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STRUCTURE AND APPLICATION OF HCA IN IMAGE ANALYSIS

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ABSTRACT. In this paper, we study the neighborhood structure of hexagonal cellular automata with null boundary conditions over the field $\mathbb{Z}_2$. Rule matrix with null boundary condition and application of $\mathcal{HCA}$ in the field of image analysis are studied.

1. INTRODUCTION

The concept of Cellular Automata($\mathcal{CA}$) was initiated in the early 1950's by John Von Neumann and Stan Ulam [8, 10]. Afterwards, Stephen Wolfram developed the $\mathcal{CA}$ theory [11].

The Hexagonal Cellular Automata ($\mathcal{HCA}$) are 2D $\mathcal{CA}$ whose cells are of the form of a hexagonal. Morita et al.[6] introduced this type of cellular automaton and they called it $\mathcal{HCA}$. Image processing are excess more important compared with serial algorithms [9]. $\mathcal{CA}$ are widely used by researchers in the domain of image processing. So, $\mathcal{CA}$ can be used as a parallel method for any image processing task [3].

The paper is organized as follows.. In this second section, the concept used in the paper are formally defined. In this third section, the neighbor structure of 2D $\mathcal{HCA}$ is explained. In this forth section,rule matrix of $\mathcal{HCA}$ is studied. In this fifth section, we dispute about a few application in the field of image analysis using $\mathcal{HCA}$.

2. PRELIMINARIES

Definition 2.1. [5]A Null Boundary $\mathcal{CA}$ is the one in which the extreme cells are connected to logic zero state.

Definition 2.2. [7] **Uniform** $\mathcal{CA}$: The same rule applied to all the cells.

Definition 2.3. [1] **Hybrid** $\mathcal{CA}$: The different rules have to implement the different cells.

Definition 2.4. [2] **Cellular Automata**:($\mathcal{CA}$): $\mathcal{CA}$ is defined as a quadruplets $\mathcal{M} = \{d, \mathbf{Q}, \mathbf{N}, f\}$
* $d \in \mathbb{Z}_+$ is the dimension of the $\mathcal{CA}$.
* $\mathbf{Q} = \{1, 2, ..., p\}$ is a countable set of states.

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* $\mathbf{N} = (\vec{n_1}, \vec{n_2}, ..., \vec{n_m})$ is the neighbor vector
* $f : \mathbf{Q}^m \to \mathbf{Q}$ is the local rule. f given the new states of a cell from the old neighbors states of the cells.

A mapping $\mathbb{C} : \mathbb{Z}^d \to \mathbf{Q}$. $\mathbb{C}^t$ is denote the time t, the cell move to next state at time t+1.

$\mathbb{C}^{t+1}(\vec{n}) = f(\mathbb{C}^t(\vec{n_1}), \mathbb{C}^t(\vec{n_2}), ..., \mathbb{C}^t(\vec{n_m}))$

now we consider f is a local rule of linear function

$\mathbb{C}^{t+1}(\vec{n}) = \lambda_1 \mathbb{C}^t(\vec{n_1}) + \lambda_2 \mathbb{C}^t(\vec{n_2}) + ... + \lambda_m \mathbb{C}^t(\vec{n_m}))$

λ_i is the co-efficient for neighborhood.

In [4] the state of the cell $(\mathcal{K}, \mathcal{L})$ at time t is denoted by $\mathrm{S}^{(t)}_{(\mathcal{K},\mathcal{L})}$. The state of the cell $(\mathcal{K}, \mathcal{L})$ at time (t+1) is denoted by $\mathrm{S}^{(t+1)}_{(\mathcal{K},\mathcal{L})} = \mathrm{R}^{(t)}_{(\mathcal{K},\mathcal{L})}$.

