_p} \sqcap \widehat{\mathcal{V}_q}\right)^c = \widehat{\mathcal{U}_p^c} \sqcup \widehat{\mathcal{V}_q^c}$.

Proof. The proof follows from Definition 3.6 and 3.8.

Theorem 3.3. *Let $\widehat{\mathcal{U}_p}$, $\widehat{\mathcal{V}_q}$ and $\widehat{\mathcal{W}_r}$ are three GSFSSs over (X, E). Then the associative laws and distributive laws of GSFSSs are holds:*
(1) $\widehat{\mathcal{U}_p} \sqcup (\widehat{\mathcal{V}_q} \sqcup \widehat{\mathcal{W}_r}) = (\widehat{\mathcal{U}_p} \sqcup \widehat{\mathcal{V}_q}) \sqcup \widehat{\mathcal{W}_r}$
(2) $\widehat{\mathcal{U}_p} \sqcap (\widehat{\mathcal{V}_q} \sqcap \widehat{\mathcal{W}_r}) = (\widehat{\mathcal{U}_p} \sqcap \widehat{\mathcal{V}_q}) \sqcap \widehat{\mathcal{W}_r}$
(3) $\widehat{\mathcal{U}_p} \sqcup (\widehat{\mathcal{V}_q} \sqcap \widehat{\mathcal{W}_r}) = (\widehat{\mathcal{U}_p} \sqcup \widehat{\mathcal{V}_q}) \sqcap (\widehat{\mathcal{U}_p} \sqcup \widehat{\mathcal{W}_r})$
(4) $\widehat{\mathcal{U}_p} \sqcap (\widehat{\mathcal{V}_q} \sqcup \widehat{\mathcal{W}_r}) = (\widehat{\mathcal{U}_p} \sqcap \widehat{\mathcal{V}_q}) \sqcup (\widehat{\mathcal{U}_p} \sqcap \widehat{\mathcal{W}_r})$.
(5) $(\widehat{\mathcal{U}_p} \sqcup \widehat{\mathcal{V}_q}) \sqcap \widehat{\mathcal{U}_p} = \widehat{\mathcal{U}_p}$
(6) $(\widehat{\mathcal{U}_p} \sqcap \widehat{\mathcal{V}_q}) \sqcup \widehat{\mathcal{U}_p} = \widehat{\mathcal{U}_p}$.

Proof. The proof follows from Definition 3.6 and 3.8.

Definition 3.12. Let $(\widehat{\mathcal{U}_p}, A)$ and $(\widehat{\mathcal{V}_q}, B)$ be two GSFSSs on (X, E). Then the operation "$(\widehat{\mathcal{U}_p}, A)$ AND $(\widehat{\mathcal{V}_q}, B)$" is denoted by $(\widehat{\mathcal{U}_p}, A) \wedge (\widehat{\mathcal{V}_q}, B)$ and is defined by $(\widehat{\mathcal{U}_p}, A) \wedge (\widehat{\mathcal{V}_q}, B) = (\widehat{\mathcal{W}_r}, A \times B)$, where $\widehat{\mathcal{W}_r(\alpha, \beta)} = \left(\widehat{\mathcal{W}(\alpha, \beta)(x)}, \widehat{r(\alpha, \beta)}\right)$ such that $\widehat{\mathcal{W}(\alpha, \beta)} = \widehat{\mathcal{U}(\alpha)} \sqcap \widehat{\mathcal{V}(\beta)}$ and $\widehat{r(\alpha, \beta)} = \widehat{p(\alpha)} \sqcap \widehat{q(\beta)}$, for all $(\alpha, \beta) \in A \times B$.

Definition 3.13. Let $(\widehat{\mathcal{U}_p}, A)$ and $(\widehat{\mathcal{V}_q}, B)$ be two GSFSSs on (X, E). Then the operation "$(\widehat{\mathcal{U}_p}, A)$ OR $(\widehat{\mathcal{V}_q}, B)$" is denoted by $(\widehat{\mathcal{U}_p}, A) \vee (\widehat{\mathcal{V}_q}, B)$ and is defined by $(\widehat{\mathcal{U}_p}, A) \vee (\widehat{\mathcal{V}_q}, B) = (\widehat{\mathcal{W}_r}, A \times B)$, where $\widehat{\mathcal{W}_r(\alpha, \beta)} = \left(\widehat{\mathcal{W}(\alpha, \beta)(x)}, \widehat{r(\alpha, \beta)}\right)$ such that $\widehat{\mathcal{W}(\alpha, \beta)} = \widehat{\mathcal{U}(\alpha)} \sqcup \widehat{\mathcal{V}(\beta)}$ and $\widehat{r(\alpha, \beta)} = \widehat{p(\alpha)} \sqcup \widehat{q(\beta)}$, for all $(\alpha, \beta) \in A \times B$.

Theorem 3.4. *Let $(\widehat{\mathcal{U}_p}, A)$ and $(\widehat{\mathcal{V}_q}, B)$ be two GSFSSs on (X, E), then*
(i) $\left((\widehat{\mathcal{U}_p}, A) \wedge (\widehat{\mathcal{V}_q}, B)\right)^c = \left(\widehat{\mathcal{U}_p}, A\right)^c \vee \left(\widehat{\mathcal{V}_q}, B\right)^c$
(ii) $\left((\widehat{\mathcal{U}_p}, A) \vee (\widehat{\mathcal{V}_q}, B)\right)^c = \left(\widehat{\mathcal{U}_p}, A\right)^c \wedge \left(\widehat{\mathcal{V}_q}, B\right)^c$.

Proof. (i) Suppose that $(\widehat{\mathcal{U}_p}, A) \wedge (\widehat{\mathcal{V}_q}, B) = (\widehat{\mathcal{W}_r}, A \times B)$ and $\left((\widehat{\mathcal{U}_p}, A) \wedge (\widehat{\mathcal{V}_q}, B)\right)^c = (\widehat{\mathcal{W}_r^c}, A \times B)$. Now, $\widehat{\mathcal{W}_r^c(\alpha, \beta)} = \left(\widehat{\mathcal{W}^c(\alpha, \beta)(x)}, \widehat{r^c(\alpha, \beta)}\right)$, for all $(\alpha, \beta) \in A \times B$.

By Theorem 3.2 and Definition 3.12, $\widehat{\mathcal{W}^c(\alpha,\beta)} = \left(\widehat{\mathcal{U}(\alpha)} \sqcap \widehat{\mathcal{V}(\beta)}\right)^c = \widehat{\mathcal{U}^c(\alpha)} \sqcup \widehat{\mathcal{V}^c(\beta)}$ and $\widehat{r^c(\alpha,\beta)} = \left(\widehat{p(\alpha)} \sqcap \widehat{q(\beta)}\right)^c = \widehat{p^c(\alpha)} \sqcup \widehat{q^c(\beta)}$. Also, $\left(\widehat{\mathcal{U}_p}, A\right)^c \vee \left(\widehat{\mathcal{V}_q}, B\right)^c = (\widehat{\Lambda_o}, A \times B)$, where $\widehat{\Lambda_o(\alpha,\beta)} = \left(\widehat{\Lambda(\alpha,\beta)(x)}, \widehat{o(\alpha,\beta)}\right)$ such that $\widehat{\Lambda(\alpha,\beta)} = \widehat{\mathcal{U}^c(\alpha)} \sqcup \widehat{\mathcal{V}^c(\beta)}$ and $\widehat{o(\alpha,\beta)} = \widehat{p^c(\alpha)} \sqcup \widehat{q^c(\beta)}$ for all $(\alpha,\beta) \in A \times B$. Thus, $\widehat{\mathcal{W}_r^c} = \widehat{\Lambda_o}$. Hence $\left((\widehat{\mathcal{U}_p}, A) \wedge (\widehat{\mathcal{V}_q}, B)\right)^c = \left(\widehat{\mathcal{U}_p}, A\right)^c \vee \left(\widehat{\mathcal{V}_q}, B\right)^c$.

