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REGULAR SEMIOPEN SOFT SETS AND THEIR APPLICATIONS

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ABSTRACT. The purpose of this paper is to introduce the notion of soft regular semi compactness, connectedness, and separation axioms using regular semiopen soft sets in soft topological spaces. Moreover, we investigate soft RS-regular space and soft RS-normal space are soft topological properties under bijection, soft regular semi irresolute and soft regular semi irresolute open functions. Also, we show that the properties of being soft regular semi T_i-spaces ($i = 1, 2, 3, 4$) are hereditary properties.

1. INTRODUCTION

The concept of soft sets was first introduced by Molodtsov [17] in 1999 as a general mathematical tool for dealing with uncertain objects. In [17, 18], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [12], the properties and applications of soft set theory have been studied increasingly [3, 11, 17, 20]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1, 4, 5, 7, 12, 13, 14, 15, 18, 19, 22, 26]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [8]. Recently, in 2011, Shabir and Naz [23] initiated the study of soft topological spaces. They defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Min in [16] investigate some properties of these soft separation axioms mentioned in [23]. Banu and Halis in [6] studied some properties of soft Hausdorff space. Recently Vadivel and Elavarasan [24] introduced the concept of regular semiopen soft set.

The main purpose of this paper is to introduce the notion of soft regular semi compactness, connectedness, and separation axioms using regular semiopen soft sets in soft topological spaces. In particular we study the properties of the soft RS-regular spaces and soft RS-normal spaces. We show that if (x, E) is regular semiclosed soft set for all

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$x \in X$ in a soft topological space (X, τ, E), then (X, τ, E) is soft regular semi T_1-space. Also, we show that if a soft topological space (X, τ, E) is soft regular semi T_3-space, then $\forall\, x \in X$, (x, E) is regular semiclosed soft set. This paper, not only can form the theoretical basis for further applications of topology on soft sets, but also lead to the development of information systems.

2. PRELIMINARIES

In this section, we recall some definition and concepts discussed in [9, 16, 23, 25]. Throughout this study X and Y denote universal sets, $E, E^{'}$ denote two sets of parameters, $A, B, C, D, B^{'}, D^{'} \subseteq E$ or $E^{'}$. Let X be an initial universe and E be a set of parameters. Let $\mathbb{P}(X)$ denote the power set of X and A be a nonempty subset set of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to \mathbb{P}(X)$. For two soft sets (F, A) and (G, B) over common universe X, we say that (F, A) is a soft subset (G, B) if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$. In this case, we write $(F, A)\widetilde{\subseteq}(G, B)$ and (G, B) is said to be a soft super set of (F, A). Two soft sets (F, A) and (G, B) over a common universe X are said to be soft equal if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$. The soft set (F, A) over X such that $F(e) = \{x\}\ \forall\ e \in E$ is called singleton soft point and denoted by x_E or (x, E). A soft set (F, A) over X is called null soft set, denoted by (Φ, A), if for each $e \in A,\ F(e) = \Phi$. Similarly, it is called absolute soft set, denoted by $\widetilde{X}$, if for each $e \in A,\ F(e) = X$.

The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for each $e \in C$,

$$H(e) = \begin{cases} F(e)\ e \in A - B \\ G(e)\ e \in B - A \\ F(e) \cup G(e)\ e \in A \cap B \end{cases}$$

We write $(F, A) \cup (G, B) = (H, C)$. Moreover, the intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X, denoted by $(F, A) \cap (G, B)$, is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for each $e \in C$. The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E) - (G, E)$, is defined as $H(e) = F(e) - G(e)$, for each $e \in E$. Let Y be nonempty subset of X. Then $\widetilde{Y}$ denotes the soft set (Y, E) over X where $Y(e) = Y$ for each $e \in E$. In particular, (X, E) will be denoted by $\widetilde{X}$. Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$, whenever $x \in F(e)$, for each $e \in E$ [21].

The relative complement of a soft set (F, A) is denoted by $(F, A)^{'}$ and is defined by $(F, A)^{'} = (F, A)$ where $F^{'} : A \to \mathbb{P}(X)$ is defined by following

$$F^{'}(e) = X - F(e),\ \forall e \in A$$

In this paper, for convenience, let $SS(X, E)$ be the family of soft sets over X with set of parameters E. We will apply two next propositions so much in the proofs.

Proposition 2.1. [21] *Let $(F, E),\ (G, E),\ (H, E)$ and (I, E) be soft sets (X, E). Then the following holds:*

(1) $(F, E)\widetilde{\subseteq}(G, E)$ *if and only if* $(F, E) \cap (G, E) = (F, E)$;
(ii) $(F, E)\widetilde{\subseteq}(G, E),\ (H, E)$ *if and only if* $(F, E)\widetilde{\subseteq}(G, E) \cap (H, E)$;
(iii) If $(F, E)\widetilde{\subseteq}(H, E)$ *and* $(G, E)\widetilde{\subseteq}(I, E)$*, then* $(F, E) \cup (G, E)\widetilde{\subseteq}(H, E) \cup (I, E)$;
(iv) $(F, E) \cap (F, E)' = (\Phi, E)$;
(v) $(F, E) \cap (G, E) = (\Phi, E)$ *if and only if* $(F, E)\widetilde{\subseteq}(G, E)'$;

(vi) $(F,E)\tilde{\subseteq}(G,E)$ *if and only if* $(G,E)'\tilde{\subseteq}(F,E)'$.

Also we can obtain the following easily.

Proposition 2.2. [21] *Let* (F,E), (G,E) *and* (H,E) *be soft sets and* $\{(F_i,E)|i\in I\}$ *be a family of soft sets in* (X,E). *Then the following holds.*

(i) $(F,E)\cap(F,E)'=(\Phi,E)$;
(ii) $(F,E)\cup(\Phi,E)=(F,E)$;
(iii) $(F,E)\cap(\bigcup_{i\in I}(F_i,E))=\bigcup_{i\in I}((F,E)\cap(F_i,E))$;
(iv) *If* $(F,E)\tilde{\subseteq}(G,E)$ *and* $(G,E)\cap(H,E)=(\Phi,E)$, *then* $(F,E)\cap(H,E)=(\Phi,E)$;
(v) $(\Phi,E)'=\widetilde{X}$;
(vi) $\widetilde{X'}=(\Phi,E)$.

Let τ be the collection of soft sets over X. Then τ is called a soft topology [23] on X if τ satisfies the following axioms:

(i) (Φ,E) and $\widetilde{X}$ belongs to τ.
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.

The triple (X,τ,E) is called soft topological space (briefly, sts) over X. The members of τ are said to be soft open in X, and the soft set (F,E) is called soft closed in X if its relative complement $(F,E)'$ belongs to τ. Let (X,τ,E) be a soft topological space and (F,A) be a soft set over X. Soft closure of a soft set (F,A) in X is denoted by $cl(F,A)=\widetilde{\bigcap}\{(F,E)\tilde{\supseteq}(F,A):(F,E)$ is a soft closed set of $X\}$. Soft interior of a soft set (F,A) in X is denoted by $int(F,A)=\widetilde{\bigcup}\{(O,A)\tilde{\subseteq}(F,A):(O,A)$ is a soft open set of $X\}$.

The proof of the following proposition is an easy application of De Morgan's laws with the definition of a soft topology on X (see Proposition 3.3 of [25]).

Proposition 2.3. *Let* (X,τ,E) *be a soft space over* X. *Then*

(i) (Φ,E) *and* $\widetilde{X}$ *are closed soft sets over* X.
(ii) The intersection of any number of soft closed sets is a soft closed set over X.
(iii) The union of any two soft closed sets is a soft closed set over X.

Theorem 2.4. [23] *Let* (Y,τ_Y,E) *be a soft subspace of a sts* (X,τ,E) *and* $(F,E)\in SS(X,E)$. *Then*

(1) If (F,E) *is open soft set in* Y *and* $\tilde{Y}\in\tau$, *then* $(F,E)\in\tau$.
(2) (F,E) *is open soft set in* Y *if and only if* $(F,E)=\tilde{Y}\tilde{\cap}(G,E)$ *for some* $(G,E)\in\tau$.
(3) (F,E) *is closed soft set in* Y *if and only if* $(F,E)=\tilde{Y}\tilde{\cap}(H,E)$ *for some* (H,E) *is* τ*-closed soft set.*

Definition 2.1. [2] Let $(X,\ \tau,\ E)$ be a sts and $(F,E)\in SS(X,E)$. Then (F,E) is called a (i) regular closed soft set if $(F,E)=cl(int(F,E))$ and (ii) regular open soft set if $(F,E)=int(cl(F,E))$.

Definition 2.2. [24] In a sts (X,τ,E), a soft set

(1) (G,C) is said to be regular semiopen soft (briefly, RSO soft) set if $\exists$ an regular open soft set (H,B) such that $(H,B)\tilde{\subseteq}(G,C)\tilde{\subseteq}cl(H,B)$.
(2) (L,A) is said to be regular semiclosed soft (briefly, RSC soft) set if $\exists$ an regular closed soft set (K,D) such that $int(K,D)\tilde{\subseteq}(L,A)\tilde{\subseteq}(K,D)$.

We shall denote the family of all regular semiopen soft sets (regular semiclosed soft sets) of a sts (X, τ, E) by $RSOSS(X, E)$, $(RSCSS(X, E))$.

