ANNALS OF COMMUNICATIONS IN MATHEMATICS
Volume 2, Number 2 (2019), 121-140
ISSN: 2582-0818

INTUITIONISTIC BIPOLAR NEUTROSOPHIC SET AND ITS APPLICATION TO INTUITIONISTIC BIPOLAR NEUTROSOPHIC GRAPHS

S. SATHAM HUSSAIN*, SAID BROUMI, YOUNG BAE JUN AND DURGA NAGARAJAN

ABSTRACT. This manuscript is devoted to study a new concept of intuitionistic bipolar neutrosophic set with the operations like union, intersection and complement. Also, an application to intuitionistic bipolar neutrosophic graph with examples are developed. Further, we presented the Cartesian product, cross product, lexicographic product and strong product with suitable examples.

1. INTRODUCTION

The neutrosophic set has three independent parts, namely truth-membership degree, indeterminacy-membership degree and falsity-membership degree provided the sum of these values lies between 0 and 3; therefore, it is applied to many different areas, such as algebra [21, 22] and decision-making problems (see [26] and references therein). Author Smarandache [25] remarks the difference between neutrosophic set and logic, and intuitionistic fuzzy set and logic. Interval neutrosophic sets with applications in BCK/BCI-algebra and KU-algebras are developed in [1, 2, 18, 22, 24]. Single valued neutrosophic graphs with their degree, order and size are established in [12, 13]. Intuitionistic fuzzy set is initiated by Atanassov as a significant generalization of fuzzy set. Intuitionistic fuzzy sets are very useful while representing a problem by a linguistic variable, given in terms of a membership function only, seems too complicated. Recently intuitionistic fuzzy sets have been applied to many fields such as logic programming, medical diagnosis, decision making problems etc. On the other hand, bipolar fuzzy sets are extension of fuzzy sets whose membership degree ranges from $[-1, 1]$. The membership degree $(0, 1]$ represents that an object satisfies a certain property whereas the membership degree $[-1, 0)$ represents that the element satisfies the implicit counter-property. The positive information indicates that the consideration to be possible and negative information indicates that the consideration is granted to be impossible. Application to decision making of bipolar neutrosophic sets and bipolar neutrosophic graph structures are studied in [3, 4], respectively. Neutrosophic bipolar vague sets and its application to graph theory are analysed in [19, 20]. Similarity

2010 *Mathematics Subject Classification.* 05C76, 05C99, 05C72.
Key words and phrases. Intuitionistic bipolar neutrosophic set, Intuitionistic bipolar neutrosophic graph, Cartesian product, Lexicographic product.
*Corresponding author.

measures of bipolar neutrosophic sets and its application to decision making are established in [26]. In [10, 15], intuitionistic neutrosophic sets and its relations are discussed. Furthermore, intuitionistic neutrosophic graph structures are extensively studied in [6, 7]. Motivated by these works, we established intuitionistic bipolar neutrosophic set and its application to intuitionistic bipolar neutrosophic graphs.

The major contribution of this work as follows:

- Newly introduced intuitionistic bipolar neutrosophic set with the operations like union, intersection and complement.
- Its application to Intuitionistic Bipolar Neutrosophic Graph (IBNG) with example are developed. Also neutrosophic bipolar vague subgraph, induced subgraph, strong and complete IBNG are established.
- Further we presented the Cartesian product, cross product, lexicographic product and strong product with suitable examples. The obtained results give the generalization of above mentioned works.

2. PRELIMINARIES

Definition 2.1. [12] Let X be a space of points (objects), with a generic element in X denoted by x. A Single Valued Neutrosophic Set (SVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership-function $F_A(x)$. For each point x in X, $T_A(x), F_A(x), I_A(x) \in [0,1]$,

$$A = \{\langle x, T_A(x), F_A(x), I_A(x)\rangle,\ x \in X\} \text{ and } 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 2.2. [13] A neutrosophic graph is defined as a pair $G^* = (V, E)$ where (i) $V = \{v_1, v_2, .., v_n\}$ such that $T_1 : V \to [0,1]$, $I_1 : V \to [0,1]$ and $F_1 : V \to [0,1]$ denote the degree of truth-membership function, indeterminacy function and falsity-membership function, respectively and

$$0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3, \text{ for } u \in V.$$

(ii) $E \subseteq V \times V$ where $T_2 : E \to [0,1]$, $I_2 : E \to [0,1]$ and $F_2 : E \to [0,1]$ are such that

$$T_2(uv) \leq \min\{T_1(u), T_1(v)\},\ I_2(uv) \leq \min\{I_1(u), I_1(v)\},$$
$$F_2(uv) \leq \max\{F_1(u), F_1(v)\} \quad \text{and } 0 \leq T_2(uv) + I_2(uv) + F_2(uv) \leq 3,\ \forall uv \in E.$$

Definition 2.3. [16] A bipolar neutrosophic set A in X is defined as an object of the form

$$A = \{\langle x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x)\rangle,\ x \in X\},$$

where $T^P, I^P, F^P : X \to [0,1]$ and $T^N, I^N, F^N : X \to [-1,0]$. The Positive membership degree $T^P(x), I^P(x), F^P(x)$ denote the truth membership, indeterminate membership and false membership of an element $x \in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^N(x), I^N(x), F^N(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set A.

Definition 2.4. [16] Let X be a non-empty set. Then we call

$$A = \{\langle x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x)\rangle, x \in X\}$$

a bipolar single valued neutrosophic relation on X such that $T_A^P(x,y) \in [0,1]$, $I_A^P(x,y) \in [0,1]$, $F_A^P(x,y) \in [0,1]$ and $T_A^N(x,y) \in [-1,0]$, $I_A^N(x,y) \in [-1,0]$, $F_A^N(x,y) \in [-1,0]$.