The rule matrix T_R that changes set of states of $\mathcal{CA}$ from (t) to (t+1) such that

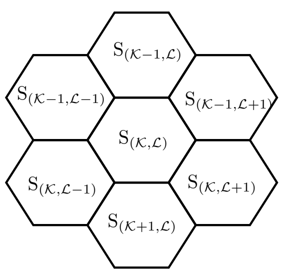
$[\mathrm{S}]_{1\times mn}.(T_R)_{mn\times mn} = [\mathrm{R}]_{mn\times 1}$,

where

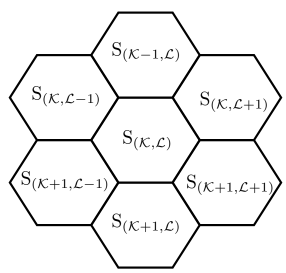
$$([\mathrm{R}]_{mn\times 1}) = (\mathrm{S}^{(t+1)}_{11}, \mathrm{S}^{(t+1)}_{12}, \cdots, \mathrm{S}^{(t+1)}_{1n}, \cdots, \mathrm{S}^{(t+1)}_{m1}, \cdots, \mathrm{S}^{(t+1)}_{mn})$$
$$=(\mathrm{R}^{(t)}_{11}, \mathrm{R}^{(t)}_{12}, \cdots, \mathrm{R}^{(t)}_{1n}, \cdots, \mathrm{R}^{(t)}_{m1}, \cdots, \mathrm{R}^{(t)}_{mn}).$$

3. The Neighborhood Structure of 2D $\mathcal{HCA}$

In this section, we show the neighborhood structure $\mathcal{HCA}$ over the field $\mathbb{Z}_2$ under the null boundary.



a. $\mathcal{L}$ is even positive integer



b. $\mathcal{L}$ is odd positive integer

FIGURE 1. Two configuration of the $\mathcal{HCA}$.

In Figure 1, we show the $\mathcal{HCA}$ which comprises 6 cells surrounding the center cell $\mathrm{S}_{(\mathcal{K},\mathcal{L})}$ time t . The state of $\mathrm{S}_{(\mathcal{K},\mathcal{L})}$ at time $(t+1)$ is a function $f : \mathbb{Z}_2^6 \to \mathbb{Z}_2$ defined as follows.

If $\mathcal{L}$ is an even integer figure 1.a, then we have,

$\mathrm{S}_{(\mathcal{K}-1,\mathcal{L})} + \mathrm{S}_{(\mathcal{K}-1,\mathcal{L}+1)} + \mathrm{S}_{(\mathcal{K},\mathcal{L}+1)} + \mathrm{S}_{(\mathcal{K}+1,\mathcal{L})} + \mathrm{S}_{(\mathcal{K},\mathcal{L}-1)} + \mathrm{S}_{(\mathcal{K}-1,\mathcal{L}-1)}$ ...(1)

If $\mathcal{L}$ is an odd integer figure 1.b, then we have,

$\mathrm{S}_{(\mathcal{K}-1,\mathcal{L})} + \mathrm{S}_{(\mathcal{K},\mathcal{L}+1)} + \mathrm{S}_{(\mathcal{K}+1,\mathcal{L}+1)} + \mathrm{S}_{(\mathcal{K}+1,\mathcal{L})} + \mathrm{S}_{(\mathcal{K}+1,\mathcal{L}-1)} + \mathrm{S}_{(\mathcal{K},\mathcal{L}-1)}$ ...(2)

4. Rule Matrix of the $\mathcal{HCA}$ with Null Boundary

In this section, we discuss with the rule matrix of 2D $\mathcal{HCA}$ with null boundary over the field $\mathbb{Z}_2$.
Case (i). We take n is even positive integer and the following theorem.