Definition 2.3. [24] Let $(X,\ \tau,\ E)$ be a sts and (G, C) be a soft set over X. Then

(1) the soft regular semiclosure of (G, C), $rsscl(G, C) = \widetilde{\bigcap}\{(S, F) | (G, C)\widetilde{\subseteq} (S, F)$ and $(S, F) \in RSCSS(X, E)\}$ is a soft set.
(2) the soft regular semiinterior of (G, C), $rssint(G, C) = \widetilde{\bigcup}\{(S, F) | (S, F) \widetilde{\subseteq}(G, C)$ and $(S, F) \in RSOSS(X, E)\}$ is a soft set.

Thus $rsscl(G, C)$ is the smallest rscs set containing (G, C) and $rssint(G, C)$ is the largest RSO soft set contained in (G, C).

Theorem 2.5. [10] *Let $SS(X, A)$ and $SS(Y, B)$ be families of soft sets. For the soft function $fpu : SS(X, A) \to SS(Y, B)$, the following statements hold,*

(a) $f_{pu}^{-1}((G, B)') = (f_{pu}^{-1}(G, B))'\ \forall (G, B) \in SS(Y, B)$.
(b) $f_{pu}(f_{pu}^{-1}((G, B)))\widetilde{\subseteq}(G, B)\ \forall\ (G, B) \in SS(Y, B)$. *If f_{pu} is surjective, then the equality holds.*
(c) $(F, A)\widetilde{\subseteq}f_{pu}^{-1}(f_{pu}((F, A)))\ \forall\ (F, A) \in SS(X, A)$. *If f_{pu} is injective, then the equality holds.*
(d) $f_{pu}(\widetilde{X})\widetilde{\subseteq}\widetilde{Y}$. *If f_{pu} is surjective, then the equality holds.*
(e) $f_{pu}^{-1}(\widetilde{Y}) = \widetilde{X}$ *and* $f_{pu}(\Phi, A) = (\Phi, B)$.
(f) If $(F, A)\widetilde{\subseteq}(G, A)$*, then* $f_{pu}(F, A)\widetilde{\subseteq}f_{pu}(G, A)$.
(g) If $(F, B)\widetilde{\subseteq}(G, B)$*, then* $f_{pu}^{-1}(F, B)\widetilde{\cup}f_{pu}^{-1}(G, B)\ \forall\ (F, B), (G, B) \in SS(Y, B)$.
(h) $f_{pu}^{-1}[(F, B)\widetilde{\cup}(G, B)] = f_{pu}^{-1}(F, B)\widetilde{\cup}f_{pu}^{-1}(G, B)$ *and* $f_{pu}^{-1}[(F, B)\widetilde{\cap}(G, B)] = f_{pu}^{-1}(F, B)\widetilde{\cap}f_{pu}^{-1}(G, B)\ \forall\ (F, B), (G, B) \in SS(Y, B)$.
(i) $f_{pu}[(F, A)\widetilde{\cup}(G, A)] = f_{pu}(F, A)\widetilde{\cup}f_{pu}(G, A)$ *and* $f_{pu}[(F, A)\widetilde{\cap}(G, A)]\widetilde{\subseteq} f_{pu}(F, A) \widetilde{\cap}f_{pu}(G, A)\ \forall\ (F, A), (G, A) \in SS(X, A)$. *If f_{pu} is injective, then the equality holds.*

Definition 2.4. [24] Let $(X,\ \tau,\ E)$ and $(Y,\ \tau',\ E^{'})$ be two sts's. A soft function $f : SS(X, E) \to SS(Y, E^{'})$ is said to be

(i) Soft regular semi continuous (briefly, SRS-continuous) if for each open soft set (G, C) of $(Y, E^{'})$, the inverse image $f^{-1}(G, C)$ is a RSO soft set of (X, E).
(ii) Soft regular semi irresolute (briefly, SRS-irresolute) if for each RSO soft set (G, C) of $(Y, E^{'})$, the inverse image $f^{-1}(G, C)$ is a RSO soft set of (X, E).
(ii) Soft regular semi irresolute open (briefly, $SRSI$-open) if for each RSO soft set (G, C) of (X, E), the image $f(G, C)$ is a RSO soft set of $(Y, E^{'})$.

3. Soft regular semi compactness

The study on compactness (which depends on open sets) for a soft topological space was initiated by Zorlutuna et al. in [25]. This section is devoted to introduce regular semi compactness in sts along with its characterization.

Definition 3.1. A cover of a soft set is said to be a soft RSO-cover if every member of the cover is a RSO soft set.

Definition 3.2. A sts (X, τ, E) is said to be soft RS-compact if each soft RSO-cover of (X, E) has a finite subcover.

Remark. *Every soft RS-compact soft topological space is also soft semi compact.*

Theorem 3.1. *A sts (X,τ,E) is soft RS-compact $\Leftrightarrow$ each family of RSC soft sets in (X,E) with the FIP has a nonempty intersection.*

Proof. Let $\{(L_i,A)|i\in I\}$ be a collection of RSC soft sets with the FIP. If possible, assume $\widetilde{\bigcap\limits_{i\in I}}(L_i,A)=(\Phi,E)\Rightarrow \widetilde{\bigcup\limits_{i\in I}}(L_i,A)'=(X,E)$. So, the collection $\{((L_i,A))'\mid i\in I\}$ forms a soft RSO-cover of (X,E), which is soft RS-compact. So, $\exists$ a finite subcollection I_0 of I which also covers (X,E). i.e., $\widetilde{\bigcup\limits_{i\in I}}(L_i,A)'=(X,E)\Rightarrow \widetilde{\bigcap\limits_{i\in I}}(L_i,A)=(\Phi,E)$, a contradiction.

For the converse, if possible, let (X,τ,E) be not soft RS-compact. Then $\exists$ a soft RSO-cover $\{(G_i,C)\mid i\in I\}$ of (X,U), such that for every finite sub collection I_0 of I we have $\widetilde{\bigcup\limits_{i\in I_0}}(G_i,C)\neq(X,E)\Rightarrow \widetilde{\bigcap\limits_{i\in I_0}}(G_i,C)'\neq(\Phi,E)$. Hence $\{(G_i,C)'\mid i\in I\}$ has the FIP. So, by hypothesis $\widetilde{\bigcap\limits_{i\in I_0}}(G_i,C)'\neq(\Phi,E)\Rightarrow \widetilde{\bigcup\limits_{i\in I_0}}(G_i,C)\neq(X,E)$ a contradiction. □

Theorem 3.2. *A sts (X,τ,E) is soft RS-compact iff for every family $\mathcal{A}$ of soft sets with FIP, $\tilde{\bigcap\limits_{(G,C)\in\mathcal{A}}} rsscl(G,C)\neq(\Phi,E)$.*

Proof. Let (X,τ,E) be soft RS-compact and if possible, let $\tilde{\bigcap\limits_{(G,C)\in\mathcal{A}}} rsscl(G,C)=(\Phi,E)$ for some family $\mathcal{A}$ of soft sets with the FIP. So, $\tilde{\bigcup\limits_{(G,C)\in\mathcal{A}}}(rsscl(G,C))'=(X,E)\Rightarrow \mathcal{B}=\{(rsscl(G,C))'\mid (G,C)\in\mathcal{A}\}$ is a soft RSO-cover of (X,E). Then by soft RS-compactness of (X,E), $\exists$ a finite subcover $\mathcal{B}_0$ of $\mathcal{B}$. i.e., $\tilde{\bigcup\limits_{(G,C)\in\mathcal{B}_0}}(rsscl(G,C))'=(X,E)\Rightarrow \tilde{\bigcup\limits_{(G,C)\in\mathcal{B}_0}}(G,C)'=(X,E)\Rightarrow \tilde{\bigcap\limits_{(G,C)\in\mathcal{B}_0}}(G,C)=(\Phi,E)$, a contradiction. Hence $\tilde{\bigcap\limits_{(G,C)\in\mathcal{A}}} rsscl(G,C)\neq(\Phi,E)$.

Conversely, we have $\tilde{\bigcap\limits_{(G,C)\in\mathcal{A}}} rsscl(G,C)\neq(\Phi,E)$, for every family $\mathcal{A}$ of soft sets with FIP. Assume (X,τ,E) is not soft RS-compact. Then $\exists$ a family $\mathcal{B}$ of RSO soft sets covering X without a finite subcover. So, for every finite subfamily $\mathcal{B}_0$ of $\mathcal{B}$ we have $\tilde{\bigcup\limits_{(G,C)\in\mathcal{B}_0}}(G,C)\neq(X,E)\Rightarrow \tilde{\bigcap\limits_{(G,C)\in\mathcal{B}_0}}(G,C)'\neq(\Phi,E)\Rightarrow\{(G,C)'|(G,C)\in\mathcal{B}$ is a family of soft sets with FIP. Now $\tilde{\bigcup\limits_{(G,C)\in\mathcal{B}}}(G,C)=(X,E)\Rightarrow \tilde{\bigcap\limits_{(G,C)\in\mathcal{B}}}(G,C)'=(\Phi,E)$ $\Rightarrow \tilde{\bigcap\limits_{(G,C)\in\mathcal{B}}} rss(G,C)'=(\Phi,E)$, a contradiction. □

Theorem 3.3. *SRS-continuous image of a soft RS-compact space is soft compact.*

Proof. Let $f:SS(X,E)\to SS(Y,E')$ be a SRS-continuous function where (X,τ,E) is a soft RS-compact sts and (Y,δ,E') is another sts. Take a soft open cover $\{(G_i,C)\mid i\in I\}$ of $(Y,E')\Rightarrow\{f^{-1}((G_i,C))\mid i\in I\}$ forms a soft RSO-cover of $(X,E)\Rightarrow$ $\exists$ a finite subset I_0 of I such that $\{f^{-1}((G_i,C))\mid i\in I_0\}$ forms a soft RSO-cover of $(X,E)\Rightarrow\{(G_i,C))\mid i\in I_0\}$ forms a finite soft open cover of (Y,E'). □

Theorem 3.4. *Soft RSC subspace of a soft RS-compact sts is soft RS-compact.*

Proof. Let (Y, B) be a soft RSC subspace of a soft RS-compact sts (X, τ, A) and $\{(G_i, C) \mid i \in I\}$ be a soft RSO-cover of (Y, B). Then the family $\{(G_i, C) \mid i \in I\}\tilde{\bigcup}((X, A) - (Y, B))$ is a soft RSO-cover of (X, A), which has a finite subcover, as (X, A) is soft RS-compact. So, $\{(G_i, C) \mid i \in I\}$ has a finite subfamily to cover (Y, B). Hence (Y, B) is soft RS-compact. □

Let (X, τ_1, E) and (X, τ_2, E) be soft topological spaces. If $\tau_1 \subseteq \tau_2$, then τ_2 is soft finer than τ_1. If $\tau_1 \subseteq \tau_2$ or $\tau_2 \subseteq \tau_1$, then τ_1 is soft comparable with τ_2. Then, we have the following.