Definition 2.5. [3, 4] Let $A = (T_A^P, I_A^P, F_A^P, T_A^N, I_A^N, F_A^N)$ and $B = (T_B^P, I_B^P, F_B^P, T_B^N, I_B^N, F_B^N)$ be bipolar single valued neutrosophic set on X. If $B = (T_B^P, I_B^P, F_B^P, T_B^N, I_B^N, F_B^N)$ is a bipolar single valued neutrosophic relation on $A = (T_A^P, I_A^P, F_A^P, T_A^N, I_A^N, F_A^N)$ then

$$\begin{aligned} T_B^P(xy) &\leq \min(T_A^P(x), T_A^P(y)), & T_B^N(xy) &\geq \max(T_A^N(x), T_A^N(y)),\\ I_B^P(xy) &\geq \max(I_A^P(x), I_A^P(y)), & I_B^N(xy) &\leq \min(I_A^N(x), I_A^N(y)),\\ F_B^P(xy) &\geq \max(F_A^P(x), F_A^P(y)), & F_B^N(xy) &\leq \min(F_A^N(x), F_A^N(y)). \end{aligned}$$

A bipolar single valued neutrosophic relation B on X is called symmetric if $T_B^P(xy) = T_B^P(yx), I_B^P(xy) = I_B^P(yx), F_B^P(xy) = F_B^P(yx)$ and $T_B^N(xy) = T_B^N(yx), I_B^N(xy) = I_B^N(yx), F_B^N(xy) = F_B^N(yx)$ for all $xy \in X$.

Definition 2.6. [3, 4] A bipolar single-valued neutrosophic graph on a nonempty set X is a pair $G = (C, D)$, where C is a bipolar single-valued neutrosophic set on X and D is a bipolar single-valued neutrosophic relation in X such that

$$\text{(i) } T_D^P(xy) \leq \min(T_C^P(x), T_C^P(y)), I_D^P(xy) \leq \min(I_C^P(x), I_C^P(y)),$$
$$F_D^P(xy) \leq \max(F_C^P(x), F_C^P(y)),$$
$$\text{(ii) } T_D^N(xy) \geq \max(T_C^N(x), T_C^P(y)), I_D^N(xy) \geq \max(I_C^N(x), I_C^P(y)),$$
$$F_D^N(xy) \geq \min(F_C^N(x), F_C^P(y)),$$

for all $x, y \in X$.

Definition 2.7. [10, 15] An element x of X is called significant with respect to neutrosophic set A of X if the degree of truth-membership or indeterminacy-membership or falsity membership value, i.e $T_A(x)$ or $I_A(x)$ or $F_A(x) \geq 0.5$. Otherwise, we call it insignificant. Also, for neutrosophic set the truth-membership, indeterminacy-membership and falsity- membership all can not be significant.
we define an intuitionistic neutrosophic set by $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, where $\min\{T_A(x), F_A(x)\} \leq 0.5$, $\min\{T_A(x), I_A(x)\} \leq 0.5$, & $\min\{I_A(x), F_A(x)\} \leq 0.5$, for all $x \in X$ with the condition $0 \leq \{T_A(x) + I_A(x) + F_A(x)\} \leq 2$

Definition 2.8. [10, 15] A INS Relation (INSR) is defined as a intuitionistic subset of $X \times Y$, having the form

$$R = \{< (x, y), T_R(x, y), I_R(x, y), F_R(x, y) >: x \in X, y \in Y\}$$

where,

$$T_R : X \times Y \to [0, 1], I_R : X \times Y \to [0, 1], F_R : X \times Y \to [0, 1]$$

satisfies the conditions
(i) at least one of this $T_R(x, y), I_R(x, y)$ and $F_R(x, y)$ is ≥ 0.5 and
(ii) $0 \leq \{T_A(x) + I_A(x) + F_A(x)\} \leq 2$. The colllection of all INSR on $X \times Y$ is denoted as $GR(X \times Y.)$

Definition 2.9. [6, 7] An intuitionistic neutrosophic graph is a pair $G = (A, B)$ with underlying set V, where $T_A, F_A, I_A : V \to [0, 1]$ denote the truth, falsity and indeterminacy membership values of the vertices in V and $T_B, F_B, I_B : E \subseteq V \times V \to [0, 1]$ denote the truth, falsity and indeterminacy membership values of the edges $kl \in E$ such that

$$(i) T_B(kl) \leq T_A(k) \wedge T_A(l), I_B(kl) \leq I_A(k) \wedge I_A(l), F_B(kl) \geq F_A(k) \wedge F_A(l)$$
$$(ii) T_B(kl) \wedge I_B(kl) \leq 0.5, T_B(kl) \wedge F_B(kl) \leq 0.5, I_B(kl) \wedge F_B(kl) \leq 0.5,$$
$$(iii) 0 \leq T_B(kl) + I_B(kl) + F_B(kl) \leq 2 \ \forall k, l \in V.$$

3. INTUITIONISTIC BIPOLAR NEUTROSOPHIC SET

Definition 3.1. An element x of X is called significant with respect to neutrosophic set A of X if the degree of truth-membership or indeterminacy-membership or falsity membership value, i.e $T_A(x)$ or $I_A(x)$ or $F_A(x) \geq 0.5$. Otherwise, we call it insignificant. Also, for neutrosophic set the truth-membership, indeterminacy-membership and falsity-membership all can not be significant. we define an intuitionistic bipolar neutrosophic set by

$$A = \langle x, T_A^P(x), I_A^P(x), F_A^P(x), T_A^N(x), I_A^N(x), F_A^N(x)\rangle$$

where

$$\min\{T_A^P, F_A^P\} \leq 0.5, \max\{T_A^N, F_A^N\} \geq -0.5, \min\{T_A^P, I_A^P\} \leq 0.5,$$
$$\max\{T_A^N, I_A^N\} \geq -0.5, \min\{F_A^P, I_A^P\} \leq 0.5, \max\{F_A^N, I_A^N\} \geq -0.5$$

$T_A^P : X \to [0,1], T_A^N : X \to [-1,0], I_A^P : X \to [0,1]$,
$I_A^N : X \to [-1,0], F_A^P : X \to [0,1], F_A^N : X \to [-1,0]$, with the conditions

$$0 \leq T_A^P(x) + I_A^P(x) + F_A^P(x) \leq 2, -2 \geq T_A^P(x) + I_A^P(x) + F_A^P(x) \geq 0.$$

Definition 3.2. A IBNS relation (IBNSR) is defined as a intuitionistic bipolar subset of $X \times Y$, having the form

$$R = \{< (x,y), T_R^P(x,y), I_R^P(x,y), F_R^P(x,y), T_R^N(x,y), I_R^N(x,y), F_R^N(x,y) >: x \in X, y \in Y\}$$

where,

$$T_R^P : X \times Y \to [0,1], I_R^P : X \times Y \to [0,1], F_R^P : X \times Y \to [0,1]$$
$$T_R^N : X \times Y \to [-1,0], I_R^N : X \times Y \to [-1,0], F_R^N : X \times Y \to [-1,0]$$

satisfy the conditions (i) at least one of this $T_R^P(x,y), I_R^P(x,y)$ and $F_R^P(x,y)$ is ≥ 0.5 at least one of this $T_R^N(x,y), I_R^N(x,y)$ and $F_R^N(x,y)$ is ≤ -0.5 and
(ii) $0 \leq T_A^P(x) + I_A^P(x) + F_A^P(x) \leq 2$, $-2 \geq T_A^P(x) + I_A^P(x) + F_A^P(x) \geq 0$.