(ii) Suppose that $(\widehat{\mathcal{U}_p}, A) \vee (\widehat{\mathcal{V}_q}, B) = (\widehat{\mathcal{W}_r}, A \times B)$ and $\left((\widehat{\mathcal{U}_p}, A) \vee (\widehat{\mathcal{V}_q}, B)\right)^c = (\widehat{\mathcal{W}_r^c}, A \times B)$. Now, $\widehat{\mathcal{W}_r^c(\alpha,\beta)} = \left(\widehat{\mathcal{W}^c(\alpha,\beta)(x)}, \widehat{r^c(\alpha,\beta)}\right)$, for all $(\alpha,\beta) \in A \times B$. By Theorem 3.2 and Definition 3.12, $\widehat{\mathcal{W}^c(\alpha,\beta)} = \left(\widehat{\mathcal{U}(\alpha)} \sqcup \widehat{\mathcal{V}(\beta)}\right)^c = \widehat{\mathcal{U}^c(\alpha)} \sqcap \widehat{\mathcal{V}^c(\beta)}$ and $\widehat{r^c(\alpha,\beta)} = \left(\widehat{p(\alpha)} \sqcup \widehat{q(\beta)}\right)^c = \widehat{p^c(\alpha)} \sqcap \widehat{q^c(\beta)}$. Also, $\left(\widehat{\mathcal{U}_p}, A\right)^c \wedge \left(\widehat{\mathcal{V}_q}, B\right)^c = (\widehat{\Lambda_o}, A \times B)$, where $\widehat{\Lambda_o(\alpha,\beta)} = \left(\widehat{\Lambda(\alpha,\beta)(x)}, \widehat{o(\alpha,\beta)}\right)$ such that $\widehat{\Lambda(\alpha,\beta)} = \widehat{\mathcal{U}^c(\alpha)} \sqcap \widehat{\mathcal{V}^c(\beta)}$ and $\widehat{o(\alpha,\beta)} = \widehat{p^c(\alpha)} \sqcap \widehat{q^c(\beta)}$ for all $(\alpha,\beta) \in A \times B$. Thus, $\widehat{\mathcal{W}_r^c} = \widehat{\Lambda_o}$. Hence $\left((\widehat{\mathcal{U}_p}, A) \vee (\widehat{\mathcal{V}_q}, B)\right)^c = \left(\widehat{\mathcal{U}_p}, A\right)^c \wedge \left(\widehat{\mathcal{V}_q}, B\right)^c$.

4. SIMILARITY MEASURE BETWEEN TWO GSFSSS

In this section, finding similarity measure between GSFSSs is given below.

Definition 4.1. Let $X = \{x_1, x_2, ..., x_m\}$ be a non-empty set of the universe and $E = \{e_1, e_2, ..., e_n\}$ be a set of parameters. Suppose that $\widehat{\mathcal{U}_p}$ and $\widehat{\mathcal{V}_q}$ are two GSFSSs on (X, E). The similarity measure between two GSFSSs $\widehat{\mathcal{U}_p}$ and $\widehat{\mathcal{V}_q}$ is denoted by $Sim(\widehat{\mathcal{U}_p}, \widehat{\mathcal{V}_q})$ and is defined as $Sim(\widehat{\mathcal{U}_p}, \widehat{\mathcal{V}_q}) = \varphi(\widehat{\mathcal{U}}, \widehat{\mathcal{V}}) \cdot \psi(p,q)$ such that $\varphi(\widehat{\mathcal{U}}, \widehat{\mathcal{V}}) = \frac{1}{m}\sum_{j=1}^m min\left\{T_1\left(\widehat{\mathcal{U}(e)(x_j)}, \widehat{\mathcal{V}(e)(x_j)}\right), T_2\left(\widehat{\mathcal{U}(e)(x_j)}, \widehat{\mathcal{V}(e)(x_j)}\right), S\left(\widehat{\mathcal{U}(e)(x_j)}, \widehat{\mathcal{V}(e)(x_j)}\right)\right\}$ and $\psi(p,q) = 1 - \frac{\sum_{i=1}^n |p(e_i) - q(e_i)|}{\sum_{i=1}^n |p(e_i) + q(e_i)|}$, where

$T_1\left(\widehat{\mathcal{U}(e)(x_j)}, \widehat{\mathcal{V}(e)(x_j)}\right) = \frac{\sum_{i=1}^n (\vartheta_{\mathcal{U}(e_i)}(x_j) \cdot \vartheta_{\mathcal{V}(e_i)}(x_j))}{\sum_{i=1}^n (1 - \sqrt{(1-\vartheta^2_{\mathcal{U}(e_i)}(x_j)) \cdot (1-\vartheta^2_{\mathcal{V}(e_i)}(x_j))})}$,

$T_2\left(\widehat{\mathcal{U}(e)(x_j)}, \widehat{\mathcal{V}(e)(x_j)}\right) = \frac{\sum_{i=1}^n (\varpi^2_{\mathcal{U}(e_i)}(x_j) \cdot \varpi^2_{\mathcal{V}(e_i)}(x_j))}{\sum_{i=1}^n (1 - \sqrt{(1-\varpi^4_{\mathcal{U}(e_i)}(x_j)) \cdot (1-\varpi^4_{\mathcal{V}(e_i)}(x_j))})}$,

$S\left(\widehat{\mathcal{U}(e)(x_j)}, \widehat{\mathcal{V}(e)(x_j)}\right) = \sqrt{1 - \frac{\sum_{i=1}^n |\tau^2_{\mathcal{U}(e_i)}(x_j) - \tau^2_{\mathcal{V}(e_i)}(x_j)|}{\sum_{i=1}^n 1 + ((\tau^2_{\mathcal{U}(e_i)}(x_j)) \cdot (\tau^2_{\mathcal{V}(e_i)}(x_j)))}}$, for $j = 1, 2, ..., m$.