Proposition 3.5. *Let (X, τ_2, E) be a soft RS-compact space and $\tau_1 \subseteq \tau_2$. Then (X, τ_1, E) is soft RS-compact.*

Proof. Let $\{(F_i, E)|i \in I\}$ be a soft RSO-cover of $\tilde{X}$ by RSO soft sets of (X, τ_1, E). Since $\tau_1 \subseteq \tau_2$, then $\{(F_i, E)|i \in I\}$ is a soft RSO-cover of $\tilde{X}$ by RSO soft sets of (X, τ_2, E). But (X, τ_2, E) is soft RS-compact. Therefore $(X, E)\widetilde{\subseteq}(F_{i_1}, E)\widetilde{\cup}, \ldots, (F_{i_n}, E)$, for some $i_1, \ldots, i_n \in I$. Hence (X, τ_1, E) is soft RS-compact. □

Let (F, E) be a soft set over X and Y be a nonempty subset of X. Then the sub-soft set of (F, E) over Y denoted by $({}^Y F, E)$ is defined as follows ${}^Y F(e) = Y \cap F(e)$, for each $e \in E$. In other words $({}^Y F, E) = \widetilde{Y} \cap (F, E)$. Now, suppose that (X, τ, E) is a sts over X and Y is a nonempty subset of X. Then $\tau_Y = \{({}^Y F, E) \mid (F, E) \in \tau\}$, is said to be soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E). Here, we exhibit a criterion that implies $\widetilde{Y}$ is soft RS-compact by soft RSO covers of $\widetilde{Y}$, that all of members are RSO soft sets in X.

Theorem 3.6. *Let (Y, τ_Y, E) be a soft subspace of a soft space (X, τ, E). Then (Y, τ_Y, E) is soft RS-compact if and only if every cover of $\widetilde{Y}$ by RSO soft sets in X contains a finite subcover.*

Proof. Let (Y, τ_Y, E) be soft RS-compact and $\{(F_i, E)|i \in I\}$ be a cover of $\widetilde{Y}$ by RSO soft sets in X. By Propositions 2.1 and 2.2, we can see that $\{({}^Y F_i, E)|i \in I\}$ is a soft RSO cover of $\widetilde{Y}$. Therefore

$$(Y, E)\widetilde{\subseteq}({}^Y F_{i_1}, E)\cup, \ldots, \cup({}^Y F_{i_n}, E),$$

for some $i_1, \ldots, i_n \in I$. This implies that $\{(F_{i_k}, E)\}_{k=1}^{n}$ is a subcover of $\widetilde{Y}$ by RSO soft sets in X.

Conversely, let $\{({}^Y F_i, E)|i \in I\}$ be a soft RSO cover of $\widetilde{Y}$. It is easy to see that $\{(F_i, E)|i \in I\}$ is a cover of $\widetilde{Y}$ by RSO soft sets in X. Then we can write

$$Y\widetilde{\subseteq}(F_{i_1}, E)\cup, \ldots, \cup(F_{i_n}, E),$$

for some $i_1, \ldots, i_n \in I$. Therefore $\{({}^Y F_{i_k}, E)\}_{k=1}^{n}$ is a subcover of $\widetilde{Y}$. Hence (Y, τ_Y, E) is soft RS-compact. □

Definition 3.3. A soft space (X, τ, E) is said to be soft RS-Hausdorff if for each pair $x,\ y$ of distinct points of X, there exist disjoint RSO soft sets containing x and y, respectively.

Theorem 3.7. *Every soft RS-compact subspace of a soft RS-Hausdorff space is soft RSC.*

Proof. Let (Y, τ_Y, E) be a soft RS-compact subspace of soft RS-Hausdorff space (X, τ, E). Let $x \in (X, E) - (Y, E)$. Then for all $y \in (Y, E)$, $x \neq y$. Therefore, there exist RSO soft sets (U_y, E) and (U_{xy}, E) containing x and y, respectively such that $(U_y, E) \cap (U_{xy}, E) = \Phi$. Obviously, $\{(U_{xy}, E) | y \in Y\}$ is a cover of $\widetilde{Y}$ by RSO soft sets in X. By Theorem 3.6, we have $(Y, E) \widetilde{\subseteq} (U_{xy_1}, E) \cup, \ldots, \cup (U_{xy_n}, E)$ for some $y_1, \ldots, y_n \in Y$. Now, $x \in (U_{y_1}, E) \cap \cdots \cap (U_{y_n}, E) = (U_x, E)$ and Proposition 2.2 implies that $(U_x, E) \cap (Y, E) = (\Phi, E)$. Hence $x \in (U_x, E) \widetilde{\subseteq} (X, E) - (Y, E)$. Then $(X, E) - (Y, E) = \bigcup_{x \in X - Y} (U_x, E)$. Therefore $(X, E) - (Y, E)$ is soft RSO. Hence (Y, E) is soft RSC. □

Now, we consider the countably soft RS-compact spaces in soft topology. A soft topological space (X, τ, E) is said to be countably soft RS-compact if every countable soft RSO cover of $\widetilde{X}$ contains a finite subcover of $\widetilde{X}$. Obviously, every soft RS-compact space is countably soft RS-compact.

There is a criterion for a soft space to be countable soft RS-compact in terms of soft RSC sets rather than soft RSO sets. First we have a definition.

A collection $\mathcal{A}$ of soft set is said to have the FIP if for every finite sub-collection $\{(A_1, E), \ldots, (A_n, E)\}$ of $\mathcal{A}$, the intersection $(A_1, E) \cap \ldots \cap (A_n, E)$ is non-null.

Theorem 3.8. *A sts is countably soft RS-compact if and only if every countable family of soft RSC sets with the FIP has a non-null intersection.*

Proof. Let the soft space (X, τ, E) be countably soft RS-compact. Let the family $\{(F_i, E)\}_{i=1}^{\infty}$ of RSC soft sets have the FIP. If $\bigcap_{i=1}^{\infty} (F_i, E) = (\Phi, E)$ by Proposition 2.2, $\{(F_i, E)'\}_{i=1}^{\infty}$ is a countable soft RSO cover of $\widetilde{X}$. Therefore $\widetilde{X} = (F_{i_1}, E)' \cup \ldots \cup (F_{i_k}, E)'$, for some $i_1, \ldots, i_k \in N$. Now, De Morgan laws and Proposition 2.2 imply that $(F_{i_1}, E) \cap \ldots \cap (F_{i_k}, E) = (\Phi, E)$. This is a contradiction.

Conversely, Let $\{(F_i, E)\}_{i=1}^{\infty}$ be a countable soft RSO-cover of $\widetilde{X}$ without any subcover. Then $\{(F_i, E)'\}_{i=1}^{\infty}$ is a family of RSC soft sets over X such that $\bigcap_{i=1}^{\infty} (F_i, E)' = (\Phi, E)$. Let $i_1, \ldots, i_k$ be arbitrary positive integers. If $(F_{i_1}, E)' \cap \ldots \cap (F_{i_k}, E)' = (\Phi, E)$ then $\widetilde{X} = (F_{i_1}, E) \cup \ldots \cup (F_{i_k}, E)$, that is impossible. Therefore $(F_{i_1}, E)' \cap \ldots \cap (F_{i_k}, E)' \neq (\Phi, E)$ for each $i_1, \ldots, i_k \in N$. This shows that $\{(F_i, E)'\}_{i=1}^{\infty}$ have the FIP. Therefore $\bigcap_{i=1}^{\infty} (F_i, E)' \neq (\Phi, E)$. This is a contradiction. □

An immediate result of previous theorem is the following.

Corollary 3.9. *A soft space (X, τ, E) is countably soft RS-compact if and only if every nested sequence $(F_1, E) \widetilde{\supseteq} (F_2, E) \widetilde{\supseteq} \ldots$ of nonnull RSC soft sets over X has a non-null intersection.*

Proof. Let (X, τ, E) is countably soft RS-compact. The collection $\{(F_i, E)\}_{i=1}^{\infty}$ have the FIP. Therefore $\bigcap_{i=1}^{\infty} (F_i, E)' \neq (\Phi, E)$. Conversely, let $\{(C_i, E)\}_{i=1}^{\infty}$ be a collection of RSC soft sets with the FIP. By Proposition 2.2, we construct nested sequence $(F_1, E) \widetilde{\supseteq} (F_2, E) \widetilde{\supseteq} \ldots$ of non-null RSC soft sets by setting $(F_i, E) = (C_1, E) \cap \cap \cdots \cap (C_i, E)$, for each positive integer i. By the hypothesis $\bigcap_{i=1}^{\infty} (F_i, E) = \bigcap_{i=1}^{\infty} (C_i, E) \neq (\Phi, E)$. Now, Theorem 3.8 implies that (X, τ, E) is countably soft RS-compact. □

4. Soft regular semi connectedness

Definition 4.1. Two soft sets (L, A) and (H, B) are said to be disjoint if $(L, A)(x) \cap (H, B)(y) = \Phi \; \forall \; x \in A, \; y \in B$.