Definition 3.3. Let $A_1 =< x, T_{A_1}^P(x), I_{A_1}^P(x), F_{A_1}^P(x), T_{A_1}^N(x), I_{A_1}^N(x), F_{A_1}^N(x) >$ and $A_2 =< x, T_{A_2}^P(x), I_{A_2}^P(x), F_{A_2}^P(x), T_{A_2}^N(x), I_{A_2}^N(x), F_{A_2}^N(x) >$ be two IBNSs. then $A_1 \subset A_2$ if any only if

$$T_{A_1}^P(x) \leq T_{A_2}^P(x), T_{A_1}^N(x) \geq T_{A_2}^N(x).$$
$$I_{A_1}^P(x) \leq I_{A_2}^P(x), I_{A_1}^N(x) \geq I_{A_2}^N(x).$$
$$F_{A_1}^P(x) \geq F_{A_2}^P(x), F_{A_1}^N(x) \leq F_{A_2}^N(x). \forall x \in X.$$

Definition 3.4. The union of two IBNSs A and B is also IBNS, whose truth membership, intermediate membership and false membership functions are,

$$T_{(A\cup B)}^P(x) = \max\{T_A^P(x), T_B^P(x)\}$$
$$I_{(A\cup B)}^P(x) = \min\{I_A^P(x), I_B^P(x)\}$$
$$F_{(A\cup B)}^P(x) = \min\{F_A^P(x), F_B^P(x)\},$$

and

$$T_{(A\cup B)}^N(x) = \min\{T_A^N(x), T_B^N(x)\}$$
$$T_{(A\cup B)}^N(x) = \max\{T_A^N(x), T_B^N(x)\}$$
$$T_{(A\cup B)}^N(x) = \max\{T_A^N(x), T_B^N(x)\},$$

for all $x \in X$.

Example 3.5. Let $A = \{((x_1, 0.7, 0.3, 0.4)^P(-0.6, -0.4, -0.3)^N), ((x_2, 0.5, 0.5, 0.8)^P (-0.6, -0.5, -0.4)^N)\}$ and $B = \{((x_1, 0.4, 0.7, 0.4)^P(-0.4, -0.7, -0.3)^N), ((x_2, 0.4, 0.3, 0.9)^P (-0.5, -0.6, -0.2)^N)\}$ be two IBNSs of X. Then by definition of union we get,

$A \cup B = \{((x_1, 0.7, 0.3, 0.3)^P(-0.6, -0.7, -0.3)^N), ((x_2, 0.5, 0.3, 0.8)^P(-0.6, -0.5, -0.2)^N)\}$

Definition 3.6. The intersection of two IBNSs A and B is also IBNS, whose truth-membership, indeterminacy-membership and falsity-membership functions are,

$$T^P_{(A\cap B)}(x) = \min\{T^P_A(x), T^P_B(x)\}$$
$$I^P_{(A\cap B)}(x) = \max\{I^P_A(x), I^P_B(x)\}$$
$$F^P_{(A\cap B)}(x) = \max\{F^P_A(x), F^P_B(x)\},$$

and

$$T^N_{(A\cap B)}(x) = \max\{T^N_A(x), T^N_B(x)\}$$
$$T^N_{(A\cap B)}(x) = \min\{T^N_A(x), T^N_B(x)\}$$
$$T^N_{(A\cap B)}(x) = \min\{T^N_A(x), T^N_B(x)\},$$

for all $x \in X$.

Example 3.7. For above example, then by definition of intersection, we obtain

$A \cap B = \{((x_1, 0.4, 0.3, 0.4)^P(-0.4, -0.4, -0.3)^N), ((x_2, 0.4, 0.3, 0.9)^P(-0.5, -0.5, -0.4)^N)\}$

Definition 3.8. The complement of IBNSs
$A =< x, T^P_A(x), I^P_A(x), F^P_A(x), T^N_A(x), I^N_A(x), F^N_A(x) >$ for all $x \in X$, is defined as

$$(T^P(x))^C = F^P(x), (I^P(x))^C = 1 - I^P(x), (F^P(x))^C = T^P(x),$$

and

$$(T^N(x))^C = F^N(x), (I^N(x))^C = -1 - I^N(x), (F^N(x))^C = T^N(x),$$

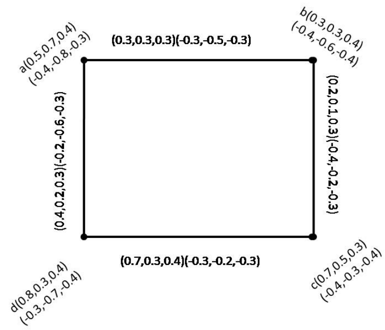
for all $x \in X$.