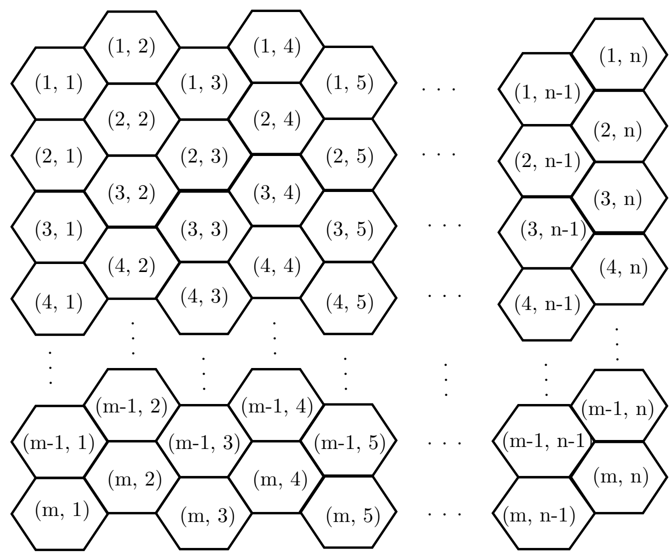


FIGURE 2. $\mathcal{HCA}$ of order $m \times n$
and n is even

Theorem 4.1. *Let $\mathcal{CA} = (d, S, N, f)$ be the $\mathcal{HCA}$. Let $m \geq 3$ and n be an even positive integer. We prove that there exist a rule matrix T_R^E from $\mathbb{Z}_2^{mn} \to \mathbb{Z}_2^{mn}$ corresponding to the 2D $\mathcal{HCA}$ which takes from configuration the state $\mathbb{C}^t$ of order $m \times n$ to the $(t+1)^{th}$ state $\mathbb{C}^{(t+1)}$ is given by,*

$$T_R^E = \begin{pmatrix} \mathcal{A}^E & \mathcal{B}^E & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \mathcal{C}^E & \mathcal{A}^E & \mathcal{B}^E & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & \mathcal{C}^E & \mathcal{A}^E & \mathcal{B}^E & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \mathcal{C}^E & \mathcal{A}^E & \mathcal{B}^E & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \mathcal{C}^E & \mathcal{A}^E & \mathcal{B}^E \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & \mathcal{C}^E & \mathcal{A}^E \end{pmatrix}_{(mn \times mn)}$$

Where each sub matrix,

$$\mathcal{A}^E = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 0 \end{pmatrix}_{(n \times n)}$$

$$\mathcal{B}^E = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix}_{(n\times n)}$$

$$\mathcal{C}^E = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 1 \end{pmatrix}_{(n\times n)} \qquad \textit{and}$$