Definition 4.2. A soft RS-separation of sts (X, τ, E) is a pair $(L, A), \; (H, B)$ of disjoint nonnull RSO soft sets whose union is (X, E).

If there doesn exist a soft RS-separation of (X, E), then the sts is said to be soft RS-connected, otherwise soft RS-disconnected.

Example 4.3. Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$, and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E)\}$, where $(F_1, E), \; (F_2, E)$ are soft sets over X defined as follows:
$F_1(e_1) = \{h_1\}, \; F_1(e_2) = \{h_2\}$
$F_2(e_1) = \{h_2\}, \; F_2(e_2) = \{h_1\}$
Then τ defines a soft topology on X. So $(F_1, E), (F_2, E)$ are RSO soft sets in (X, E) but $(F_1, E)\widetilde{\cap}(F_2, E) = (\Phi, E)$, so there is a soft RS-separation of (X, E) and hence is soft RS-disconnected.

Theorem 4.1. *If the soft sets (L, A) and (G, C) form a soft RS-separation of (X, E), and if (Y, B) is a soft RS-connected subspace of (X, E), then $(Y, B)\widetilde{\subseteq}(L, A)$ or $(Y, B)\widetilde{\subseteq}(G, C)$.*

Proof. Since (L, A) and (G, C) are disjoint RSO soft sets, so are $(L, A)\widetilde{\cap}(Y, B)$ and $(G, C)\widetilde{\cap}(Y, B)$ and their soft union gives (Y, B), i.e., they would constitute a soft RS-separation of (Y, B), a contradiction. Hence, one of $(L, A)\widetilde{\cap}(Y, B)$ and $(G, C)\widetilde{\cap}(Y, B)$ is empty and so (Y, B) is entirely contained in one of them. □

Theorem 4.2. *Let (Y, B) be a soft RS-connected subspace of (X, E) and (K, D) be a soft set in (X, E) such that $(Y, B)\widetilde{\subseteq}(K, D)\widetilde{\subseteq}cl(Y, B)$, then (K, D) is also soft RS-connected.*

Proof. Let the soft set (K, D) satisfies the hypothesis. If possible, let (F, A) and (G, C) form a soft RS-separation of (K, D). Then, by Theorem 4.1, $(Y, B)\widetilde{\subseteq}(F, A)$ or $(Y, B)\widetilde{\subseteq}(G, C)$. Let $(Y, B)\widetilde{\subseteq}(F, A) \Rightarrow rsscl(Y, B)\widetilde{\subseteq}rsscl(F, A)$; since $rsscl(F, A)$ and (G, C) are disjoint, (Y, B) cannot intersect (G, C). This contradicts the fact that (G, C) is a nonempty subset of $(Y, B) \Rightarrow \nexists$ a soft RS-separation of (K, D) and hence is soft RS-connected. □

Theorem 4.3. *A sts (X, τ, E) is soft RS-disconnected $\Leftrightarrow \exists$ a nonnull proper soft subset of (X, E) which is both RSO and RSC soft sets.*

Proof. Let (K, D) be a nonnull proper soft subset of (X, E) which is both RSO and RSC soft sets. Now $(H, C) = (K, D)'$ is nonnull proper subset of (X, E) which is also both RSO and RSC soft sets $\Rightarrow rsscl(K, D) = (K, D)$ and $rsscl(H, C) = (H, C) \Rightarrow (X, E)$ can be expressed as the soft union of two RS-separated soft sets $(K, D), \; (H, C)$ and so, is soft RS-disconnected.

Conversely, let (X, E) be soft RS-disconnected $\Rightarrow \exists$ nonnull soft subsets (K, D) and (H, C) such that $rsscl(K, D)\tilde{\cap}(H, C) = (\Phi, E), \; (K, D)\tilde{\cap}rsscl(H, C) = (\Phi, E)$ and $(K, D)\tilde{\cup}(H, C) = (X, E)$. Now $(K, D)\tilde{\subseteq}rsscl(K, D)$ and $rsscl(K, D)\tilde{\cap}(H, C) = (\Phi, E) \Rightarrow (K, D)\tilde{\cap}(H, C) = (\Phi, E) \Rightarrow (H, C) = (K, D)'$. Then $(K, D)\tilde{\cup}rsscl(H, C) = (X, E)$ and $(K, D)\tilde{\cap}rsscl(H, C) = (\Phi, E) \Rightarrow (K, D) = (rsscl(H, C))'$ and similarly $(H, C) = (rsscl(K, D))' \Rightarrow (K, D), \; (H, C)$ are RSO soft sets being the complements of RSC soft sets. Also $(H, C) = (K, D)' \Rightarrow$ they are also soft RSC. □

Theorem 4.4. *SRS-continuous image of a soft RS-connected sts is soft connected.*

Proof. Let $f : SS(X,E) \to SS(Y,E)$ be a SRS-continuous function where (X,τ,E) a soft RS-connected sts and (Y,δ,E) is a sts. We wish to show $f(X,E)$ is soft connected. Suppose $f(X,E) = (K,D)\tilde{\cup}(H,C)$ be a soft separation. i.e., (K,D) and (H,C) are disjoint soft open sets whose union is $f(X,E) \Rightarrow f^{-1}(K,D)$ and $f^{-1}(H,C)$ are disjoint RSO soft sets whose union is (X,E). So, $f^{-1}(K,D)$ and $f^{-1}(H,C)$ form a soft RS-separation of (X,E), a contradiction. □

Theorem 4.5. *SRS-irresolute image of a soft RS-connected sts is soft RS-connected.*

Proof. Similar to that of Theorem 4.4 □

5. SOFT REGULAR SEMI SEPARATION AXIOMS

Soft separation axioms for sts were studied by Shabir et al. [23]. Here we consider separation axioms for sts's using RSO and RSC soft sets.

Definition 5.1. Two soft sets (G,C) and (H,B) are said to be distinct if $G(e) \cap H(e) = \Phi,\ \forall\, e \in B \cap C$.

Definition 5.2. A sts (X,τ,E) is said to be a soft RS-T_0 space if for two disjoint soft points $G(e)$ and $F(e)$, $\exists$ a RSO soft set containing one but not the other.

Example 5.3. Let $X = \{x_1,x_2\}$, $E = \{e_1,e_2\}$, and $\tau = \{\Phi, \widetilde{X}, (F_1,E), (F_2,E)\}$, where $(F_1,E),\ (F_2,E)$ are soft sets over X defined as follows:
$F_1(e_1) = \{x_1\},\ F_1(e_2) = \{x_1\}$
$F_2(e_1) = \{x_2\},\ F_2(e_2) = \{x_2\}$
Then τ defines a soft topology on X. Also (X,τ,E) is soft RS-T_0 space, since $x_1,x_2 \in X$, $x_1 \neq x_2$, $\exists$ a RSO soft sets $(F_1,E),\ (F_2,E)$ such that $x_1 \in (F_1,E)$ and $x_2 \notin (F_1,E)$ or $x_1 \notin (F_2,E)$ and $x_2 \in (F_1,E)$.

Theorem 5.1. *Let (X,τ,E) be a sts and the soft points $F(e),\ G(e)$ of (X,E) such that $F(e) \neq G(e)$. If there exists RSO soft sets (G,C) and (H,D) such that $F(e)\widetilde{\in}(G,C)$ and $G(e)\widetilde{\notin}(H,D)$ or $G(e)\widetilde{\in}(H,D)$ and $F(e)\widetilde{\notin}(G,C)$. Then (X,τ,E) is soft RS-T_0 space.*

Proof. Let $F(e)$ and $G(e)$ be two distinct soft points in a sts (X,τ,E). Let (G,C) and (H,D) be RSO soft sets such that either $F(e)\widetilde{\in}(G,C)$ and $G(e)\widetilde{\in}(G,C)'$ or $G(e)\widetilde{\in}(H,D)$ and $F(e)\widetilde{\in}(G,C)'$. If $F(e)\widetilde{\in}(G,C)$ and $G(e)\widetilde{\in}(G,C)'$. Then $G(e)\widetilde{\in}(G,C)'$ for all $e \in C$, this implies that $G(e)\widetilde{\notin}(G,C)$ for all $e \in C$. Therefore, $G(e)\widetilde{\notin}(G,C)$. Similarly, if $G(e)\widetilde{\in}(H,D)$ and $F(e)\widetilde{\in}(H,D)'$, then $F(e)\widetilde{\notin}(H,D)$. Hence (X,τ,E) is soft RS-T_0 space. □

Theorem 5.2. *A sts is a soft RS-T_0 space if the soft regular semiclosures of distinct soft points are distinct.*

Proof. Let $F(e)$ and $H(e)$ be two distinct soft points with distinct soft regular semiclosures in a sts (X,τ,E). If possible, suppose we had $F(e)\widetilde{\in}rsscl(H(e))$, then $rsscl(F(e))\widetilde{\subseteq}rsscl(H(e))$, a contradiction. So $F(e)\widetilde{\notin}rsscl(H(e)) \Rightarrow (rsscl(H(e)))'$ is a RSO soft set containing $F(e)$ but not $H(e)$. Hence (X,τ,E) is a soft RS-T_0 space. □

Theorem 5.3. *A soft subspace of a soft RS-T_0 space is soft RS-T_0.*

Proof. Let (Y,B) be a soft subspace of a soft RS-T_0 space (X,E) and let $F(e),\ G(e)$ be two distinct soft points of (Y,B). Then these soft points are also in $(X,E) \Rightarrow \exists$ a RSO soft set (H,D) containing one of these soft points, say $F(e)$, but not the other $\Rightarrow (H,D)\widetilde{\cap}(Y,B)$ is a RSO soft set containing $F(e)$ but not the other. □

Definition 5.4. A sts (X, τ, E) is said to be a soft RS-T_1-space if for two distinct soft points $F(e),\ G(e)$ of (X, E), $\exists\ RSO$ soft sets (H, D) and (G, C) such that
$F(e)\widetilde{\in}(H, D)$ and $G(e)\widetilde{\notin}(H, D)$;
$G(e)\widetilde{\in}(G, C)$ and $F(e)\widetilde{\notin}(G, C)$.