Theorem 4.1. *Let $\widehat{\mathcal{U}_p}$, $\widehat{\mathcal{V}_q}$ and $\widehat{\mathcal{W}_r}$ be the any three GSFSSs over (X, E). Then the following statements are holds:*

(i) $Sim(\widehat{\mathcal{U}_p}, \widehat{\mathcal{V}_q}) = Sim(\widehat{\mathcal{V}_q}, \widehat{\mathcal{U}_p})$

(ii) $0 \leq Sim(\widehat{\mathcal{U}_p}, \widehat{\mathcal{V}_q}) \leq 1$
(iii) $\widehat{\mathcal{U}_p} = \widehat{\mathcal{V}_q} \implies Sim(\widehat{\mathcal{U}_p}, \widehat{\mathcal{V}_q}) = 1$
(iv) $\widehat{\mathcal{U}_p} \sqsubseteq \widehat{\mathcal{V}_q} \sqsubseteq \widehat{\mathcal{W}_r} \implies Sim(\widehat{\mathcal{U}_p}, \widehat{\mathcal{W}_r}) \leq Sim(\widehat{\mathcal{V}_q}, \widehat{\mathcal{W}_r})$
(v) $\widehat{\mathcal{U}_p} \sqcap \widehat{\mathcal{V}_q} = \{\phi\} \Leftrightarrow Sim(\widehat{\mathcal{U}_p}, \widehat{\mathcal{V}_q}) = 0.$

Proof. The proof (i), (ii) and (v) are trivial. (iii) Suppose that $\widehat{\mathcal{U}_p} = \widehat{\mathcal{V}_q}$ implies that $\vartheta_{\mathcal{U}(e_i)}(x_j) = \vartheta_{\mathcal{V}(e_i)}(x_j)$, $\varpi_{\mathcal{U}(e_i)}(x_j) = \varpi_{\mathcal{V}(e_i)}(x_j)$, $\tau_{\mathcal{U}(e_i)}(x_j) = \tau_{\mathcal{V}(e_i)}(x_j)$ and $p(e_i) = q(e_i)$.
Now, $T_1\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right) = \frac{\sum_{i=1}^{n} \vartheta^2_{\mathcal{U}(e_i)}(x_1)}{\sum_{i=1}^{n}(1-1+\vartheta^2_{\mathcal{U}(e_i)}(x_1))} = \frac{\sum_{i=1}^{n} \vartheta^2_{\mathcal{U}(e_i)}(x_1)}{\sum_{i=1}^{n} \vartheta^2_{\mathcal{U}(e_i)}(x_1)} = 1$
and $T_2\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right) = \frac{\sum_{i=1}^{n} \varpi^4_{\mathcal{U}(e_i)}(x_1)}{\sum_{i=1}^{n}(1-1+\varpi^4_{\mathcal{U}(e_i)}(x_1))} = \frac{\sum_{i=1}^{n} \varpi^4_{\mathcal{U}(e_i)}(x_1)}{\sum_{i=1}^{n} \varpi^4_{\mathcal{U}(e_i)}(x_1)} = 1$
and $S\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right) = \sqrt{(1-0)} = 1.$
Now,
$\min\left\{T_1\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right), T_2\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right), S\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right)\right\} = 1.$
Also, if we replace x_1 by $\{x_2, x_3, ..., x_m\}$, we get the sequence $\{1, 1, 1, ..., 1(\text{m}-1\, times)\}$.
Thus, $\varphi(\widehat{\mathcal{U}}, \widehat{\mathcal{V}}) = \frac{1}{m}\{1 + 1 + 1 + ... + 1(\text{m}\, times)\} = \frac{m}{m} = 1$ and $\psi(p, q) = 1$.
Hence $Sim(\widehat{\mathcal{U}_p}, \widehat{\mathcal{V}_q}) = 1$.
(iv) For $j = 1, 2, ..., m$

$$\left\{\begin{array}{lll} \widehat{\mathcal{U}_p} \sqsubseteq \widehat{\mathcal{V}_q} & \implies & \vartheta_{\mathcal{U}(e_i)}(x_j) \leq \vartheta_{\mathcal{V}(e_i)}(x_j), \quad \varpi_{\mathcal{U}(e_i)}(x_j) \leq \varpi_{\mathcal{V}(e_i)}(x_j) \\ & & \tau_{\mathcal{U}(e_i)}(x_j) \geq \tau_{\mathcal{V}(e_i)}(x_j), \quad p(e_i) \leq q(e_i) \\ \widehat{\mathcal{U}_p} \sqsubseteq \widehat{\mathcal{W}_r} & \implies & \vartheta_{\mathcal{U}(e_i)}(x_j) \leq \vartheta_{\mathcal{W}(e_i)}(x_j), \quad \varpi_{\mathcal{U}(e_i)}(x_j) \leq \varpi_{\mathcal{W}(e_i)}(x_j) \\ & & \tau_{\mathcal{U}(e_i)}(x_j) \geq \tau_{\mathcal{W}(e_i)}(x_j), \quad p(e_i) \leq r(e_i) \\ \widehat{\mathcal{V}_q} \sqsubseteq \widehat{\mathcal{W}_r} & \implies & \vartheta_{\mathcal{V}(e_i)}(x_j) \leq \vartheta_{\mathcal{W}(e_i)}(x_j), \quad \varpi_{\mathcal{V}(e_i)}(x_j) \leq \varpi_{\mathcal{W}(e_i)}(x_j) \\ & & \tau_{\mathcal{V}(e_i)}(x_j) \geq \tau_{\mathcal{W}(e_i)}(x_j), \quad q(e_i) \leq r(e_i) \end{array}\right\} \tag{4.1}$$

Clearly,
$$\vartheta_{\mathcal{U}(e_i)}(x_j) \cdot \vartheta_{\mathcal{W}(e_i)}(x_j) \leq \vartheta_{\mathcal{V}(e_i)}(x_j) \cdot \vartheta_{\mathcal{W}(e_i)}(x_j)$$
implies that
$$\sum_{i=1}^{n}(\vartheta_{\mathcal{U}(e_i)}(x_j) \cdot \vartheta_{\mathcal{W}(e_i)}(x_j)) \leq \sum_{i=1}^{n}(\vartheta_{\mathcal{V}(e_i)}(x_j) \cdot \vartheta_{\mathcal{W}(e_i)}(x_j)), \tag{4.2}$$
for $j = 1, 2, ..., m$
Clearly,
$$\vartheta^2_{\mathcal{U}(e_i)}(x_j) \leq \vartheta^2_{\mathcal{V}(e_i)}(x_j)$$
implies that
$$-\vartheta^2_{\mathcal{U}(e_i)}(x_j) \geq -\vartheta^2_{\mathcal{V}(e_i)}(x_j)$$
and
$$(1 - (\vartheta^2_{\mathcal{U}(e_i)}(x_j))) \cdot (1 - (\vartheta^2_{\mathcal{W}(e_i)}(x_j))) \geq (1 - (\vartheta^2_{\mathcal{V}(e_i)}(x_j))) \cdot (1 - (\vartheta^2_{\mathcal{W}(e_i)}(x_j)))$$

and $\sqrt{(1-(\vartheta^2_{\mathcal{U}(e_i)}(x_j)))\cdot(1-(\vartheta^2_{\mathcal{W}(e_i)}(x_j)))} \geq \sqrt{(1-(\vartheta^2_{\mathcal{V}(e_i)}(x_j)))\cdot(1-(\vartheta^2_{\mathcal{W}(e_i)}(x_j)))}$
and
$1-\sqrt{(1-(\vartheta^2_{\mathcal{U}(e_i)}(x_j)))\cdot(1-(\vartheta^2_{\mathcal{W}(e_i)}(x_j)))} \leq 1-\sqrt{(1-(\vartheta^2_{\mathcal{V}(e_i)}(x_j)))\cdot(1-(\vartheta^2_{\mathcal{W}(e_i)}(x_j)))}$
and

$$\sum_{i=1}^{n} 1-\sqrt{(1-(\vartheta^2_{\mathcal{U}(e_i)}(x_j)))\cdot(1-(\vartheta^2_{\mathcal{W}(e_i)}(x_j)))} \leq \sum_{i=1}^{n} 1-\sqrt{(1-(\vartheta^2_{\mathcal{V}(e_i)}(x_j)))\cdot(1-(\vartheta^2_{\mathcal{W}(e_i)}(x_j)))} \tag{4.3}$$