4. Intuitionistic Bipolar Neutrosophic Graphs

Definition 4.1. An Intuitionistic Bipolar Neutrosophic Graph (IBNG) is defined as a pair $G = (R, S)$, $R = (A^P, A^N)$ and $S = (B^P, B^N)$ where

(i) $R = \{r_1, r_2, ..., r_n\}$ such that, $T^P_A : R \to [0, 1], I^P_A : R \to [0, 1], F^P_A : R \to [0, 1], T^N_A : R \to [-1, 0], I^N_A : R \to [-1, 0]$, and $F^N_A : R \to [-1, 0]$ denote the degree of truth-membership, indeterminacy-membership and falsity-membership functions, respectively,
(ii) $S \subseteq R \times R$ where $T^P_B : R \times R \to [0, 1], I^P_B : R \times R \to [0, 1], F^P_B : R \times R \to [0, 1], T^N_B : R \times R \to [-1, 0], I^N_B : R \times R \to [-1, 0]$, and $F^N_B : R \times R \to [-1, 0]$
(iii) $T^P_B(rs) \le \min(T^P_A(r), T^P_A(s)), I^P_B(rs) \le \min(I^P_A(r), I^P_A(s))$, $F^P_B(rs) \le \max(F^P_A(r), F^P_A(s))$,
(iv) $T^P_B(rs) \wedge I^P_B(rs) \le 0.5, T^P_B(rs) \wedge F^P_B(rs) \le 0.5, I^P_B(rs) \wedge F^P_B(rs) \le 0.5$.
(v) $0 \le T^P_B(rs) + I^P_B(rs) + F^P_B(rs) \le 2$.
(vi) $T^N_B(rs) \ge \max(T^N_A(r), T^N_A(s)), I^N_B(rs) \ge \max(I^N_A(r), I^N_A(s))$, $F^N_B(rs) \ge \min(F^N_A(r), F^N_A(s))$,
(vii) $T^N_B(rs) \vee I^N_B(rs) \ge -0.5, T^N_B(rs) \vee F^N_B(rs) \ge -0.5, I^N_B(rs) \vee F^N_B(rs) \ge -0.5$
(viii) $0 \ge T^N_B(rs) + I^N_B(rs) + F^N_B(rs) \ge -2$.

Example 4.2. Consider a IBNGs such that $A = \{a, b, c, d\}$, $B = \{ab, bc, cd\}$ by routine condition we have,



$Figure\,1$: INTUITIONISTIC BIPOLAR NEUTROSOPHIC GRAPH

Definition 4.3. [3] A graph $G^{'} = (R^{'}, S^{'})$ is said to be subgraph of $G = (R, S)$ if

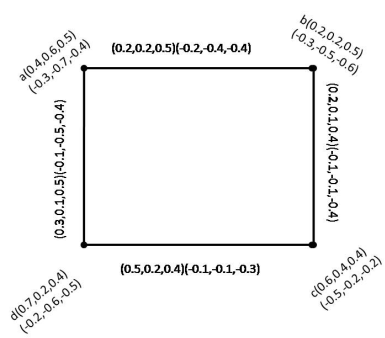
$$(T_A^{'})^P(r) \leq T_A^P(r), (I_A^{'})^P(r) \leq I_A^P(r), (F_A^{'})^P(r) \geq F_A^P(r)$$
$$(T_A^{'})^N(r) \geq T_A^N(r), (I_A^{'})^N(r) \geq T_A^N(r), (F_A^{'})^N(r) \leq F_A^N(r),$$

for all $r \in R$ and

$$(T_B^{'})^P(rs) \leq T_B^P(rs), (I_A^{'})^P(rs) \leq I_B^P(rs), (F_B^{'})^P(rs) \geq F_B^P(rs)$$
$$(T_B^{'})^N(rs) \geq T_B^N(rs), (I_B^{'})^N(rs) \geq T_B^N(rs), (F_B^{'})^N(rs) \leq F_B^N(rs),$$

for all $rs \in S$

Example 4.4. An IBNG subgraph is represented as Figure 2



$Figure\,2$: INTUITIONISTIC BIPOLAR NEUTROSOPHIC SUBGRAPH

Definition 4.5. A graph $G^{'} = (R', S')$ is said to be induced subgraph of $G = (R, S)$ if

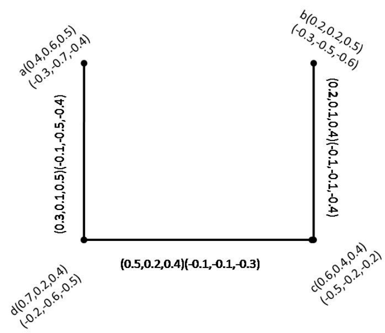
$$(T_A^{'})^P(r) = T_A^P(r), (I_A^{'})^P(r) = I_A^P(r), (F_A^{'})^P(r) = F_A^P(r)$$
$$(T_A^{'})^N(r) = T_A^N(r), (I_A^{'})^N(r) = T_A^N(r), (F_A^{'})^N(r) = F_A^N(r),$$

for all $r \in R$ and

$$(T_B^{'})^P(rs) = T_B^P(rs), (I_A^{'})^P(rs) = I_B^P(rs), (F_B^{'})^P(rs) = F_B^P(rs)$$
$$(T_B^{'})^N(rs) = T_B^N(rs), (I_B^{'})^N(rs) = T_B^N(rs), (F_B^{'})^N(rs) = F_B^N(rs),$$

for $rs \in S$

Example 4.6. An IBNG induced subgraph is represented as Figure 3.



$Figure\,3$: INTUITIONISTIC BIPOLAR NEUTROSOPHIC INDUCED SUBGRAPH

Definition 4.7. A graph $G^{'} = (R', S')$ is said to be spanning subgraph of $G = (R, S)$ if

$$(T_B^{'})^P(rs) \leq T_B^P(rs), (I_A^{'})^P(rs) \leq I_B^P(rs), (F_B^{'})^P(rs) \geq F_B^P(rs)$$
$$(T_B^{'})^N(rs) \geq T_B^N(rs), (I_B^{'})^N(rs) \geq T_B^N(rs), (F_B^{'})^N(rs) \leq F_B^N(rs),$$

for all $rs \in S$

Definition 4.8. An IBNG $G = (R, S)$ is called strong IBNG if

$$(T_B^{'})^P(rs) = T_B^P(rs), (I_A^{'})^P(rs) = I_B^P(rs), (F_B^{'})^P(rs) = F_B^P(rs)$$
$$(T_B^{'})^N(rs) = T_B^N(rs), (I_B^{'})^N(rs) = T_B^N(rs), (F_B^{'})^N(rs) = F_B^N(rs),$$

for all $rs \in S$. S is the set of edges.