Example 5.5. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$ where
$F_1(e_1) = \{x_1, x_2\},\ F_1(e_2) = \{x_1, x_3\}$
$F_2(e_1) = \{x_2, x_3\},\ F_2(e_2) = \{x_1, x_2\}$
$F_3(e_1) = \{x_1, x_3\},\ F_3(e_2) = \{x_2, x_3\}$
$F_4(e_1) = \{x_2\},\ F_4(e_2) = \{x_1\}$
$F_5(e_1) = \{x_3\},\ F_5(e_2) = \{x_2\}$
$F_6(e_1) = \{x_1\},\ F_2(e_2) = \{x_3\}$
Then (X, τ, E) is a sts over X. We note that (X, τ, E) is soft RS-T_1 space, because there exists RSO soft sets $(F_1, E), (F_2, E), (F_3, E)$ such that $x_1 \in (F_1, E),\ x_2 \notin (F_1, E)$ and $x_2 \in (F_2, E),\ x_1 \notin (F_2, E)$; $x_1 \in (F_1, E),\ x_3 \notin (F_1, E)$ and $x_3 \in (F_3, E),\ x_1 \notin (F_3, E)$; $x_2 \in (F_2, E),\ x_3 \notin (F_2, E)$ and $x_3 \in (F_3, E),\ x_2 \notin (F_3, E)$.

Theorem 5.4. *Let (X, τ, E) be a sts and the soft points $F(e),\ G(e)$ of (X, E) such that $F(e) \neq G(e)$. If there exists RSO soft sets (G, C) and (H, D) such that $F(e)\widetilde{\in}(G, C)$ and $G(e)\widetilde{\notin}(H, D)$ and $G(e)\widetilde{\in}(H, D)$ and $F(e)\widetilde{\notin}(G, C)$. Then (X, τ, E) is soft RS-T_1 space.*

Proof. It is similar to the proof of Theorem 5.1 □

Theorem 5.5. *If every soft point of a sts (X, τ, E) is a RSC soft set then (X, τ, E) is a soft RS-T_1 space.*

Proof. Let $F(e)$ and $G(e)$ be two distinct soft points of $(X, E) \Rightarrow (F(e))'$, $(G(e))'$ are RSO soft sets such that $G(e)\widetilde{\in}(F(e))'$ and $G(e)\widetilde{\notin}F(e)$; $F(e)\widetilde{\in}(G(e))'$ and $F(e)\widetilde{\notin}G(e)$. □

Theorem 5.6. *A soft subspace of a soft RS-T_1 space is soft RS-T_1.*

Proof. It is similar to the proof of Theorem 5.3 □

Definition 5.6. A sts (X, τ, E) is said to be a soft RS-T_2 space if and only if for distinct soft points $F(e),\ G(e)$ of (X, E), $\exists$ disjoint RSO soft sets (H, B) and (G, C) such that $F(e)\widetilde{\in}(H, B)$ and $G(e)\widetilde{\in}(G, C)$.

Example 5.7. In Example 5.3, (X, τ, E) is also soft RS-T_2 space.

Example 5.8. Let us consider the soft topology (X, τ, E) on Exanple 5.5. Now we show that (X, τ, E) is not a soft RS-T_2 space. For $x_1 \neq x_2$, $x_1 \in (F_1, E),\ x_2 \in (F_2, E)$ and $(F_1, E)\widetilde{\cap}(F_2, E) = \{\{x_2\}, \{x_1\}\} \neq (\Phi, E)$. Then (X, τ, E) is not soft RS-T_2 space.

Theorem 5.7. *A soft subspace of a soft RS-T_2 space is soft RS-T_2.*

Proof. Let (X, τ, E) be a soft RS-T_2 space and (Y, B) be a soft subspace of (X, E), where $B \subseteq E$ and $Y \subseteq X$. Let $F(e)$ and $G(e)$ be two distinct soft points of (Y, B). (X, E) is soft RS-$T_2 \Rightarrow \exists$ two disjoint RSO soft sets (H, D) and (G, C) such that $F(e)\widetilde{\in}(H, D),\ G(e)\widetilde{\in}(G, C)$. Then $(H, D)\widetilde{\cap}(Y, B)$ and $(G, C)\widetilde{\cap}(Y, B)$ are RSO soft sets satisfying the requirements for (Y, B) to be a soft RS-T_2 space. □

Theorem 5.8. *A sts (X, τ, E) is soft RS-T_2 if and only if for distinct soft points $G(e),\ F(e)$ of (X, E), $\exists$ a RSO soft set (F, A) containing $G(e)$ but not $F(e)$ such that $F(e)\widetilde{\notin} rsscl(F, A)$.*

Proof. Let $G(e),\ F(e)$ be distinct soft points in a soft RS-T_2 space (X, τ, E).

($\Rightarrow$) $\exists$ distinct RSO soft sets (G, C) and (H, D) such that $F(e)\widetilde{\in}(G, C),\ G(e)\widetilde{\in}(H, D)$. This implies $G(e)\widetilde{\in}(G, C)'$. So, $(G, C)'$ is a RSC soft set containing $G(e)$ but not $F(e)$ and $rsscl(G, C)' = (G, C)'$.

($\Leftarrow$) Take a pair of distinct soft points $G(e)$ and $F(e)$ of (X, E), $\exists$ a RSO soft set (H, D) containing $G(e)$ but not $F(e)$ such that $F(e)\widetilde{\notin} rsscl(H, D) \Rightarrow F(e)\widetilde{\in}(rsscl(H, D))'$ $\Rightarrow (H, D)$ and $(rsscl(H, D))'$ are disjoint RSO soft set containing $G(e)$ and $F(e)$ respectively. □

Definition 5.9. A sts (X, τ, E) is said to be a soft RS-regular space if for every soft point $F(e)$ and RSC soft set (L, A) not containing $F(e)$, $\exists$ disjoint RSO soft sets (G, C) and (H, D) such that $F(e)\widetilde{\in}(G, C)$ and $(L, A)\widetilde{\subseteq}(H, D)$, where $C, D \in E$.

A soft RS-regular with soft RS-T_1 space is called a soft RS-T_3 space.

Example 5.10. Let us consider the soft topology (X, τ, E) on Exanple 5.5. We know that (X, τ, E) is soft RS-T_1 space from Example 5.5. Now we show that (X, τ, E) is not a soft RS-T_3 space. For this,
$\tau' = \{\Phi, X, (F_1, E)', (F_2, E)', (F_3, E)', (F_4, E)', (F_5, E)', (F_6, E)'\}$
where,
$F_1'(e_1) = \{x_3\},\ F_1'(e_2) = \{x_2\}$
$F_2'(e_1) = \{x_1\},\ F_2'(e_2) = \{x_3\}$
$F_3'(e_1) = \{x_2\},\ F_3'(e_2) = \{x_1\}$
$F_4'(e_1) = \{x_1, x_3\},\ F_4'(e_2) = \{x_2, x_3\}$
$F_5'(e_1) = \{x_1, x_2\},\ F_5'(e_2) = \{x_1, x_3\}$
$F_6'(e_1) = \{x_2, x_3\},\ F_6'(e_2) = \{x_1, x_2\}$
Then, $x_1 \in (F_1, E)$, a RSC soft set $(G, E) = (F_1, E)'$ and $x_1 \notin (G, E)$, then there exists a RSO soft sets (F_2, E) and (F_5, E) such that $x_1 \in (F_2, E)$ and $(G, E)\widetilde{\subseteq}(F_5, E)$ but $(F_2, E)\widetilde{\cap}(F_5, E) = \{\{x_3\}, \{x_2\}\} \neq (\Phi, E)$. Then (X, τ, E) is not soft RS-regular space, so (X, τ, E) is not a soft RS-T_3 space.

Remark. *It can be shown that the property of being soft RS-T_3 is hereditary.*

Remark. *Soft RS-$T_3 \Rightarrow$ soft RS-$T_2 \Rightarrow$ soft RS-$T_1 \Rightarrow$ soft RS-T_0.*

Definition 5.11. A cover of a soft set is said to be a soft RSO-cover if every member of the cover is a RSO soft set.

Definition 5.12. A sts (X, τ, E) is said to be soft RS-compact if each soft RSO-cover of (X, E) has a finite subcover.