Equation 4.2 is divided by 4.3,

$$\frac{\sum_{i=1}^{n}(\vartheta_{\mathcal{U}(e_i)}(x_j)\cdot\vartheta_{\mathcal{W}(e_i)}(x_j))}{\sum_{i=1}^{n} 1-\sqrt{(1-(\vartheta^2_{\mathcal{U}(e_i)}(x_j)))\cdot(1-(\vartheta^2_{\mathcal{W}(e_i)}(x_j)))}} \leq \frac{\sum_{i=1}^{n}(\vartheta_{\mathcal{V}(e_i)}(x_j)\cdot\vartheta_{\mathcal{W}(e_i)}(x_j))}{\sum_{i=1}^{n} 1-\sqrt{(1-(\vartheta^2_{\mathcal{V}(e_i)}(x_j)))\cdot(1-(\vartheta^2_{\mathcal{W}(e_i)}(x_j)))}} \tag{4.4}$$

Clearly, $\varpi^2_{\mathcal{U}(e_i)}(x_j)\cdot\varpi^2_{\mathcal{W}(e_i)}(x_j) \leq \varpi^2_{\mathcal{V}(e_i)}(x_j)\cdot\varpi^2_{\mathcal{W}(e_i)}(x_j)$ implies that

$$\sum_{i=1}^{n}(\varpi^2_{\mathcal{U}(e_i)}(x_j)\cdot\varpi^2_{\mathcal{W}(e_i)}(x_j)) \leq \sum_{i=1}^{n}(\varpi^2_{\mathcal{V}(e_i)}(x_j)\cdot\varpi^2_{\mathcal{W}(e_i)}(x_j)) \tag{4.5}$$

,for $j = 1, 2, ..., m$
Clearly, $\varpi^4_{\mathcal{U}(e_i)}(x_j) \leq \varpi^4_{\mathcal{V}(e_i)}(x_j)$ implies that $-\varpi^4_{\mathcal{U}(e_i)}(x_j) \geq -\varpi^4_{\mathcal{V}(e_i)}(x_j)$ and
$(1-(\varpi^4_{\mathcal{U}(e_i)}(x_j)))\cdot(1-(\varpi^4_{\mathcal{W}(e_i)}(x_j))) \geq (1-(\varpi^4_{\mathcal{V}(e_i)}(x_j)))\cdot(1-(\varpi^4_{\mathcal{W}(e_i)}(x_j)))$ and
$\sqrt{(1-(\varpi^4_{\mathcal{U}(e_i)}(x_j)))\cdot(1-(\varpi^4_{\mathcal{W}(e_i)}(x_j)))} \geq \sqrt{(1-(\varpi^4_{\mathcal{V}(e_i)}(x_j)))\cdot(1-(\varpi^4_{\mathcal{W}(e_i)}(x_j)))}$
and
$1-\sqrt{(1-(\varpi^4_{\mathcal{U}(e_i)}(x_j)))\cdot(1-(\varpi^4_{\mathcal{W}(e_i)}(x_j)))} \leq 1-\sqrt{(1-(\varpi^4_{\mathcal{V}(e_i)}(x_j)))\cdot(1-(\varpi^4_{\mathcal{W}(e_i)}(x_j)))}$
and

$$\sum_{i=1}^{n} 1-\sqrt{(1-(\varpi^4_{\mathcal{U}(e_i)}(x_j)))\cdot(1-(\varpi^4_{\mathcal{W}(e_i)}(x_j)))} \leq \sum_{i=1}^{n} 1-\sqrt{(1-(\varpi^4_{\mathcal{V}(e_i)}(x_j)))\cdot(1-(\varpi^4_{\mathcal{W}(e_i)}(x_j)))} \tag{4.6}$$

Equation 4.5 is divided by 4.6,

$$\frac{\sum_{i=1}^{n}(\varpi^2_{\mathcal{U}(e_i)}(x_j)\cdot\varpi^2_{\mathcal{W}(e_i)}(x_j))}{\sum_{i=1}^{n} 1-\sqrt{(1-(\varpi^4_{\mathcal{U}(e_i)}(x_j)))\cdot(1-(\varpi^4_{\mathcal{W}(e_i)}(x_j)))}} \leq \frac{\sum_{i=1}^{n}(\varpi^2_{\mathcal{V}(e_i)}(x_j)\cdot\varpi^2_{\mathcal{W}(e_i)}(x_j))}{\sum_{i=1}^{n} 1-\sqrt{(1-(\varpi^4_{\mathcal{V}(e_i)}(x_j)))\cdot(1-(\varpi^4_{\mathcal{W}(e_i)}(x_j)))}} \tag{4.7}$$

Clearly, $\tau^2_{\mathcal{U}(e_i)}(x_j) \geq \tau^2_{\mathcal{V}(e_i)}(x_j)$ and $\tau^2_{\mathcal{U}(e_i)}(x_j) - \tau^2_{\mathcal{W}(e_i)}(x_j) \geq \tau^2_{\mathcal{V}(e_i)}(x_j) - \tau^2_{\mathcal{W}(e_i)}(x_j)$.
Hence

$$\sum_{i=1}^{n}\left|\tau^2_{\mathcal{U}(e_i)}(x_j) - \tau^2_{\mathcal{W}(e_i)}(x_j)\right| \geq \sum_{i=1}^{n}\left|\tau^2_{\mathcal{V}(e_i)}(x_j) - \tau^2_{\mathcal{W}(e_i)}(x_j)\right| \tag{4.8}$$