Definition 4.9. An IBNG $G = (R, S)$ is called complete IBNG if

$$(T_B^{'})^P(rs) = T_B^P(rs), (I_A^{'})^P(rs) = I_B^P(rs), (F_B^{'})^P(rs) = F_B^P(rs)$$
$$(T_B^{'})^N(rs) = T_B^N(rs), (I_B^{'})^N(rs) = T_B^N(rs), (F_B^{'})^N(rs) = F_B^N(rs),$$

for all $rs \in S$. R is the set of nodes.

Definition 4.10. The Cartesian product of two IBNGs G_1 and G_2 is denoted by the pair $G_1 \times G_2 = (R_1 \times R_2, S_1 \times S_2)$ and defined as

$$\begin{aligned}
T^P_{A_1\times A_2}(kl) &= T^P_{A_1}(k) \wedge T^P_{A_2}(l)\\
I^P_{A_1\times A_2}(kl) &= I^P_{A_1}(k) \wedge I^P_{A_2}(l)\\
F^P_{A_1\times A_2}(kl) &= F^P_{A_1}(k) \vee F^P_{A_2}(l)\\
T^N_{A_1\times A_2}(kl) &= T^N_{A_1}(k) \vee T^N_{A_2}(l)\\
I^N_{A_1\times A_2}(kl) &= I^N_{A_1}(k) \vee I^N_{A_2}(l)\\
F^N_{A_1\times A_2}(kl) &= F^N_{A_1}(k) \wedge F^N_{A_2}(l),
\end{aligned}$$

for all $kl \in R_1 \times R_2$. The membership value of the edges in $G_1 \times G_2$ can be calculated as,

$$\begin{aligned}
(1) T^P_{B_1\times B_2}(k,l_1)(k,l_2) &= T^P_{A_1}(k) \wedge T^P_{B_2}(l_1l_2)\\
T^N_{B_1\times B_2}(k,l_1)(k,l_2) &= T^N_{A_1}(k) \vee T^N_{B_2}(l_1l_2),
\end{aligned}$$

$$\begin{aligned}
(2) I^P_{B_1\times B_2}(k,l_1)(k,l_2) &= I^P_{A_1}(k) \wedge I^P_{B_2}(l_1l_2)\\
I^N_{B_1\times B_2}(k,l_1)(k,l_2) &= I^N_{A_1}(k) \vee I^N_{B_2}(l_1l_2),
\end{aligned}$$

$$\begin{aligned}
(3) F^P_{B_1\times B_2}(k,l_1)(k,l_2) &= F^P_{A_1}(k) \vee F^P_{B_2}(l_1l_2)\\
F^N_{B_1\times B_2}(k,l_1)(k,l_2) &= F^N_{A_1}(k) \wedge F^N_{B_2}(l_1l_2),
\end{aligned}$$

for all $k \in R_1, l_1l_2 \in S_2$.

$$\begin{aligned}
(4) T^P_{B_1\times B_2}(k_1,l)(k_2,l) &= T^P_{A_2}(l) \wedge T^P_{B_2}(k_1k_2)\\
T^N_{B_1\times B_2}(k_1,l)(k_2,l) &= T^N_{A_2}(l) \vee T^N_{B_2}(k_1k_2),
\end{aligned}$$

$$\begin{aligned}
(5) I^P_{B_1\times B_2}(k_1,l)(k_2,l) &= I^P_{A_2}(l) \wedge I^P_{B_2}(k_1k_2)\\
I^N_{B_1\times B_2}(k_1,l)(k_2,l) &= I^N_{A_2}(l) \vee I^N_{B_2}(k_1k_2),
\end{aligned}$$

$$\begin{aligned}
(6) F^P_{B_1\times B_2}(k_1,l)(k_2,l) &= F^P_{A_2}(l) \vee F^P_{B_2}(k_1k_2)\\
F^N_{B_1\times B_2}(k_1,l)(k_2,l) &= F^N_{A_2}(l) \wedge F^N_{B_2}(k_1k_2),
\end{aligned}$$

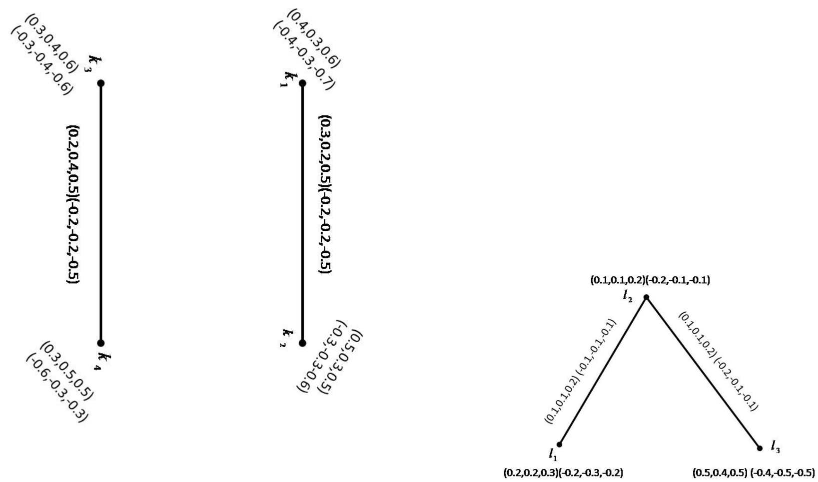
for all $k_1k_2 \in S_1, l \in R_2$.

Example 4.11. Consider $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ are two IBNG of $G = (R, S)$ respectively, as represented in Figure 4, now we get $G_1 \times G_2$ as follows Figure 5

Theorem 4.1. *The Cartesian product $G_1 \times G_2 = (R_1 \times R_2, S_1 \times S_2)$ of IBNG of IBNG G_1 and G_2 is an IBNG of $G_1 \times G_2$.*