Theorem 5.9. *A sts which is both soft RS-compact and soft RS-T_2 is soft RS-T_3.*

Proof. It suffices to show every soft RS-compact sts is soft RS-regular. Let $F(e)$ be a soft point and (H, D) be a RSC soft set not containing the point $\Rightarrow\ F(e)\widetilde{\in}(H, D)'$. Now for each soft point $G(e)$, $\exists$ disjoint RSO soft sets (G, C_1) and (G, C_2) such that $G(e)\widetilde{\in}(G, C_1)$ and $F(e)\widetilde{\in}(G, C_2)$. So, the collection $\{(G_i, C) | i \in I\}$ forms a RSO-cover of (H, D). Now (H, D) is a RSC soft set $\Rightarrow (H, D)$ is soft RS-compact. Hence $\exists$ a finite subfamily I_0 of I such that $(H, D)\widetilde{\subseteq} \bigcup_{i \in I_0} (G_i, C)$. Take $(K, B) = \bigcap_{i=1}^{n} (K_i, B)$ and

$(G,C)=\bigcup_{i=1}^{n}(G_i,C)$. Then $(K,B),\ (G,C)$ are disjoint RSO sets such that $F(e)$ is a soft point of (K,B) and $(L,A)\widetilde{\subseteq}(G,C)$. □

Proposition 5.10. *Let (X,τ,E) be a sts, (G,C) be a RSC soft set in (X,E) and $F(e)$ be a soft point such that $F(e)\widetilde{\notin}(G,C)$. If (X,τ,E) is soft RS-regular space, then there exists a RSO soft set (K,D) such that $F(e)\widetilde{\in}(K,D)$ and $(K,D)\widetilde{\cap}(G,C)=(\Phi,E)$.*

Proof. It is obvious from Definition 5.9 □

Proposition 5.11. *Let (X,τ,E) be a sts, $(G,C)\widetilde{\in}\ SS(X,E)$ and $x\in X$. Then:*

(i) $x\in(G,C)$ if and only if $(x,E)\widetilde{\subseteq}(G,C)$.
(ii) If $(x,E)\widetilde{\cap}(G,C)=(\Phi,E)$, then $x\notin(G,C)$.

Proof. Obvious. □

Theorem 5.12. *Let (X,τ,E) be a sts and $x\in X$. If (X,τ,E) is soft RS-regular space, then:*

(1) $x\notin(G,C)$ if and only if $(x,E)\widetilde{\cap}(G,C)=(\Phi,E)$ for every RSC soft set (G,C).
(2) $x\notin(H,D)$ if and only if $(x,E)\widetilde{\cap}(H,D)=(\Phi,E)$ for every RSO soft set (H,D).

Proof. (i) Let (G,C) be a RSC soft set such that $x\notin(G,C)$. Since (X,τ,E) is soft RS-regular space. Then by Proposition 5.10 there exists a RSO soft set (H,D) such that $x\in(H,D)$ and $(G,C)\widetilde{\cap}(H,D)=(\Phi,E)$. It follows that $(x,E)\widetilde{\subseteq}(H,D)$ from Proposition 5.11(1). Hence $(x,E)\widetilde{\cap}(G,C)=(\Phi,E)$. Conversely, if $(x,E)\widetilde{\cap}(G,C)=(\Phi,E)$, then $x\notin(G,C)$ from Proposition 5.11(2).

(ii) Let (H,D) be a RSO soft set such that $x\notin(H,D)$. If $x\notin F(e)$ for all $e\in E$, then we get the proof. If $x\notin F(e_1)$ and $x\in F(e_2)$ for some $e_1,e_2\in E$, then $x\in(F(e_1))'$ and $x\notin(F(e_2))'$ for some $e_1,e_2\in E$. This means that, $(x,E)\widetilde{\cap}(H,D)\neq(\Phi,E)$. Hence $(H,D)'$ is RSC soft set such that $x\notin(H,D)'$. It follows by (1) $(x,E)\widetilde{\cap}(H,D)'=(\Phi,E)$. This implies that, $(x,E)\widetilde{\subseteq}(H,D)$ and so $x\in(H,D)$, which is contradiction with $x\notin F(e_1)$ for some $e_1\in E$. Therefore, $(x,E)\widetilde{\cap}(H,D)=(\Phi,E)$. Conversely, if $(x,E)\widetilde{\cap}(H,D)=(\Phi,E)$, then it is obvious that $x\notin(H,D)$. This completes the proof. □

Corollary 5.13. *Let (X,τ,E) be a sts and $x\in X$. If (X,τ,E) is soft RS-regular space, then the following are equivalent:*

(1) (X,τ,E) is soft RS-T_1 space.
(2) $\forall\ x,y\in X$ such that $x\neq y$, there exist RSO soft sets (G,C) and (H,D) such that $(x,E)\widetilde{\subseteq}(G,C)$ and $(y,E)\widetilde{\cap}(G,C)=(\Phi,E)$ and $(y,E)\widetilde{\subseteq}(H,D)$ and $(x,E)\widetilde{\cap}(H,D)=(\Phi,E)$.

Proof. It is obvious from Theorem 5.12 □

Theorem 5.14. *Let (X,τ,E) be a sts and $x\in X$. Then the following are equivalent:*

(1) (X,τ,E) is soft RS-regular space.
(2) For every RSC soft set (G,C) such that $(x,E)\widetilde{\cap}(G,C)=(\Phi,E)$, there exist RSO soft sets (L,A) and (H,D) such that $(x,E)\widetilde{\subseteq}(L,A)$, $(G,C)\widetilde{\subseteq}(H,D)$ and $(L,A)\widetilde{\cap}(H,D)=(\Phi,E)$.

Proof. (1)⇒(2): Let (G, C) be a RSC soft set such that $(x, E)\widetilde{\cap}(G, C) = (\Phi, E)$. Then $x \notin (G, C)$ from Theorem 5.12(1). It follows by (1), there exist RSO soft sets (L, A) and (H, D) such that $x \in (L, A)$, $(G, C)\widetilde{\subseteq}(H, D)$ and $(L, A)\widetilde{\cap}(H, D) = (\Phi, E)$. This means that, $(x, E)\widetilde{\subseteq}(L, A)$, $(G, C)\widetilde{\subseteq}(H, D)$ and $(L, A)\widetilde{\cap}(H, D) = (\Phi, E)$.

(2)⇒(1): Let (G, C) be a RSC soft set such that $x \notin (G, C)$. Then $(x, E)\widetilde{\cap}(G, C) = (\Phi, E)$ from Theorem 5.12(1). It follows by (2), there exist RSO soft sets (L, A) and (H, D) such that $(x, E)\widetilde{\subseteq}(L, A)$, $(G, C)\widetilde{\subseteq}(H, D)$ and $(L, A)\widetilde{\cap}(H, D) = (\Phi, E)$. Hence $(x, E)\widetilde{\subseteq}(L, A)$, $(G, C)\widetilde{\subseteq}(H, D)$ and $(L, A)\widetilde{\cap}(H, D) = (\Phi, E)$. Thus, (X, τ, E) is soft RS-regular space. □

Theorem 5.15. *Let (X, τ, E) be a sts. If (X, τ, E) is soft RS-T_3 space, then $\forall\ x \in X$, (x, E) is RSC soft set.*

Proof. We want to prove that (x, E) is RSC soft set, which is suficient to prove that $(x, E)'$ is RSO soft set for all $y \in \{x\}'$. Since (X, τ, E) is soft RS-T_3-space, then there exist RSO soft sets $(H, D)_y$ and (G, C) such that $(y, E)\widetilde{\subseteq}(H, D)_y$ and $(x, E)\widetilde{\cap}(H, D)_y = (\Phi, E)$ and $(x, E)\widetilde{\subseteq}(G, C)$ and $(y, E)\widetilde{\cap}(G, C) = (\Phi, E)$. It follows that $\widetilde{\bigcup}_{y\in\{x\}'} (H, D)_y \widetilde{\subseteq}(x, E)'$. Now we want to prove that $(x, E)'\widetilde{\subseteq} \widetilde{\bigcup}_{y\in\{x\}'} (H, D)_y$. Let $\widetilde{\bigcup}_{y\in\{x\}'} (H, D)_y = (F, A)$, where $F(e) = \widetilde{\bigcup}_{y\in\{x\}'} (F(e)_y)$ for all $e \in E$. Since $(x, E)'(e) = \{x\}'$ for all $e \in E$ from Definition 15 in [23]. So, for all $y \in \{x\}'$ and $e \in E$, $(x, E)'(e) = \{x\} = \widetilde{\bigcup}_{y\in\{x\}'} \{y\} = \widetilde{\bigcup}_{y\in\{x\}'} (y, E)(e)\widetilde{\subseteq} \widetilde{\bigcup}_{y\in\{x\}'} F(e)_y = F(e)$. Thus, $(x, E)'\widetilde{\subseteq} \widetilde{\bigcup}_{y\in\{x\}'} (H, D)_y$ from definition of soft subsets and so $(x, E)' = \widetilde{\bigcup}_{y\in\{x\}'} (H, D)_y$. This means that, $(x, E)'$ is RSO soft set for all $y \in \{x\}'$. Therefore, (x, E) is RSC soft set. □

Theorem 5.16. *Every soft RS-T_3 space is soft RS-T_2 space.*

Proof. Let (X, τ, E) be a soft RS-T_3 space and $x,\ y \in X$ such that $x \neq y$. By Theorem 5.15, (y, E) is RSC soft set and $x \notin (y, E)$. It follows from the soft RS-regularity, there exist RSO soft sets (G, C) and (H, D) such that $x \in (G, C)$, $(y, E)\widetilde{\subseteq}(H, D)$ and $(G, C)\widetilde{\cap}(H, D) = (\Phi, E)$. Thus, $x \in (G, C)$, $y \in (y, E)\widetilde{\subseteq}(H, D)$ and $(G, C)\widetilde{\cap}(H, D) = (\Phi, E)$. Therefore, (X, τ, E) is soft RS-T_2 space. □