Also, $\tau^2_{\mathcal{U}(e_i)}(x_j)\cdot\tau^2_{\mathcal{W}(e_i)}(x_j) \geq \tau^2_{\mathcal{V}(e_i)}(x_j)\cdot\tau^2_{\mathcal{W}(e_i)}(x_j)$ implies that

$$\sum_{i=1}^{n} 1+(\tau^2_{\mathcal{U}(e_i)}(x_j)\cdot\tau^2_{\mathcal{W}(e_i)}(x_j)) \geq \sum_{i=1}^{n} 1+(\tau^2_{\mathcal{V}(e_i)}(x_j)\cdot\tau^2_{\mathcal{W}(e_i)}(x_j)) \tag{4.9}$$

for $j = 1, 2, ..., m$
Equation 4.8 is divided by 4.9, we get

$$\frac{\sum_{i=1}^{n} |\tau^2_{\mathcal{U}(e_i)}(x_j) - \tau^2_{\mathcal{W}(e_i)}(x_j)|}{\sum_{i=1}^{n} 1 + (\tau^2_{\mathcal{U}(e_i)}(x_j) \cdot \tau^2_{\mathcal{W}(e_i)}(x_j))} \geq \frac{\sum_{i=1}^{n} |\tau^2_{\mathcal{V}(e_i)}(x_j) - \tau^2_{\mathcal{W}(e_i)}(x_j)|}{\sum_{i=1}^{n} 1 + (\tau^2_{\mathcal{V}(e_i)}(x_j) \cdot \tau^2_{\mathcal{W}(e_i)}(x_j))}$$

and

$$1 - \frac{\sum_{i=1}^{n} |\tau^2_{\mathcal{U}(e_i)}(x_j) - \tau^2_{\mathcal{W}(e_i)}(x_j)|}{\sum_{i=1}^{n} 1 + (\tau^2_{\mathcal{U}(e_i)}(x_j) \cdot \tau^2_{\mathcal{W}(e_i)}(x_j))} \leq 1 - \frac{\sum_{i=1}^{n} |\tau^2_{\mathcal{V}(e_i)}(x_j) - \tau^2_{\mathcal{W}(e_i)}(x_j)|}{\sum_{i=1}^{n} 1 + (\tau^2_{\mathcal{V}(e_i)}(x_j) \cdot \tau^2_{\mathcal{W}(e_i)}(x_j))}$$

and

$$\sqrt{1 - \frac{\sum_{i=1}^{n} |\tau^2_{\mathcal{U}(e_i)}(x_j) - \tau^2_{\mathcal{W}(e_i)}(x_j)|}{\sum_{i=1}^{n} 1 + (\tau^2_{\mathcal{U}(e_i)}(x_j) \cdot \tau^2_{\mathcal{W}(e_i)}(x_j))}} \leq \sqrt{1 - \frac{\sum_{i=1}^{n} |\tau^2_{\mathcal{V}(e_i)}(x_j) - \tau^2_{\mathcal{W}(e_i)}(x_j)|}{\sum_{i=1}^{n} 1 + (\tau^2_{\mathcal{V}(e_i)}(x_j) \cdot \tau^2_{\mathcal{W}(e_i)}(x_j))}} \tag{4.10}$$

Minimum of Equations 4.4, 4.7 and 4.10 and adding for each $j = 1, 2, ..., m$.
Finally, divided by m, we get

$$\varphi(\overbrace{\mathcal{U}}, \overbrace{\mathcal{W}}) \leq \varphi(\overbrace{\mathcal{V}}, \overbrace{\mathcal{W}}) \tag{4.11}$$

By Equation 4.1, Clearly $p(e_i) \leq q(e_i) \leq r(e_i)$ and $p(e_i) - r(e_i) \leq q(e_i) - r(e_i)$.
Hence $\Big|q(e_i) - r(e_i)\Big| \leq \Big|p(e_i) - r(e_i)\Big|$ and $-\Big|p(e_i) - r(e_i)\Big| \leq -\Big|q(e_i) - r(e_i)\Big|$ and

$$-\sum_{i=1}^{n} \Big|p(e_i) - r(e_i)\Big| \leq -\sum_{i=1}^{n} \Big|q(e_i) - r(e_i)\Big| \tag{4.12}$$

Since $\Big|p(e_i) + r(e_i)\Big| \leq \Big|q(e_i) + r(e_i)\Big|$ implies that

$$\sum_{i=1}^{n} \Big|p(e_i) + r(e_i)\Big| \leq \sum_{i=1}^{n} \Big|q(e_i) + r(e_i)\Big| \tag{4.13}$$

Equation 4.12 is divided by 4.13, we get

$$-\frac{\sum_{i=1}^{n} \Big|p(e_i) - r(e_i)\Big|}{\sum_{i=1}^{n} \Big|p(e_i) + r(e_i)\Big|} \leq -\frac{\sum_{i=1}^{n} \Big|q(e_i) - r(e_i)\Big|}{\sum_{i=1}^{n} \Big|q(e_i) + r(e_i)\Big|}$$

implies

$$1 - \frac{\sum_{i=1}^{n} \Big|p(e_i) - r(e_i)\Big|}{\sum_{i=1}^{n} \Big|p(e_i) + r(e_i)\Big|} \leq 1 - \frac{\sum_{i=1}^{n} \Big|q(e_i) - r(e_i)\Big|}{\sum_{i=1}^{n} \Big|q(e_i) + r(e_i)\Big|}$$

Thus,

$$1 - \frac{\sum_{i=1}^{n} \Big|p(e_i) - r(e_i)\Big|}{\sum_{i=1}^{n} \Big|p(e_i) + r(e_i)\Big|} \leq 1 - \frac{\sum_{i=1}^{n} \Big|q(e_i) - r(e_i)\Big|}{\sum_{i=1}^{n} \Big|q(e_i) + r(e_i)\Big|}$$

Hence

$$\psi(p, r) \leq \psi(q, r) \tag{4.14}$$

Multiply by Equations 4.11 and 4.14,

$$\varphi(\overbrace{\mathcal{U}}, \overbrace{\mathcal{W}}) \cdot \psi(p, r) \leq \varphi(\overbrace{\mathcal{V}}, \overbrace{\mathcal{W}}) \cdot \psi(q, r)$$

Hence $Sim(\overbrace{\mathcal{U}_p}, \overbrace{\mathcal{W}_r}) \leq Sim(\overbrace{\mathcal{V}_q}, \overbrace{\mathcal{W}_r})$. This proves (iv).

Example 4.2. Calculate the similarity measure between the two GSFSSs namely $\widehat{\mathcal{U}_p}$ and $\widehat{\mathcal{V}_q}$. We choose the universal set $X = \{x_1, x_2\}$ and parameter $E = \{e_1, e_2, e_3, e_4\}$ can be defined as below:

$\widehat{\mathcal{U}_p(e)}$	e_1	e_2	e_3	e_4
$\widehat{\mathcal{U}(e)(x_1)}$	$(0.45, 0.1, 0.55)$	$(0.35, 0.4, 0.45)$	$(0.25, 0.3, 0.65)$	$(0.55, 0.4, 0.35)$
$\widehat{\mathcal{U}(e)(x_2)}$	$(0.6, 0.1, 0.5)$	$(0.3, 0.4, 0.4)$	$(0.2, 0.3, 0.6)$	$(0.7, 0.2, 0.3)$
$p(e)$	$(0.2, 0.4, 0.25)$	$(0.1, 0.15, 0.1)$	$(0.6, 0.65, 0.2)$	$(0.5, 0.55, 0.3)$

