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CUBE SUM LABELING OF GRAPHS

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ABSTRACT. Here, we define a cube sum labeling and cube sum graph. Let G be a (p,q) graph. G is said to be a cube sum graph if there exist a bijection $f : V(G) \to \{0,1,\dots,p-1\}$ such that the induced function $f^* : E(G) \to N$ given by

$$f^*(uv) = |f(u)^3 + f(v)^3| \; for \; all \; uv \in E(G)$$

are all distinct. In this paper, we developed the concept of cube sum labeling of some family of graphs like paths, cycle, stars, wheel graph, fan graphs are discussed in this paper.

1. INTRODUCTION

Graphs are one of the prime objects of study in discrete mathematics. In general, a graph is represented by set of nodes connected by arcs. Graphs are therefore mathematical structures used to model pairwise relations between objects. They are found on road maps, constellations, when constructing schemes and drawings.

Graph labeling is one of the fascinating area of graph theory with wide ranging application. Graph labeling was first introduced in 1960's. Labeled graph serve as useful methods for a circuit design, coding theory, communication network addressing, data base management, data mining etc., An enormous body of literature has grown around graph labeling in the last four decades.

The concept of cube difference labeling was introduced by J.Shiama [8]. J.Shiama proved that the following graphs paths, cycle, stars and trees admits cube difference labeling. Sharon Philomena.V [7] proved that the Square and cube difference labeling of graphs.

All the graphs in this paper are simple, finite and undirected. The symbol $V(G)$ and $E(G)$ be the vertex set and edge set of the graph respectively. We introduced the concept of quad difference labelling and proved that some graph admits this kind of labeling. Also we will discuss about some important theorems and examples based on those theorems.

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2. BASIC TERMINOLOGY

Definition 2.1. A Labeled graph is a graph whose vertices are each assigned an element from a set of symbols (letters, usually, but this is unimportant). The important thing to note is that the vertices can be distinguished one from another.

Definition 2.2. A Star graph S_k is defined to be the tree of order k with maximum diameter 2; in which case a star $k > 2$ has $k-1$ leaves.

Definition 2.3. A fan graph $F_{m,n}$ is defined as the graph join $\overline{K}_m + P_n$, where $\overline{K}_m$ is the empty graph on m nodes and P_n is the path on nodes.

Definition 2.4. Let $G = (V(G), E(G))$ be a graph. G is said to be cube difference labeling if there exist a bijection $f : V(G) \to \{0, 1, 2, \ldots .p-1\}$ such that the induced function $f* : E(G) \to N$ given by $f^*(uv) = \left|f(u)^3 - f(v)^3\right|$ is injective.

Definition 2.5. Let $G = (V(G), E(G))$ be a graph. G is said to be cube sum labeling if there exist a bijection $f : V(G) \to \{0, 1, 2, \ldots .p-1\}$ such that the induced function $f* : E(G) \to N$ given by $f^*(uv) = \left|f(u)^3 + f(v)^3\right|$ is injective.

Definition 2.6. Let G be a graph and is said to be QDL if there exist a one to one and onto function from vertices to $\{0, 1, .., p-1\}$ such that f induces the mapping $f^\star : E(G) \to N$ is given by

$$f^\star(uv) = |[f(u)]^4 - [f(v)]^4|$$

is injective.

3. CONCEPTS OF CUBE SUM LABELING

Theorem 3.1. *The Path P_n is a cube sum labelling*

Proof. Let the graph G be a path P_n
Let $V(P_n) = \{u_i \ : 1 \le i \le n\}$
Let $E(P_n) = \{u_i u_{i+1} \ : 1 \le i \le n-1\}$
Define an injective function
$f : V(P_n) \to (0, 1, 2, \ldots, n-1\}$ by $f(u_i) = i, \ \ 0 \le i \le n-1$
Then f induces the function $f^* : E(P_n) \to N$ defined by $f^*(uv) = \left|f(u)^3 + f(v)^3\right|$ for every $uv \in E(G)$ are all distinct such that $f(e_i) \ne f(e_j)$ for every $e_i \ne e_j$.
The Edge label is given below:

$$f^*(u_i u_{i+1}) = 2i^3 + 3i(i+1) + 1, \ \ 0 \le i \le n-1$$

Here we get all the edges with distinct weights.
Hence path P_n admits cube sum labeling. □

Example 3.1. The Path P_8 graph is cube sum labeling is shown in Figure 1.

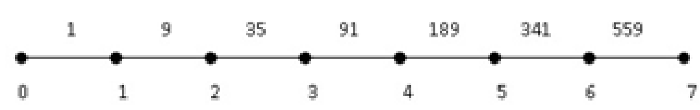

FIGURE 1.

Theorem 3.2. *The Cycle graph C_n admits cube sum labelling.*

Proof. Let C_n be the cycle graph with n vertices and n edges.
Define an injective function $f : V(P_n) \to (0, 1, 2, \ldots, n-1\}$ by $f(u_i) = i, \ \ 0 \leq i \leq n-1$
Then f induces the function $f^* : E(P_n) \to N$the induced edge labeling function $f^* : E(G) \to N$ defined by $f^*(uv) = \left|f(u)^3 + f(v)^3\right|$ for every $uv \in E(G)$ are all distinct such that $f(e_i) \neq f(e_j)$ for every $e_i \neq e_j$
The edge sets are

$$E_1 = \{u_i u_{i+1} \mid 0 \leq i \leq n-1\}$$
$$E_2 = \{u_{n-1} u_0\}$$

The Edge labels are given below:
In E_1

$$f^*(\mathrm{u_i} u_{i+1}) = 2i^3 + 3i(i+1) + 1, 0 \leq i \leq n-1$$

In E_2

$$f^*(u_{n-1} u_0) = (n-1)^3$$

Here all the edge labels are distinct.
Hence the cycle C_n admits cube sum labeling. □

Example 3.2. The cycle graph C_5 admits cube sum labeling is shown in Figure 2.