Theorem 5.17. *A soft subspace (Y, τ_Y, E) of a soft RS-T_3 space (X, τ, E) is soft RS-T_3.*

Proof. By Theorem 5.3 (Y, τ_Y, E) is soft RS-T_1 space. Now we want to prove that (Y, τ_Y, E) is soft RS-regular space. Let $y \in Y$ and (G, C) be a RSC soft set in Y such that $y \notin (G, C)$. Then $(G, C) = (Y, E)\widetilde{\cap}(H, D)$ for some RSC soft set (H, D) in X from Theorem 2.4 Hence $y \notin (Y, E)\widetilde{\cap}(H, D)$. But $y \in (Y, E)$, so $y \notin (H, D)$. Since (X, τ, E) is soft RS-T_3-space, so there exist RSO soft sets (H_1, D) and (H_2, D) in X such that $y \in (H_1, D)$, $(H, D)\widetilde{\subseteq}(H_2, D)$ and $(H_1, D)\widetilde{\cap}(H_2, D) = (\Phi, E)$. Take $(G_1, C) = (Y, E)\widetilde{\cap}(H_1, D)$ and $(G_2, C) = (Y, E)\widetilde{\cap}(H_2, D)$, then (G_1, C), (G_2, C) are RSO soft sets in Y such that $y \in (G_1, C)$, $(G, C)\widetilde{\subseteq}(Y, E)\widetilde{\cap}(H_2, D) = (G_2, C)$ and $(G_1, C)\widetilde{\cap}(G_2, C)\widetilde{\subseteq}(H_1, D)\widetilde{\cap}(H_2, D) = (\Phi, E)$. Thus, (Y, τ_Y, E) is soft RS-T_3 space. □

Definition 5.13. A sts (X, τ, E) is said to be a soft RS-normal space if for every pair of disjoint RSC soft sets (G_1, C) and (G_2, C), $\exists$ two disjoint RSO soft sets $(H_1, D),\ (H_2, D)$ such that $(G_1, C)\widetilde{\subseteq}(H_1, D)$ and $(G_2, C)\widetilde{\subseteq}(H_2, D)$.

A soft RS-normal with T_1-space is called a soft RS-T_4-space.

Example 5.14. Let us consider the soft topology (X, τ, E) on Example 5.5. We know that (X, τ, E) is a soft RS-T_1-space from Example 5.5. Now we show that (X, τ, E) is a soft RS-T_4-space. Here, $(F_4, E),\ (F_5, E),\ (F_6, E)$ are soft RSC sets such that $(F_5, E)\widetilde{\cap}(F_6, E) = (\Phi, E),\ (F_5, E)\widetilde{\cap}(F_4, E) = (\Phi, E),\ (F_4, E)\widetilde{\cap}(F_6, E) = (\Phi, E)$. Then there exist RSO soft sets $(F_4, E),\ (F_5, E),\ (F_6, E)$ such that $(F_5, E)\widetilde{\subseteq}(F_5, E)$, $(F_6, E)\widetilde{\subseteq}(F_6, E),\ (F_4, E)\widetilde{\subseteq}(F_4, E)$ and $(F_5, E)\widetilde{\cap}(F_6, E) = (\Phi, E),\ (F_5, E)\widetilde{\cap}(F_4, E) = (\Phi, E),\ (F_4, E)\widetilde{\cap}(F_6, E) = (\Phi, E)$. And then, (X, τ, E) is a soft RS-normal space. Therefore, (X, τ, E) is a soft RS-T_4 space.

Remark. *In Examples 5.10 and 5.14 shows that every soft RS-T_4 space is need not be soft RS-T_3.*

Theorem 5.18. *A sts (X, τ, E) is soft RS-normal if and only if for any RSC soft set (L, A) and RSO soft set (G, C) containing (L, A), there exists an RSO soft set (H, D) such that $(L, A)\widetilde{\subseteq}(H, D)$ and $rsscl(H, D)\widetilde{\subseteq}(G, C)$.*

Proof. Let (X, τ, E) be RS-normal space and (L, A) be a RSC soft set and (G, C) be a RSO soft set containing $(L, A) \Rightarrow (L, A)$ and $(G, C)'$ are disjoint RSC soft sets $\Rightarrow\ \exists$ two disjoint RSO soft sets $(H_1, D),\ (H_2, D)$ such that $(L, A)\widetilde{\subseteq}(H_1, D)$ and $(G, C)'\widetilde{\subseteq}(H_2, D)$. Now $(H_1, D)\widetilde{\subseteq}(H_2, D)' \Rightarrow rsscl(H_1, D)\widetilde{\subseteq}rsscl(H_2, D)' = (H_2, D)'$. Also, $(G, C)'$ $\widetilde{\subseteq}(H_2, D) \Rightarrow (H_2, D)'\widetilde{\subseteq}(G, C) \Rightarrow rsscl(H_1, D)\widetilde{\subseteq}(G, C)$.

Conversely, let (F, A) and (K, B) be any disjoint pair RSC soft sets $\Rightarrow (F, A)\widetilde{\subseteq}(K, B)'$, then by hypothesis there exists an RSO soft set (H, D) such that $(F, A)\widetilde{\subseteq}(H, D)$ and $rsscl(H, D)\widetilde{\subseteq}(K, B)' \Rightarrow (K, B)\widetilde{\subseteq}(rsscl(H, D))' \Rightarrow (H, D)$ and $(rsscl(H, D))'$ are disjoint RSO soft sets such that $(F, A)\widetilde{\subseteq}(H, D)$ and $(K, B)\widetilde{\subseteq}(rsscl(H, D))'$. □

Theorem 5.19. *Let $f : SS(X, E) \to SS(Y, E')$ be a soft surjective function which is both SRS-irresolute and $SRSI$-open where (X, τ, E) and (Y, σ, E') are soft topological spaces. If (X, E) is soft RS-normal space then so is (Y, E').*

Proof. Take a pair of disjoint RSC soft sets (F, A) and (K, B) of $(Y, E') \Rightarrow f^{-1}(F, A)$ and $f^{-1}(K, B)$ are disjoint RSC soft sets of $(X, E) \Rightarrow \exists$ disjoint RSO soft sets (G, C) and (H, D) such that $f^{-1}(F, A)\widetilde{\subseteq}(G, C)$ and $f^{-1}(K, B)\widetilde{\subseteq}(H, D) \Rightarrow (F, A)\widetilde{\subseteq}f(G, C)$ and $(K, B)\widetilde{\subseteq}f(H, D) \Rightarrow f(G, C)$ and $f(H, D)$ are disjoint RSO soft sets of (Y, E') containing (F, A) and (K, B) respectively. Hence the result. □

Theorem 5.20. *A regular semiclosed soft subspace of a soft RS-normal space is soft RS-normal.*

Proof. Let (Y, E') be a regular semiclosed soft subspace of a soft RS-normal space (X, E). Take a disjoint pair (F, A) and (G, C) of RSC soft sets of $(Y, E') \Rightarrow \exists$ disjoint RSC soft sets (K, B) and (H, D) such that $(F, A) = (K, B)\widetilde{\cap}(Y, E')$, $(G, C) = (H, D)\widetilde{\cap}(Y, E')$. Now by soft RS-normality of (X, E), $\exists$ disjoint RSO soft sets (K_1, B) and (H_1, D) such that $(K, B)\widetilde{\subseteq}(K_1, B)$ and $(H, D)\widetilde{\subseteq}(H_1, D) \Rightarrow (F, A)\widetilde{\subseteq}(K_1, B)\widetilde{\cap}(Y, E')$ and $(G, C)\widetilde{\subseteq}$ $(H_1, D)\widetilde{\cap}(Y, E')$. □

Theorem 5.21. *Every soft RS-compact with soft RS-T_2 space is RS-normal.*

Proof. Let (X, τ, E) be a soft RS-compact with soft RS-T_2 space. Take a disjoint pair (F, A) and (K, B) of RSC soft sets. By Theorem 5.9, for each soft point $L(e)$, $\exists$ disjoint RSO soft sets (G, C) and (H, D) such that $L(e)\widetilde{\subseteq}(G, C)$ and $(K, B)\widetilde{\subseteq}(H, D)$. So the collection $\{(G_i, C) \mid L(e)\widetilde{\subseteq}(G, C),\ i \in I\}$ is a RSO-cover of (G, C). Then by Theorem 5.9, $\exists$ a finite subfamily $\{(G_i, C) \mid i = 1, 2, \ldots, n\}$ such that $(G, C)\widetilde{\subseteq}\widetilde{\bigcup}\{(G_i, C) \mid i = 1, 2, \ldots, n\}$. Take $(G, C) = \widetilde{\bigcap}\{(G_i, C) \mid i = 1, 2, \ldots, n\}$ and $(H, D) = \widetilde{\bigcap}\{(H_i, D) \mid i = 1, 2, \ldots, n\}$. Then (G, C) and (H, D) are disjoint RSO soft sets such that $(F, A)\widetilde{\subseteq}(G, C)$ and $(K, B)\widetilde{\subseteq}(H, D)$. Hence (X, E) is soft RS-normal. $\square$

6. Some types of soft functions

Theorem 6.1. *Let (X, τ_1, A) and (Y, τ_2, B) be sts's and $f_{pu} : SS(X, A) \to SS(Y, B)$ be soft function which is bijective and $SRSI$-open. If (X, τ_1, A) is soft RS-T_0 space, then (Y, τ_2, B) is also a soft RS-T_0 space.*