0 is the zero matrix

Proof. Let $(\mathrm{S}_{(\mathcal{K},\mathcal{L})})T_R = \mathrm{R}_{(\mathcal{K},\mathcal{L})}$. $\mathrm{R}_{(\mathcal{K},\mathcal{L})} = \mathrm{S}^{(t+1)}_{(\mathcal{K},\mathcal{L})}$ is a equal to the linear combination of the neighbors in the following equation (1) and (2). The co-efficient of $\mathrm{S}_{\mathcal{K}\mathcal{L}} = 0$ if $\mathcal{K} \leq 0$ *or* $\mathcal{L} \leq 0$. By using the local rule of the $\mathcal{CA}$ we have obtain the following,
$\mathrm{R}_{(1,1)} = \mathrm{S}_{(1,2)} + \mathrm{S}_{(2,2)} + \mathrm{S}_{(2,1)}$
$\mathrm{R}_{(1,\mathcal{L})} = \mathrm{S}_{(1,\mathcal{L}+1)} + \mathrm{S}_{(2,\mathcal{L})} + \mathrm{S}_{(1,\mathcal{L}-1)}$, if $\mathcal{L}$ is even and $2 \geq \mathcal{L} > (n-1)$
$\mathrm{R}_{(1,\mathcal{L})} = \mathrm{S}_{(1,\mathcal{L}+1)} + \mathrm{S}_{(2,\mathcal{L}+1)} + \mathrm{S}_{(2,\mathcal{L})} + \mathrm{S}_{(2,\mathcal{L}-1)} + \mathrm{S}_{(1,\mathcal{L}-1)}$, if $\mathcal{L}$ is odd and $3 \geq \mathcal{L} \geq (n-1)$
$\mathrm{R}_{(1,n)} = \mathrm{S}_{(2,n)} + \mathrm{S}_{(1,n-1)}$
$\mathrm{R}_{(\mathcal{K},1)} = \mathrm{S}_{(\mathcal{K}-1,1)} + \mathrm{S}_{(\mathcal{K},2)} + \mathrm{S}_{(\mathcal{K}+1,2)} + \mathrm{S}_{(\mathcal{K}+1,1)}$, if $2 \geq \mathcal{K} \geq (n-1)$
$\mathrm{R}_{(\mathcal{K},\mathcal{L})} = \mathrm{S}_{(\mathcal{K}-1,\mathcal{L})} + \mathrm{S}_{(\mathcal{K}-1,\mathcal{L}+1)} + \mathrm{S}_{(\mathcal{K},\mathcal{L}+1)} + \mathrm{S}_{(\mathcal{K}+1,\mathcal{L})} + \mathrm{S}_{(\mathcal{K},\mathcal{L}-1)} + \mathrm{S}_{(\mathcal{K}-1,\mathcal{L}-1)}$, if $\mathcal{L}$ is even and $2 \geq \mathcal{L} > (n-1)$
$\mathrm{R}_{(\mathcal{K},\mathcal{L})} = \mathrm{S}_{(\mathcal{K}-1,\mathcal{L})} + \mathrm{S}_{(\mathcal{K},\mathcal{L}+1)} + \mathrm{S}_{(\mathcal{K}+1,\mathcal{L}+1)} + \mathrm{S}_{(\mathcal{K}+1,\mathcal{L})} + \mathrm{S}_{(\mathcal{K}+1,\mathcal{L}-1)} + \mathrm{S}_{(\mathcal{K},\mathcal{L}-1)}$, if $\mathcal{L}$ is odd and $3 \geq \mathcal{L} \geq (n-1)$
$\mathrm{R}_{(\mathcal{K},n)} = \mathrm{S}_{(\mathcal{K}+1,n)} + \mathrm{S}_{(\mathcal{K},n-1)} + \mathrm{S}_{(\mathcal{K}-1,n-1)} + \mathrm{S}_{(\mathcal{K}-1,n)}$
$\mathrm{R}_{(m,1)} = \mathrm{S}_{(m-1,1)} + \mathrm{S}_{(m,2)}$
$\mathrm{R}_{(m,\mathcal{L})} = \mathrm{S}_{(m-1,\mathcal{L})} + \mathrm{S}_{(m-1,\mathcal{L}+1)} + \mathrm{S}_{(m,\mathcal{L}+1)} + \mathrm{S}_{(m,\mathcal{L}-1)} + \mathrm{S}_{(m-1,\mathcal{L}-1)}$, if $\mathcal{L}$ is even and $2 \geq \mathcal{L} > (n-1)$
$\mathrm{R}_{(m,\mathcal{L})} = \mathrm{S}_{(m,\mathcal{L}-1)} + \mathrm{S}_{(m-1,\mathcal{L})} + \mathrm{S}_{(m,\mathcal{L}+1)}$, if $\mathcal{L}$ is odd and $2 \geq \mathcal{L} \geq (n-1)$
$\mathrm{R}_{(m,n)} = \mathrm{S}_{(m,n-1)} + \mathrm{S}_{(m-1,n-1)} + \mathrm{S}_{(m-1,n)}$
finally, we get the rule matrix of even case. $\square$

Case (ii). We take n is odd positive integer and the following theorem.

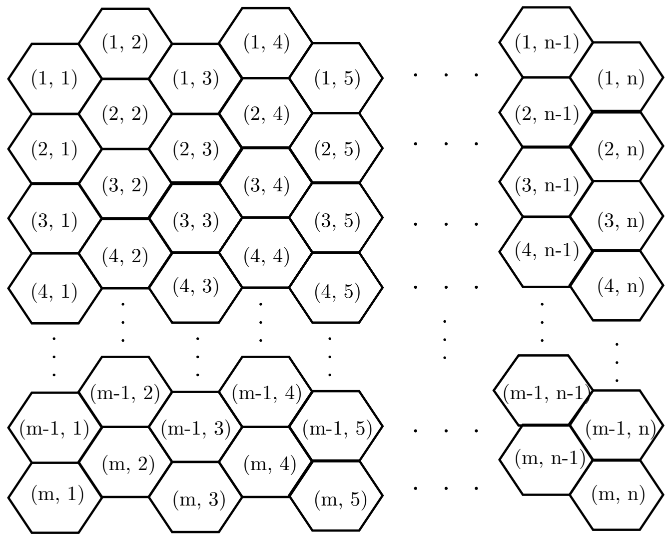


FIGURE 3. $\mathcal{HCA}$ of order $m \times n$ and n is odd

Theorem 4.2. *Let $\mathcal{CA} = (d, S, N, f)$ be the $\mathcal{HCA}$. Let $m \geq 3$ and n be an odd positive integer. We prove that there exist a rule matrix T_R^O from $\mathbb{Z}_2^{mn} \to \mathbb{Z}_2^{mn}$ corresponding to the 2D $\mathcal{HCA}$ which takes from configuration the state $\mathbb{C}^t$ of order $m \times n$ to the $(t+1)^{th}$ state $\mathbb{C}^{(t+1)}$ is given by,*