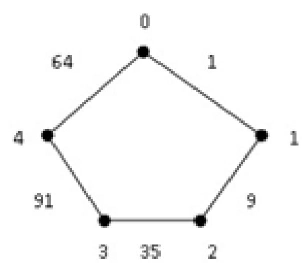

FIGURE 2.

Theorem 3.3. *The star graph $K_{1,n}$ admits a cube sum labeling.*

Proof. Consider G be the star graph $K_{1,n}$ and u be the center vertex.
Let $V(G) = \{u, \ u_i \ : 1 \leq i \leq n\}$
Define an injective function $f : V(G) \to (0, 1, 2, \ldots, n-1\}$ by

$$f(u) = 0$$
$$f(u_i) = i, \ \ 1 \leq i \leq n$$

Then f induces the function $f^* : E(G) \to N$ the induced edge labeling function $f^* : E(G) \to N$ defined by
$f^*(uv) = \left|f(u)^3 + f(v)^3\right|$ for every $uv \in E(G)$ are all distinct such that $f(e_i) \neq f(e_j)$ for every $e_i \neq e_j$
The edge set is

$$E(G) = \{uu_i \ : 1 \leq i \leq n\}$$

The edge labeling is

$$f^*(uu_i) = i^3 \ \ , \ \ 1 \leq i \leq n$$

Here the edge labels has distinct weight. Hence the star K_n is cube sum graph. □

Example 3.3. The graph $K_{1,5}$ admits cube sum graph is shown in Figure 3.

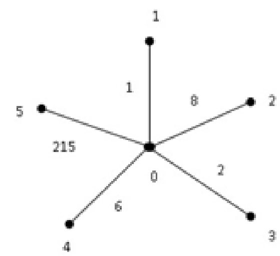

FIGURE 3.

Theorem 3.4. *The Fan graph F_n admits cube sum labeling.*

Proof. Let F_n be the fan graph with $2n+1$ vertices and $3\ n$ edges.
Define an injective function $f : V(F_n) \to (0, 1, 2, \ldots, n-1\}$ by $f(u) = 0\ ,\ \ f(u_i) = i,\ \ 0 \leq i \leq 2n$
Then f induces the function $f^* : E(F_n) \to N$the induced edge labeling function $f^* : E(G) \to N$ defined by
$f^*(uv) = \left|f(u)^3 + f(v)^3\right|$ for every $uv \in E(G)$ are all distinct such that $f(e_i) \neq f(e_j)$ for every $e_i \neq e_j$
The edge sets are

$$E_1 = \{uu_i \mid 0 \leq i \leq 2n\}$$
$$E_2 = \{u_{2i+1}u_{2i+2}\}\ ,\ 0 \leq i \leq n$$

The Edge labels are given below:
In E_1

$$f^*(u\mathrm{u_i}) = i^3, 0 \leq i \leq 2n$$

In E_2

$$f^*(u_{2i+1}u_{2i+2}) = 16i^3 + 36i^2 + 30i + 9, 0 \leq i \leq n$$

Here all the edge labels are distinct. Hence the fan graph $\mathrm{F_n}$ admits cube sum labeling. □

Theorem 3.5. *The Wheel graph W_n is a cube sum labelling.*

Proof. Let W_n be the wheel graph.
Define an injective function $f : V(W_n) \to (0, 1, 2, \ldots, n-1\}$ by $f(u_i) = i,\ \ 0 \leq i \leq n-1$
Then f induces the function $f^* : E(W_n) \to N$the induced edge labeling function $f^* : E(G) \to N$ defined by
$f^*(uv) = \left|f(u)^3 + f(v)^3\right|$ for every $uv \in E(G)$ are all distinct such that $f(e_i) \neq f(e_j)$ for every $e_i \neq e_j$
The edge sets are

$$E_1 = \{u_0u_i \mid 1 \leq i \leq n\}$$
$$E_2 = \{u_iu_{i+1} \mid 1 \leq i \leq n-1\}$$
$$E_3 = \{u_1u_n\}$$

The Edge label are given below:
In E_1

$$f^*(u_0u_i) = i^3, 1 \leq i \leq n$$

In E_2

$$f^*(\mathrm{u_i}u_{i+1}) = 2i^3 + 3i(i+1) + 1, 0 \leq i \leq n-1$$

In E_3

$$f^*(\mathrm{u_1}u_n) = n^3 + 1$$

Here all the edge labels are distinct.
Hence the wheel graph $\mathrm{W_n}$ admits cube sum labeling. □

Example 3.4. The wheel graph W_5 admits cube sum labeling is shown in Figure 4.

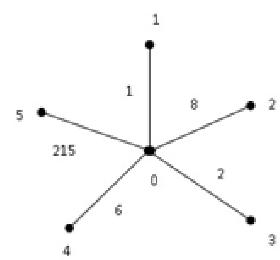

FIGURE 4.

4. CONCLUSION

We have investigated the following condition

$$f^*(uv) = |f(u)^3 + f(v)^3| \; for \; all \; uv \in E(G)$$

are all distinct. We developed the concept of cube sum labeling of some family of graphs like paths, cycle, stars, wheel graph, fan graphs are investigated.

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