$\widehat{\mathcal{V}_q(e)}$	e_1	e_2	e_3	e_4
$\widehat{\mathcal{V}(e)(x_1)}$	$(0.35, 0.25, 0.65)$	$(0.45, 0.65, 0.55)$	$(0.65, 0.55, 0.35)$	$(0.55, 0.15, 0.45)$
$\widehat{\mathcal{V}(e)(x_2)}$	$(0.7, 0.4, 0.6)$	$(0.4, 0.3, 0.5)$	$(0.6, 0.1, 0.5)$	$(0.5, 0.5, 0.4)$
$q(e)$	$(0.3, 0.3, 0.45)$	$(0.4, 0.3, 0.2)$	$(0.5, 0.15, 0.2)$	$(0.6, 0.5, 0.4)$

Using Definition 4.1 and routine calculation, we get
$T_1\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right) = \frac{0.78}{0.89} = 0.872866,$
$T_2\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right) = \frac{0.09905}{0.1712} = 0.578664$ and
$S\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right) = \sqrt{1 - \frac{0.6}{4.2656}} = 0.927006.$
Now, $\min\left\{T_1\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right), T_2\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right), S\left(\widehat{\mathcal{U}(e)(x_1)}, \widehat{\mathcal{V}(e)(x_1)}\right)\right\} =$ 0.578664 and $\psi(p,q) = 1 - \frac{1.8}{8.3} = 0.783133.$
Similarly, $T_1\left(\widehat{\mathcal{U}(e)(x_2)}, \widehat{\mathcal{V}(e)(x_2)}\right) = \frac{1.01}{1.20} = 0.876673,$
$T_2\left(\widehat{\mathcal{U}(e)(x_2)}, \widehat{\mathcal{V}(e)(x_2)}\right) = \frac{0.0269}{0.066458} = 0.404765$ and
$S\left(\widehat{\mathcal{U}(e)(x_2)}, \widehat{\mathcal{V}(e)(x_2)}\right) = \sqrt{1 - \frac{0.38}{4.2344}} = 0.954075.$
Now, $\min\left\{T_1\left(\widehat{\mathcal{U}(e)(x_2)}, \widehat{\mathcal{V}(e)(x_2)}\right), T_2\left(\widehat{\mathcal{U}(e)(x_2)}, \widehat{\mathcal{V}(e)(x_2)}\right), S\left(\widehat{\mathcal{U}(e)(x_2)}, \widehat{\mathcal{V}(e)(x_2)}\right)\right\} =$ $0.404765.$
Thus, $\varphi(\widehat{\mathcal{U}}, \widehat{\mathcal{V}}) = \frac{0.578664+0.404765}{2} = 0.491714.$
Hence, $Sim(\widehat{\mathcal{U}_p}, \widehat{\mathcal{V}_q}) = 0.491714 \times 0.783133 = 0.385077.$

5. Similarity Measure in medical diagnosis using GSFSS model

Decision making problems are a big part of human society and applied widely to practical fields like economics, management, engineering and Hospital. However, with the development of science and technology, the uncertainty also plays a dominant role at some point of the decision making analysis. In this application, we present a method for a medical diagnosis problem based on the proposed similarity measure of GSFSS's. This technique of similarity measure between two GSFSS can be applied to detect whether an ill person is suffering from a certain disease or not. We first give the following definition:

Definition 5.1. Let $\overbrace{\mathcal{U}_p}$ and $\overbrace{\mathcal{V}_q}$ be two GSFSS's over the same soft universe (X, E). We call the two GSFSS's to be significantly similar if $Sim(\overbrace{\mathcal{U}_p}, \overbrace{\mathcal{V}_q}) > 1/2$.

We first construct a GSFSS for the illness with the help of a medical person and a GSFSS for the ill person. Then, we calculate the similarity measure between two GSFSS's. If they are significantly similar, then we infer that the person may have disease, and otherwise not.

5.1. **Algorithms based on the similarity measures for GSFSS Model.** An algoritheorem for decision making problems using GSFSS model is explained.
Step 1. Input the GSFSS in tabular form.
Step 2. Input the set of choice parameters $A \subseteq E$.
Step 3. Compute the values of $T_1(x_j)$, $T_2(x_j)$ and $S(x_j)$ and $1 \leq j \leq m$.
Step 4. Calculate $\varphi = \frac{1}{m}\sum_{j=1}^{m} \min\{T_1(x_j), T_2(x_j), S(x_j)\}$.
Step 5. Determine the value $\psi(p, q) = 1 - \frac{\sum |p(e_i)-q(e_i)|}{\sum |p(e_i)+q(e_i)|}$ and $1 \leq i \leq n$.
Step 6. Compute the similarity measure = $\varphi \cdot \psi$.
Step 7. Select similarity measure, when suitable criteria for significantly similar.
Step 8. Finally, decision to the problem.
Step 9. End.

5.2. **Data Analysis.** Suppose that there are five patients $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4$ and $\mathcal{P}_5$ in a hospital with certain symptoms of dengue hemorrhagic fever. Let the universal set contain only three elements. That is $X = \{x_1$: severe, x_2: mild, x_3 : no$\}$, the set of parameters E is the set of certain symptoms of dengue hemorrhagic fever is represented by $E = \{e_1$: severe abdominal pain, e_2: persistent vomiting, e_3 : rapid breathing, e_4 : bleeding gums, e_5: restlessness and blood in vomit$\}$.

Table 1 is represented by the dengue hemorrhagic fever prepared with the help of a medical person.

Table 1
Model GSFSS for pneumonia (dengue hemorrhagic fever).

$\overbrace{\mathcal{L}_{p(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overbrace{\mathcal{L}(e)(x_1)}$	$(0.7, 0.3, 0.4)$	$(0.6, 0.35, 0.45)$	$(0.55, 0.45, 0.5)$	$(0.6, 0.35, 0.5)$	$(0.55, 0.65, 0.45)$
$\overbrace{\mathcal{L}(e)(x_2)}$	$(0.7, 0.2, 0.4)$	$(0.6, 0.35, 0.45)$	$(0.5, 0.4, 0.35)$	$(0.6, 0.3, 0.45)$	$(0.5, 0.6, 0.55)$
$\overbrace{\mathcal{L}(e)(x_3)}$	$(0.7, 0.3, 0.1)$	$(0.6, 0.3, 0.4)$	$(0.55, 0.4, 0.3)$	$(0.6, 0.3, 0.2)$	$(0.55, 0.6, 0.45)$
$\overbrace{p(e)}$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$

We construct the GSFSS's for five patients under consideration as in **Table 2**, **Table 3**, **Table 4**, **Table 5** and **Table 6**.

Table 2
GSFSS for the ill person $\mathcal{P}_1$.

$\overbrace{\mathcal{P}1_{p_1(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overbrace{\mathcal{P}_1(e)(x_1)}$	$(0.1, 0.4, 0.55)$	$(0.2, 0.6, 0.4)$	$(0.5, 0.3, 0.3)$	$(0.3, 0.5, 0.35)$	$(0.4, 0.6, 0.5)$
$\overbrace{\mathcal{P}_1(e)(x_2)}$	$(0.6, 0.6, 0.5)$	$(0.5, 0.4, 0.4)$	$(0.55, 0.4, 0.6)$	$(0.5, 0.3, 0.65)$	$(0.5, 0.4, 0.7)$
$\overbrace{\mathcal{P}_1(e)(x_3)}$	$(0.6, 0.3, 0.45)$	$(0.5, 0.5, 0.55)$	$(0.4, 0.4, 0.7)$	$(0.35, 0.6, 0.5)$	$(0.4, 0.3, 0.7)$
$\overbrace{p_1(e)}$	$(0.4, 0.2, 0.1)$	$(0.5, 0.3, 0.2)$	$(0.6, 0.1, 0.4)$	$(0.6, 0.5, 0.3)$	$(0.5, 0.3, 0.2)$

Table 3
GSFSS for the ill person $\mathcal{P}_2$.