Proof. Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then $\exists\ x_1, x_2 \in X$ such that $u(x_1) = y_1,\ u(x_2) = y_2$ and $x_1 \neq x_2$. By hypothesis, there exist RSO soft sets (G, C) and (H, D) in X such that either $x_1 \in (G, C)$ and $x_2 \notin (G, C)$ or $x_2 \in (H, D)$ and $x_1 \notin (H, D)$. So, either $x_1 \in F(e)$ and $x_2 \notin F(e)$ or $x_2 \in G(e)$ and $x_1 \notin G(e)$ for all $e \in E$. This implies that, either $y_1 = u(x_1) \in u(F(e))$ and $y_2 = u(x_2) \notin u(F(e))$ or $y_2 = u(x_2) \in u(G(e))$ and $y_1 = u(x_1) \notin u(G(e))$ for all $e \in E$. Hence either $y_1 \in f_{pu}(G, C)$ and $y_2 \notin f_{pu}(G, C)$ or $y_2 \in f_{pu}(H, D)$ and $y_1 \notin f_{pu}(H, D)$. Since f_{pu} is $SRSI$-open function, then $f_{pu}(G, C),\ f_{pu}(H, D)$ are RSO soft sets in Y. Hence (Y, τ_2, B) is also a soft RS-T_0space. $\square$

Theorem 6.2. *Let (X, τ_1, A) and (Y, τ_2, B) be sts's and $f_{pu} : SS(X, A) \to SS(Y, B)$ be soft function which is bijective and $SRSI$-open. If (X, τ_1, A) is soft RS-T_1 space, then (Y, τ_2, B) is also a soft RS-T_1 space.*

Proof. It is similar to the proof of Theorem 6.1 $\square$

Theorem 6.3. *Let (X, τ_1, A) and (Y, τ_2, B) be sts's and $f_{pu} : SS(X, A) \to SS(Y, B)$ be soft function which is bijective and $SRSI$-open. If (X, τ_1, A) is soft RS-T_2 space, then (Y, τ_2, B) is also a soft RS-T_2 space.*

Proof. $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then $\exists\ x_1, x_2 \in X$ such that $u(x_1) = y_1,\ u(x_2) = y_2$ and $x_1 \neq x_2$. By hypothesis, there exist RSO soft sets (G, C) and (H, D) in X such that $x_1 \in (G, C),\ x_2 \in (H, D)$ and $(G, C)\widetilde{\cap}(H, D) = (\Phi, E)$. So, $x_1 \in F(e),\ x_2 \in G(e)$ and $F(e)\widetilde{\cap}G(e) = (\Phi, E)$ for all $e \in E$. This implies that, $y_1 = u(x_1) \in u(F(e)),\ y_2 = u(x_2) \in u(G(e))$ for all $e \in E$. Hence $y_1 \in f_{pu}(G, C),\ y_2 \in f_{pu}(H, D)$ and $f_{pu}(G, C)\widetilde{\cap}f_{pu}(H, D) = f_{pu}[(G, C)\widetilde{\cap}(H, D)] = f_{pu}(\Phi, A) = (\Phi, B)$ from Theorem 2.5 Since f_{pu} is $SRSI$-open function, then $f_{pu}(G, C),\ f_{pu}(H, D)$ are RSO soft sets in Y. Thus, (Y, τ_2, B) is also a soft RS-T_2 space. $\square$

Theorem 6.4. *Let (X, τ_1, A) and (Y, τ_2, B) be sts's and $f_{pu} : SS(X, A) \to SS(Y, B)$ be soft function which is bijective, SRS-irresolute and $SRSI$-open. If (X, τ_1, A) is soft RS-regular space, then (Y, τ_2, B) is also a soft RS-regular space.*

Proof. Let (G, C) be a RSC soft set in Y and $y \in Y$ such that $y \notin (G, C)$. Since f_{pu} is surjective and SRS-irresolute, then $\exists\ x \in X$ such that $u(x) = y$ and $f_{pu}^{-1}(G, C)$ is RSC soft set in X such that $x \notin f_{pu}^{-1}(G, C)$. By hypothesis, there exist RSO soft sets (F, A) and (H, D) in X such that $x \in (F, A),\ f_{pu}^{-1}(G, C)\widetilde{\subseteq}(H, D)$ and $(F, A)\widetilde{\cap}(H, D) = (\Phi, E)$.

It follows that $x \in F(e)$ for all $e \in E$ and $(G,C) = f_{pu}[f_{pu}^{-1}(G,C)]\widetilde{\subseteq} f_{pu}(H,D)$ from Theorem 2.5. So, $y = u(x_1) \in u[F(e)]$ for all $e \in E$ and $(G,C)\widetilde{\subseteq} f_{pu}(H,D)$. Hence $y \in f_{pu}(F,A)$ and $(G,C)\widetilde{\subseteq} f_{pu}(H,D)$ and $f_{pu}(F,A)\widetilde{\cap} f_{pu}(H,D) = f_{pu}[(F,A)\widetilde{\cap}(H,D)] = f_{pu}(\Phi,A) = (\Phi,B)$ from Theorem 2.5 Since f_{pu} is $SRSI$-open function. Then $f_{pu}(F,A)$, $f_{pu}(H,D)$ are RSO soft sets in Y. Thus, (Y,τ_2,B) is also a soft RS-regular space. □

Theorem 6.5. *Let (X,τ_1,A) and (Y,τ_2,B) be sts's and $f_{pu} : SS(X,A) \to SS(Y,B)$ be soft function which is bijective, SRS-irresolute and $SRSI$-open. If (X,τ_1,A) is soft RS-T_3 space, then (Y,τ_2,B) is also a soft RS-T_3 space.*

Proof. Since (X,τ_1,A) is soft RS-T_3 space, then (X,τ_1,A) is soft RS-regular with soft RS-T_1 space. It follows that (Y,τ_2,B) is also a soft RS-T_1 space from Theorem 6.4 and soft RS-regular space from Theorem 6.4 Hence, (Y,τ_2,B) is also a soft RS-T_3 space. □

Theorem 6.6. *Let (X,τ_1,A) and (Y,τ_2,B) be sts's and $f_{pu} : SS(X,A) \to SS(Y,B)$ be soft function which is bijective, SRS-irresolute and $SRSI$-open. If (X,τ_1,A) is soft RS-normal space, then (Y,τ_2,B) is also a soft RS-normal space.*

Proof. Let (F,A) and (G,C) be RSC soft sets in Y such that $(F,A)\widetilde{\cap}(G,C) = (\Phi,E)$. Since f_{pu} is SRS-irresolute, then $f_{pu}^{-1}(F,A)$ and $f_{pu}^{-1}(G,C)$ are RSC soft set in X such that $f_{pu}^{-1}(F,A)\widetilde{\cap} f_{pu}^{-1}(G,C) = f_{pu}^{-1}[(F,A)\widetilde{\cap}(G,C)] = f_{pu}^{-1}[\Phi,B] = (\Phi,A)$ from Theorem 2.5 By hypothesis, there exist RSO soft sets (K,B) and (H,D) in X such that $f_{pu}^{-1}(F,A)\widetilde{\subseteq}(K,B)$, $f_{pu}^{-1}(G,C)\widetilde{\subseteq}(H,D)$ and $(K,B)\widetilde{\cap}(H,D) = (\Phi,A)$. It follows that $(F,A) = f_{pu}[f_{pu}^{-1}(F,A)]\widetilde{\subseteq} f_{pu}(K,B)$, $(G,C) = f_{pu}[f_{pu}^{-1}(G,C)]\widetilde{\subseteq} f_{pu}(H,D)$ from Theorem 2.5 and $f_{pu}(K,B)\widetilde{\cap} f_{pu}(H,D) = f_{pu}[(K,B)\widetilde{\cap}(H,D)] = f_{pu}[\Phi,A] = (\Phi,B)$ from Theorem 2.5 Since f_{pu} is $SRSI$-open function. Then $f_{pu}(K,B)$, $f_{pu}(H,D)$ are RSO soft sets in Y. Thus, (Y,τ_2,B) is also a soft RS-normal space. □

Corollary 6.7. *Let (X,τ_1,A) and (Y,τ_2,B) be sts's and $f_{pu} : SS(X,A) \to SS(Y,B)$ be soft function which is bijective, SRS-irresolute and $SRSI$-open. If (X,τ_1,A) is soft RS-T_4 space, then (Y,τ_2,B) is also a soft RS-T_4 space.*

Proof. It is obvious from Theorem 6.2 and Theorem 6.6 □

7. CONCLUSION

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [19] and easily applied to many problems having uncertainties from social life. In this paper, we introduce the notion of soft regular semi compactness, connectedness and separation axioms. In particular we study the properties of the soft RS-regular spaces and soft RS-normal spaces. We show that if (x,E) is RSC soft set for all $x \in X$ in a sts (X,τ,E), then (X,τ,E) is soft RS-T_1 space. Also, we show that if a sts (X,τ,E) is soft RS-T_3 space, then $\forall\ x \in X,\ (x,E)$ is RSC soft set. Also, we show that the property of being soft RS-T_i spaces (i = 1,2) is soft topological property under a bijection and irresolute open soft mapping. Further, the properties of being soft RS-regular and soft RS-normal are soft topological properties under a bijection, SRS-irresolute functions and $SRSI$-open functions. Finally, we show that the property of being soft RS-T_i spaces (i = 1,2,3,4) is a hereditary property. Similarly other forms of generalized open set can be applied to define different forms of compactness and connectedness. We hope that the results in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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