$$T_R^O = \begin{pmatrix} \mathcal{A}^O & \mathcal{B}^O & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \mathcal{C}^O & \mathcal{A}^O & \mathcal{B}^O & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & \mathcal{C}^O & \mathcal{A}^O & \mathcal{B}^O & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \mathcal{C}^O & \mathcal{A}^O & \mathcal{B}^O & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \mathcal{C}^O & \mathcal{A}^O & \mathcal{B}^O \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & \mathcal{C}^O & \mathcal{A}^O \end{pmatrix}_{(mn \times mn)}$$

Where each sub matrix,

$$\mathcal{A}^O = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 0 \end{pmatrix}_{(n \times n)}$$

$$\mathcal{B}^O = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 1 \end{pmatrix}_{(n\times n)}$$

$$\mathcal{C}^O = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix}_{(n\times n)} \quad \textit{and}$$

0 is the zero matrix

Proof. The proof of theorem 4.2 can receive the following alike the same steps as in the proof of theorem 4.1. □

5. Application of $\mathcal{HCA}$ Image Analysis

Two dimensional $\mathcal{HCA}$ algorithm are widely used in image processing as its shape is like to an image. In this section, we discuss the basic image processing of transition, zooming, boundary, and thinning.

5.1. **Transition.**

Transition is very important to the part of image processing. The image moving from all the direction is using the transition. In this paper, we have applied seven basic of 2D $\mathcal{HCA}$ rules. The directions for the rules is indicated in the table below.

Table 1. Translation of images using basic 2D $\mathcal{HCA}$ rules.

Rules	Direction of translation of images
1	center
2	top
4	right-top
8	right-bottom
16	bottom
32	left-bottom
64	left-top

This rules using the hexagonal grid.

Translation of an image using 2D $\mathcal{CA}$ rules represented the following figure.

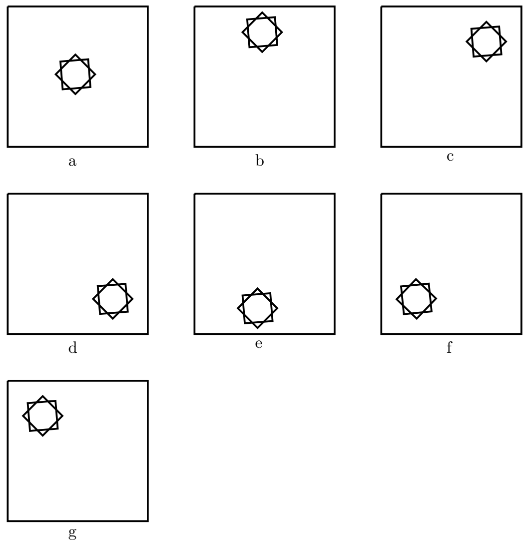
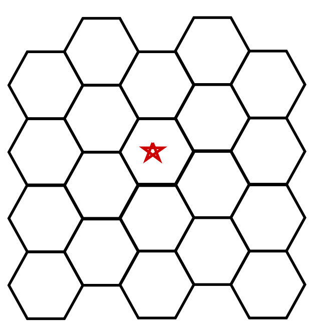


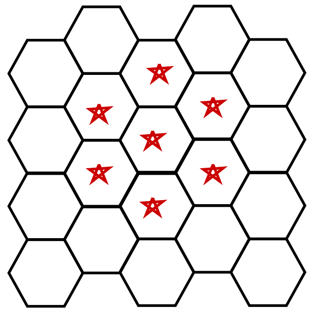
FIGURE 4. Translation of an image using 2D rules (a)Center Images (b) Top (c)right-top (d) right-bottom (e) bottom (f)left-bottom (g) left-top