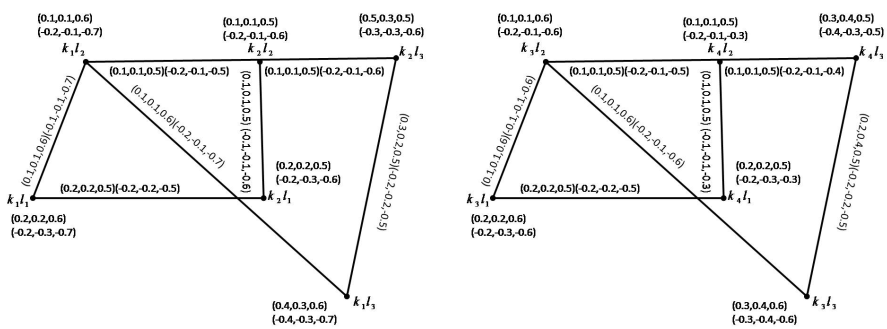
Proof. We consider:
Case 1: for $k \in R_1, l_1l_2 \in S_2$

$$\begin{aligned}
T^P_{(B_1\times B_2)}((kl_1)(kl_2)) &= T^P_{A_1}(k) \wedge T^P_{B_2}(l_1l_2)\\
&\leq T^P_{A_1}(k) \wedge [T^P_{A_2}(l_1) \wedge T^P_{A_2}(l_2)]\\
&= [T^P_{A_1}(k) \wedge T^P_{A_2}(l_1)] \wedge [T^P_{A_1}(k) \wedge T^P_{A_2}(l_2)]\\
&= T^P_{(A_1\times A_2)}(k,l_1) \wedge T^P_{(A_1\times A_2)}(k,l_2)
\end{aligned}$$



G_1 G_2

$Figure\, 4$



$G_1 \times G_2$

$Figure\, 5$: Cartesian product of IBNG

$$
\begin{aligned}
I^P_{(B_1\times B_2)}((kl_1)(kl_2)) &= I^P_{A_1}(k) \wedge I^P_{B_2}(l_1l_2)\\
&\le I^P_{A_1}(k) \wedge [I^P_{A_2}(l_1) \wedge I^P_{A_2}(l_2)]\\
&= [I^P_{A_1}(k) \wedge I^P_{A_2}(l_1)] \wedge [I^P_{A_1}(k) \wedge I^P_{A_2}(l_2)]\\
&= I^P_{(A_1\times A_2)}(k,l_1) \wedge I^P_{(A_1\times A_2)}(k,l_2)
\end{aligned}
$$

$$
\begin{aligned}
F^P_{(B_1\times B_2)}((kl_1)(kl_2)) &= F^P_{A_1}(k) \vee F^P_{B_2}(l_1l_2)\\
&\le F^P_{A_1}(k) \vee [F^P_{A_2}(l_1) \vee F^P_{A_2}(l_2)]\\
&= [F^P_{A_1}(k) \vee F^P_{A_2}(l_1)] \vee [F^P_{A_1}(k) \vee F^P_{A_2}(l_2)]\\
&= F^P_{(A_1\times A_2)}(k,l_1) \vee F^P_{(A_1\times A_2)}(k,l_2)
\end{aligned}
$$

for all $kl_1, kl_2 \in G_1 \times G_2$.
Case 2: for $k \in R_2, l_1l_2 \in S_1$

$$\begin{aligned}
T^P_{(B_1\times B_2)}((l_1k)(l_2k)) &= T^P_{A_2}(k) \wedge T^P_{B_1}(l_1l_2)\\
&\leq T^P_{A_2}(k) \wedge [T^P_{A_1}(l_1) \wedge T^P_{A_1}(l_2)]\\
&= [T^P_{A_2}(k) \wedge T^P_{A_1}(l_1)] \wedge [T^P_{A_2}(k) \wedge T^P_{A_1}(l_2)]\\
&= T^P_{(A_1\times A_2)}(l_1,k) \wedge T^P_{(A_1\times A_2)}(l_2,k)
\end{aligned}$$

$$\begin{aligned}
I^P_{(B_1\times B_2)}((l_1k)(l_2k)) &= I^P_{A_2}(k) \wedge I^P_{B_1}(l_1l_2)\\
&\leq I^P_{A_2}(k) \wedge [I^P_{A_1}(l_1) \wedge I^P_{A_1}(l_2)]\\
&= [I^P_{A_2}(k) \wedge I^P_{A_1}(l_1)] \wedge [I^P_{A_2}(k) \wedge I^P_{A_1}(l_2)]\\
&= I^P_{(A_1\times A_2)}(l_1,k) \wedge I^P_{(A_1\times A_2)}(l_2,k)
\end{aligned}$$

$$\begin{aligned}
F^P_{(B_1\times B_2)}((l_1k)(l_2k)) &= F^P_{A_2}(k) \vee F^P_{B_1}(l_1l_2)\\
&\leq F^P_{A_2}(k) \vee [F^P_{A_1}(l_1) \vee F^P_{A_1}(l_2)]\\
&= [F^P_{A_2}(k) \vee F^P_{A_1}(l_1)] \vee [F^P_{A_2}(k) \vee F^P_{A_1}(l_2)]\\
&= F^P_{(A_1\times A_2)}(l_1,k) \vee F^P_{(A_1\times A_2)}(l_2,k),
\end{aligned}$$

for all $l_1k, l_2k \in G_1 \times G_2$.
Similarly, one can prove the result for negative part also. □

Definition 4.12. The Cross product of two IBNGs G_1 and G_2 is denoted by the pair $G_1 \times G_2 = (R_1 \times R_2, S_1 \times S_2)$ and defined as

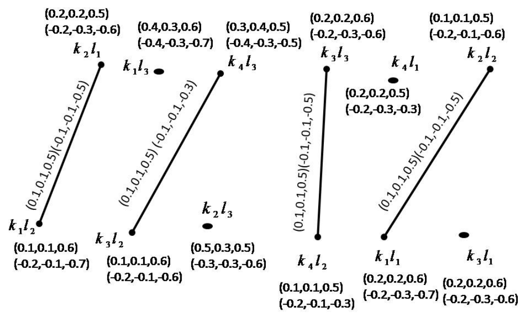
$$\begin{aligned}
(i)T^P_{A_1\times A_2}(kl) &= T^P_{A_1}(k) \wedge T^P_{A_2}(l)\\
I^P_{A_1\times A_2}(kl) &= I^P_{A_1}(k) \wedge I^P_{A_2}(l)\\
F^P_{A_1\times A_2}(kl) &= F^P_{A_1}(k) \vee F^P_{A_2}(l)\\
T^N_{A_1\times A_2}(kl) &= T^N_{A_1}(k) \vee T^N_{A_2}(l)\\
I^N_{A_1\times A_2}(kl) &= I^N_{A_1}(k) \vee I^N_{A_2}(l)\\
F^N_{A_1\times A_2}(kl) &= F^N_{A_1}(k) \wedge F^N_{A_2}(l),
\end{aligned}$$

for all $k, l \in R_1 \times R_2$.