$\overbrace{\mathcal{P}2_{p_2(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overbrace{\mathcal{P}_2(e)(x_1)}$	$(0.3, 0.3, 0.2)$	$(0.45, 0.5, 0.4)$	$(0.6, 0.4, 0.3)$	$(0.4, 0.4, 0.1)$	$(0.15, 0.45, 0.2)$
$\overbrace{\mathcal{P}_2(e)(x_2)}$	$(0.5, 0.4, 0.5)$	$(0.5, 0.35, 0.65)$	$(0.55, 0.4, 0.6)$	$(0.3, 0.45, 0.65)$	$(0.45, 0.4, 0.55)$
$\overbrace{\mathcal{P}_2(e)(x_3)}$	$(0.45, 0.25, 0.45)$	$(0.35, 0.45, 0.6)$	$(0.45, 0.55, 0.65)$	$(0.4, 0.65, 0.55)$	$(0.6, 0.2, 0.6)$
$\overbrace{p_2(e)}$	$(0.35, 0.15, 0.35)$	$(0.45, 0.4, 0.4)$	$(0.55, 0.2, 0.3)$	$(0.4, 0.1, 0.6)$	$(0.5, 0.3, 0.7)$

Table 4
GSFSS for the ill person $\mathcal{P}_3$.

$\overbrace{\mathcal{P}3_{p_3(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overbrace{\mathcal{P}_3(e)(x_1)}$	$(0.4, 0.6, 0.55)$	$(0.25, 0.6, 0.4)$	$(0.5, 0.3, 0.3)$	$(0.35, 0.5, 0.35)$	$(0.4, 0.7, 0.5)$
$\overbrace{\mathcal{P}_3(e)(x_2)}$	$(0.2, 0.7, 0.5)$	$(0.3, 0.65, 0.4)$	$(0.5, 0.5, 0.6)$	$(0.25, 0.6, 0.65)$	$(0.15, 0.7, 0.6)$
$\overbrace{\mathcal{P}_3(e)(x_3)}$	$(0.6, 0.2, 0.45)$	$(0.5, 0.5, 0.55)$	$(0.4, 0.4, 0.7)$	$(0.5, 0.7, 0.5)$	$(0.4, 0.5, 0.7)$
$\overbrace{p_3(e)}$	$(0.5, 0.25, 0.3)$	$(0.4, 0.5, 0.25)$	$(0.55, 0.15, 0.6)$	$(0.4, 0.3, 0.5)$	$(0.5, 0.25, 0.4)$

Table 5
GSFSS for the ill person $\mathcal{P}_4$.

$\overbrace{\mathcal{P}4_{p_4(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overbrace{\mathcal{P}_4(e)(x_1)}$	$(0.3, 0.4, 0.7)$	$(0.45, 0.25, 0.6)$	$(0.6, 0.2, 0.5)$	$(0.4, 0.15, 0.5)$	$(0.25, 0.6, 0.4)$
$\overbrace{\mathcal{P}_4(e)(x_2)}$	$(0.5, 0.5, 0.5)$	$(0.5, 0.35, 0.65)$	$(0.55, 0.4, 0.6)$	$(0.3, 0.45, 0.65)$	$(0.45, 0.4, 0.55)$
$\overbrace{\mathcal{P}_4(e)(x_3)}$	$(0.4, 0.25, 0.45)$	$(0.3, 0.4, 0.6)$	$(0.4, 0.5, 0.65)$	$(0.4, 0.5, 0.55)$	$(0.3, 0.2, 0.6)$
$\overbrace{p_4(e)}$	$(0.6, 0.2, 0.1)$	$(0.5, 0.1, 0.4)$	$(0.7, 0.4, 0.3)$	$(0.4, 0.35, 0.6)$	$(0.5, 0.65, 0.2)$

Table 6
GSFSS for the ill person $\mathcal{P}_5$.

$\overbrace{\mathcal{P}5}_{p5(e)}$	e_1	e_2	e_3	e_4	e_5
$\overbrace{\mathcal{P}_5(e)(x_1)}$	$(0.7, 0.6, 0.6)$	$(0.5, 0.6, 0.45)$	$(0.6, 0.35, 0.3)$	$(0.35, 0.7, 0.5)$	$(0.3, 0.8, 0.4)$
$\overbrace{\mathcal{P}_5(e)(x_2)}$	$(0.4, 0.4, 0.6)$	$(0.5, 0.5, 0.6)$	$(0.6, 0.4, 0.65)$	$(0.25, 0.3, 0.5)$	$(0.15, 0.5, 0.5)$
$\overbrace{\mathcal{P}_5(e)(x_3)}$	$(0.35, 0.6, 0.45)$	$(0.5, 0.7, 0.55)$	$(0.6, 0.4, 0.7)$	$(0.5, 0.7, 0.5)$	$(0.3, 0.6, 0.7)$
$\overbrace{p_5(e)}$	$(0.6, 0.55, 0.6)$	$(0.7, 0.35, 0.6)$	$(0.7, 0.5, 0.5)$	$(0.65, 0.5, 0.45)$	$(0.55, 0.4, 0.7)$

The generalized spherical fuzzy values in **Tables 2-6** are provided by the experts, depending on their assessment of the alternatives against the criteria under consideration. In this example, we should calculate the similarity measure of GSFSSs in Tables 2-6 with the one in **Table 1** based on Definition 4.1. Calculating the similarity measure for $\mathcal{P}_1$ to $\mathcal{P}_5$ ill persons are given below the table.

	$T_1(x_1)$	$T_2(x_1)$	$S(x_1)$	$T_1(x_2)$	$T_2(x_2)$	$S(x_2)$	$T_1(x_3)$	$T_2(x_3)$	$S(x_3)$
$\overbrace{(\mathcal{L}, \mathcal{P}_1)}$	0.689939	0.839350	0.948526	0.984220	0.623759	0.924489	0.941203	0.573321	0.873585
$\overbrace{(\mathcal{L}, \mathcal{P}_2)}$	0.822280	0.826786	0.925773	0.930784	0.777226	0.925448	0.915957	0.489043	0.882650
$\overbrace{(\mathcal{L}, \mathcal{P}_3)}$	0.851612	0.794111	0.948526	0.687532	0.602675	0.937060	0.965504	0.630628	0.873585
$\overbrace{(\mathcal{L}, \mathcal{P}_4)}$	0.851658	0.880796	0.948770	0.930784	0.714728	0.925448	0.843061	0.568141	0.882650
$\overbrace{(\mathcal{L}, \mathcal{P}_5)}$	0.944438	0.706115	0.960642	0.831444	0.877836	0.925907	0.905064	0.563243	0.873585

	φ	ψ	$Similarity$
$\overbrace{(\mathcal{L}, \mathcal{P}_1)}$	0.629006	0.514851	0.323845
$\overbrace{(\mathcal{L}, \mathcal{P}_2)}$	0.696183	0.554217	0.385836
$\overbrace{(\mathcal{L}, \mathcal{P}_3)}$	0.675805	0.561151	0.379229
$\overbrace{(\mathcal{L}, \mathcal{P}_4)}$	0.711509	0.571429	0.406577
$\overbrace{(\mathcal{L}, \mathcal{P}_5)}$	0.700267	0.715203	0.500834

From the above results, we find that the similarity measure of first four patients are $< 1/2$, but the similarity measure of fifth patient $\mathcal{P}_5$ is $\overbrace{(\mathcal{L}, \mathcal{P}_5)} = \mathbf{0.500834} > 1/2$. Hence these two GSFSS's are significantly similar. Therefore, we conclude that the patient $\mathcal{P}_5$ is suffering from dengue hemorrhagic fever.