5.2. **Zooming.** In zooming there are two operations, zooming in and zooming out. The following example of zooming demonstrates that row or column using different uniform and hybrid rules.
This example of zooming in

Example 5.1. Let us consider a 4×5 2D H with the starting configuration as shown in figure

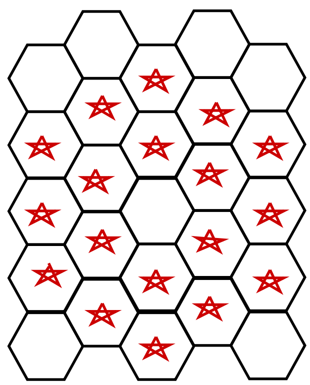


After running the $\mathcal{HCA}$ rules mentions in table foe every cells the resulting configuration is shown in below

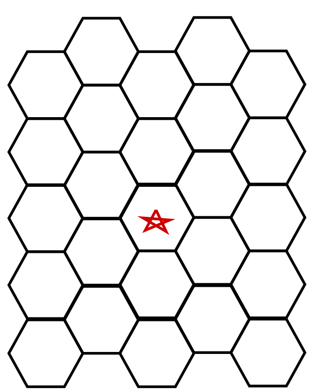


This example of zooming out.

Example 5.2. Let us consider a 5×5 2D $\mathcal{HCA}$ with the initial configuration as shown in figure

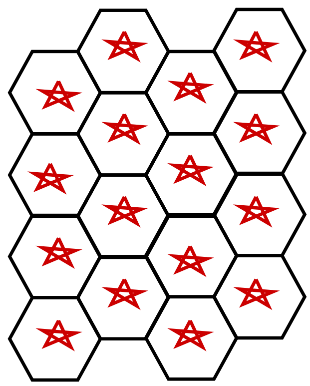


Every cell in the 1^{st} row, use the rule 1, rule 2, rule 4, rule 8, rule 16, rule 32 and rule 64.
Every cell in the 2^{nd} row, use the rule 1 and rule 16.
Every cell in the 3^{rd} row, use the rule 1 and rule 16.
Every cell in the 4^{th} row, use the rule 1 and rule 2 except the 3^{rd} cell that only use the rule 1 and 4.
In 5^{th} row 1^{st} cell we use rule 2 and rule 4. 2^{nd} cell we use rule 2, rule 4, rule 8 and rule 64. 3^{rd} cell we use rule 1, rule 2, rule 4 and rule 64. 4^{th} cell we use rule 2, rule 4, rule 32 and rule 64. 5^{th} cell using rule 2 and rule 64.
After applying this hybrid rules, the resulting configuration is shown in figure.

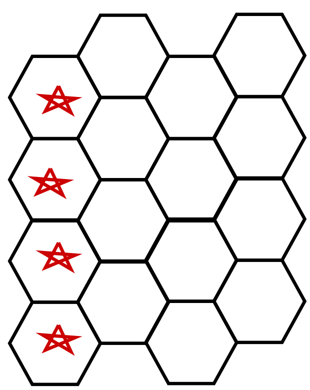


5.3. **Thinning.** Thinning is an important procedure in image analysis. The following example of thinning demonstrates that row or column can be thinned using different uniform and hybrid rules.

Example 5.3. Let us consider a 4×4 2D $\mathcal{HCA}$ containing all 1's with the starting configuration as shown in figure



Every cell in the 1^{st} column, use the rule 4, rule 8 and rule 16 if j is odd
Every cell in the 2^{nd} column, use the rule 1 and rule 8 if j is even
Every cell in the 3^{rd} column, use the rule 1 and rule 4 if j is odd
Every cell in the 4^{th} column, use the rule 1 and rule 32 if j is even



6. Conclusions

In this paper we have defined $\mathcal{HCA}$ local rule over the field $\mathbb{Z}_2$. The rule matrix associated to the 2D $\mathcal{HCA}$ has been obtained. We apply some important image process tasks such transition, zooming and thinning using 2D $\mathcal{HCA}$.

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