$$\begin{aligned}
(ii)T^P_{(B_1\times B_2)}(k_1l_1)(k_2l_2) &= T^P_{B_1}(k_1k_2) \wedge T^P_{B_2}(l_1l_2)\\
I^P_{(B_1\times B_2)}(k_1l_1)(k_2l_2) &= I^P_{B_1}(k_1k_2) \wedge I^P_{B_2}(l_1l_2)\\
F^P_{(B_1\times B_2)}(k_1l_1)(k_2l_2) &= F^P_{B_1}(k_1k_2) \vee F^P_{B_2}(l_1l_2)\\
(iii)T^N_{(B_1\times B_2)}(k_1l_1)(k_2l_2) &= T^N_{B_1}(k_1k_2) \vee T^N_{B_2}(l_1l_2)\\
I^N_{(B_1\times B_2)}(k_1l_1)(k_2l_2) &= I^N_{B_1}(k_1k_2) \vee I^N_{B_2}(l_1l_2)\\
F^N_{(B_1\times B_2)}(k_1l_1)(k_2l_2) &= F^N_{B_1}(k_1k_2) \wedge F^N_{B_2}(l_1l_2),
\end{aligned}$$

for all $k_1k_2 \in S_1, l_1l_2 \in S_2$.

Example 4.13. Consider $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ are two IBNG of $G = (R, S)$ respectively, as represented in Figure 4. Now, we get cross product $G_1 \times G_2$ as follows Figure 6.



$G_1 \times G_2$

Figure 6: CROSS PRODUCT OF INTUITIONISTIC BIPOLAR NEUTROSOPHIC GRAPH

Theorem 4.2. *Cross product $G_1 \times G_2 = (R_1 \times R_2, S_1 \times S_2)$ of two IBNG of G_1 and G_2 is an IBNG of $G_1 \times G_2$.*

Proof. For all $k_1l_1, k_2l_2 \in G_1 \times G_2$

$$\begin{aligned}
T^P_{(B_1\times B_2)}((k_1l_1)(k_2l_2)) &= T^P_{B_1}(k_1k_2) \wedge T^P_{B_2}(l_1l_2)\\
&\leq [T^P_{A_1}(k_1) \wedge T^P_{A_1}(k_2)] \wedge [T^P_{A_2}(l_1) \wedge T^P_{A_2}(l_2)]\\
&= [T^P_{A_1}(k_1) \wedge T^P_{A_2}(l_1)] \wedge [T^P_{A_1}(k_2) \wedge T^P_{A_2}(l_2)]\\
&= T^P_{(A_1\times A_2)}(k_1l_1) \wedge T^P_{(A_1\times A_2)}(k_2, l_2),
\end{aligned}$$

$$\begin{aligned}
I^P_{(B_1\times B_2)}((k_1l_1)(k_2l_2)) &= I^P_{B_1}(k_1k_2) \wedge I^P_{B_2}(l_1l_2)\\
&\leq [I^P_{A_1}(k_1) \wedge I^P_{A_1}(k_2)] \wedge [I^P_{A_2}(l_1) \wedge I^P_{A_2}(l_2)]\\
&= [I^P_{A_1}(k_1) \wedge I^P_{A_2}(l_1)] \wedge [I^P_{A_1}(k_2) \wedge I^P_{A_2}(l_2)]\\
&= I^P_{(A_1\times A_2)}(k_1l_1) \wedge I^P_{(A_1\times A_2)}(k_2, l_2),
\end{aligned}$$

$$\begin{aligned}
F^P_{(B_1\times B_2)}((k_1l_1)(k_2l_2)) &= F^P_{B_1}(k_1k_2) \vee F^P_{B_2}(l_1l_2)\\
&\leq [F^P_{A_1}(k_1) \vee F^P_{A_1}(k_2)] \vee [F^P_{A_2}(l_1) \vee F^P_{A_2}(l_2)]\\
&= [F^P_{A_1}(k_1) \vee F^P_{A_2}(l_1)] \vee [F^P_{A_1}(k_2) \vee F^P_{A_2}(l_2)]\\
&= F^P_{(A_1\times A_2)}(k_1l_1) \vee F^P_{(A_1\times A_2)}(k_2, l_2)
\end{aligned}$$

Similarly, we can prove the result for negative part also. $\square$

Definition 4.14. The lexicographic product of two IBNGs G_1 and G_2 is denoted by the pair $G_1 \bullet G_2 = (R_1 \bullet R_2, S_1 \bullet S_2)$ and defined as

$$\begin{aligned}
(i)T^P_{(A_1 \bullet A_2)}(kl) &= T^P_{A_1}(k) \wedge T^P_{A_2}(l)\\
I^P_{(A_1 \bullet A_2)}(kl) &= I^P_{A_1}(k) \wedge I^P_{A_2}(l)\\
F^P_{(A_1 \bullet A_2)}(kl) &= F^P_{A_1}(k) \vee F^P_{A_2}(l)\\
T^N_{(A_1 \bullet A_2)}(kl) &= T^N_{A_1}(k) \vee T^N_{A_2}(l)\\
I^N_{(A_1 \bullet A_2)}(kl) &= I^N_{A_1}(k) \vee I^N_{A_2}(l)\\
F^N_{(A_1 \bullet A_2)}(kl) &= F^N_{A_1}(k) \wedge F^N_{A_2}(l),
\end{aligned}$$

for all $k, l \in R_1 \times R_2$

$$\begin{aligned}
(ii)T^P_{(B_1 \bullet B_2)}(kl_1)(kl_2) &= T^P_{A_1}(k) \wedge T^P_{B_2}(l_1l_2)\\
I^P_{(B_1 \bullet B_2)}(kl_1)(kl_2) &= I^P_{A_1}(k) \wedge I^P_{B_2}(l_1l_2)\\
F^P_{(B_1 \bullet B_2)}(kl_1)(kl_2) &= F^P_{A_1}(k) \vee F^P_{B_2}(l_1l_2)\\
T^N_{(B_1 \bullet B_2)}(kl_1)(kl_2) &= T^N_{A_1}(k) \vee T^N_{B_2}(l_1l_2)\\
I^N_{(B_1 \bullet B_2)}(kl_1)(kl_2) &= I^N_{A_1}(k) \vee I^N_{B_2}(l_1l_2)\\
F^N_{(B_1 \bullet B_2)}(kl_1)(kl_2) &= F^N_{A_1}(k) \wedge F^N_{B_2}(l_1l_2),
\end{aligned}$$

for all $k \in R_1, l_1l_2 \in S_2$.