6. CONCLUSION

The main goal of this work is to present a GSFSS and studied some of its properties. The theory of generalized spherical fuzzy soft set and defined some operations such as complement, union, intersection, AND and OR. Notably, we tend to showed that De Morgan's laws, associate laws and distributive laws that are holds in generalized spherical fuzzy soft set. Also, we advocate an algorithm to solved the decision making problem based on generalized soft set model. Similarity measure of two GSFSS is discussed and an application of this to medical diagnosis has been shown. In the future direction, we will apply the generalized interval valued spherical fuzzy soft sets and generalized bipolar spherical fuzzy soft sets theory.

7. ACKNOWLEDGEMENTS

The author is obliged the thankful to the reviewer for the numerous and significant suggestions that raised the consistency of the ideas presented in this paper.

REFERENCES

[1] S. Alkhazaleh, A.R. Salleh, and N. Hassan. Possibility fuzzy soft set, Advances in Decision Sciences, (2011), 1–18.

[2] K. Atanassov. Intuitionistic fuzzy sets, Fuzzy sets and Systems, 20(1) (1986), 87–96.

[3] Bui Cong Cuong. Picture fuzzy sets, Journal of Computer Science and Cybernetics, 30(4) (2014), 409–420.

[4] Chiranjibe Jana, Tapan Senapati and Madhumangal Pal. Pythagorean fuzzy Dombi aggregation operators and its applications in multiple attribute decision making, International Journal of Intelligent Systems, 34(9) (2019), 2019–2038.

[5] Chiranjibe Jana and Madhumangal Pal. Application of bipolar intuitionistic fuzzy soft sets in decision making problem, International Journal of Fuzzy System Applications, 7 (3) (2018), 32–55.

[6] Faruk Karaaslan. Possibility neutrosophic soft sets and PNS-decision making method, Applied Soft Computing 54, (2017), 403–414.

[7] T. Mahmood, U. Kifayat, Q. Khan and J. Naeem. An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets, Neural Computing and Applications (2018).

[8] P.K. Maji, R. Biswas and A.R. Roy. Fuzzy Soft Set, Journal of Fuzzy Mathematics, 9(3) (2001), 589–602.

[9] P.K. Maji, R. Biswas and A.R. Roy. On intuitionistic Fuzzy Soft Set, Journal of Fuzzy Mathematics, 9(3) (2001), 677–692.

[10] P Majumdar, S.K. Samantab. Generalized fuzzy soft sets, Computers and Mathematics with Applications, 59, (2010), 1425–1432.

[11] D. Molodtsov. Soft set theory First results, Computers and Mathematics with Applications, 37 (1999), 19–31.

[12] Murat Kirisci. Medical decision making with respect to the fuzzy soft sets, Journal of Interdisciplinary Mathematics, (2020), 1–10.

[13] M. Palanikumar and K. Arulmozhi. On Various ideals and its Applications of Bisemirings, Gedrag And Organisatie Review, 33(2) 2020, 522–533.

[14] M. Palanikumar and K. Arulmozhi. On New Approach of Various fuzzy ideals in Ordered gamma semigroups, Gedrag And Organisatie Review, 33(2) 2020, 331–342.

[15] M. Palanikumar and K. Arulmozhi. On vague subbisemirings of bisemirings, Bulletin of the International Mathematical Virtual Institute, 11(3), (2021), 419–428.

[16] M. Palanikumar and K. Arulmozhi. New approach towards A-ideals in Ternary Semirings, Annals of Communications in Mathematics, 4(2) 2021, 114–125.

[17] M. Palanikumar and K. Arulmozhi. Different types of prime bi-ideals in Ternary Semirings, Annals of Communications in Mathematics, 4(2) 2021, 141–148.

[18] M. Palanikumar and K. Arulmozhi. m-ideals and its generators of ternary semigroups, Annals of Communications in Mathematics, 4(2) 2021, 164–171.

[19] X.D. Peng, Y. Yang, J.P. Song. Pythagorean fuzzy soft set and its application, Computer Engineering, 41(7) (2015), 224–229.

[20] Shawkat Alkhazaleh and Abdul Razak Salleh. Generalized Interval valued Fuzzy Soft Set, Journal of Applied Mathematics, (2012), 1–18.

[21] F. Smarandache. Neutrosophic set generalization of the intuitionistic fuzzy set, International Journal of Pure and Applied Mathematics, 24(3) (2005), 287–297.

[22] Ting Yu Chen. An Interval Valued Pythagorean Fuzzy Compromise Approach with Correlation Based Closeness Indices for Multiple Criteria Decision Analysis of Bridge Construction Methods, Complexity, (2018), 1–29.

[23] K. Ullah, T. Mahmood, Z. Ali and N. Jan. On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition, Complex and Intelligent Systems, (2019), 1–13.

[24] Z. Xiao, W.J. Chen, and L.L. Li. A method based on interval valued fuzzy soft set for multi attribute group decision making problems under uncertain environment, Knowledge and Information Systems, 34 (2013), 653–669.

[25] R.R. Yager, A.M Abbasov. Pythagorean membership grades, complex numbers, and decision making, International Journal of Intelligent Systems, 28 (2014), 436–452.

[26] R. R. Yager. Pythagorean membership grades in multi criteria decision making, IEEE Trans. Fuzzy Systems, 22 (2014), 958–965.

[27] Yong Yang, Chencheng Liang, Shiwei Ji and Tingting Liu. Adjustable soft discernibility matrix based on picture fuzzy soft sets and its applications in decision making, Journal of Intelligent and Fuzzy Systems, 29 (2015), 1711–1722.

[28] L. A. Zadeh. Fuzzy sets, Information and control, 8(3)(1965), 338–353.

M. PALANIKUMAR
ANNAMALAI UNIVERSITY, DEPARTMENT OF MATHEMATICS, CHIDAMBARAM, 608002, INDIA
E-mail address: palanimaths86@gmail.com

K. ARULMOZHI
BHARATH INSTITUTE OF HIGHER EDUCATION AND RESEARCH, DEPARTMENT OF MATHEMATICS, CHENNAI, 600073, INDIA
E-mail address: arulmozhiems@gmail.com