$$\begin{aligned}
(iii)T^P_{(B_1 \bullet B_2)}(k_1l_1)(k_2l_2) &= T^P_{B_1}(k_1k_2) \wedge T^P_{B_2}(l_1l_2)\\
I^P_{(B_1 \bullet B_2)}(k_1l_1)(k_2l_2) &= I^P_{B_1}(k_1k_2) \wedge I^P_{B_2}(l_1l_2)\\
F^P_{(B_1 \bullet B_2)}(k_1l_1)(k_2l_2) &= F^P_{B_1}(k_1k_2) \vee F^P_{B_2}(l_1l_2)\\
T^N_{(B_1 \bullet B_2)}(k_1l_1)(k_2l_2) &= T^N_{B_1}(k_1k_2) \vee T^N_{B_2}(l_1l_2)\\
I^N_{(B_1 \bullet B_2)}(k_1l_1)(k_2l_2) &= I^N_{B_1}(k_1k_2) \vee I^N_{B_2}(l_1l_2)\\
F^N_{(B_1 \bullet B_2)}(k_1l_1)(k_2l_2) &= F^N_{B_1}(k_1k_2) \wedge F^N_{B_2}(l_1l_2),
\end{aligned}$$

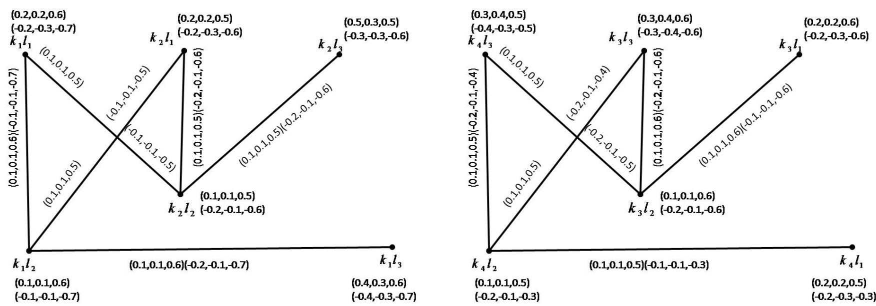
for all $k_1k_2 \in S_1, l_1l_2 \in S_2$.

Example 4.15. Lexicographic product of IBNG $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ shown in Figure 2 are defined as $G_1 \bullet G_2 = (R_1 \bullet R_2, S_1 \bullet S_2)$ and is represented in Figure 7.

Theorem 4.3. *Lexicographic product $G_1 \bullet G_2 = (R_1 \bullet R_2, S_1 \bullet S_2)$ of two IBNG of G_1 and G_2 is an IBNG of $G_1 \bullet G_2$.*

Proof. We consider two cases:
Case 1: for $k \in R_1, l_1l_2 \in S_2$

$$\begin{aligned}
T^P_{(B_1 \bullet B_2)}((kl_1)(kl_2)) &= T^P_{A_1}(k) \wedge T^P_{B_2}(l_1l_2)\\
&\leq T^P_{A_1}(k) \wedge [T^P_{A_2}(l_1) \wedge T^P_{A_2}(l_2)]\\
&= [T^P_{A_1}(k) \wedge T^P_{A_2}(l_1)] \wedge [T^P_{A_1}(k) \wedge T^P_{A_2}(l_2)]\\
&= T^P_{(A_1 \bullet A_2)}(k, l_1) \wedge T^P_{(A_1 \bullet A_2)}(k, l_2)
\end{aligned}$$



$G_1 \bullet G_2$

$Figure\,7$: LEXICOGRAPHIC PRODUCT INTUITIONISTIC BIPOLAR NEUTROSOPHIC GRAPH

$$
\begin{aligned}
I^P_{(B_1 \bullet B_2)}((kl_1)(kl_2)) &= I^P_{A_1}(k) \wedge I^P_{B_2}(l_1 l_2) \\
&\leq I^P_{A_1}(k) \wedge [I^P_{A_2}(l_1) \wedge I^P_{A_2}(l_2)] \\
&= [I^P_{A_1}(k) \wedge I^P_{A_2}(l_1)] \wedge [I^P_{A_1}(k) \wedge I^P_{A_2}(l_2)] \\
&= I^P_{(A_1 \bullet A_2)}(k, l_1) \wedge I^P_{(A_1 \bullet A_2)}(k, l_2)
\end{aligned}
$$

$$
\begin{aligned}
F^P_{(B_1 \bullet B_2)}((kl_1)(kl_2)) &= F^P_{A_1}(k) \vee F^P_{B_2}(l_1 l_2) \\
&\leq F^P_{A_1}(k) \vee [F^P_{A_2}(l_1) \vee F^P_{A_2}(l_2)] \\
&= [F^P_{A_1}(k) \vee F^P_{A_2}(l_1)] \vee [F^P_{A_1}(k) \vee F^P_{A_2}(l_2)] \\
&= F^P_{(A_1 \bullet A_2)}(k, l_1) \vee F^P_{(A_1 \bullet A_2)}(k, l_2)
\end{aligned}
$$

for all $kl_1, kl_2 \in S_1 \times S_2$.
Case 2: For all $k_1 k_2 \in S_1$, $l_1 l_2 \in S_2$

$$
\begin{aligned}
T^P_{(B_1 \bullet B_2)}((k_1 l_1)(k_2 l_2)) &= T^P_{B_1}(k_1 k_2) \wedge T^P_{B_2}(l_1 l_2) \\
&\leq [T^P_{A_1}(k_1) \wedge T^P_{A_1}(k_2)] \wedge [T^P_{A_2}(l_1) \wedge T^P_{A_2}(l_2)] \\
&= [T^P_{A_1}(k_1) \wedge T^P_{A_2}(l_1)] \wedge [T^P_{A_1}(k_2) \wedge T^P_{A_2}(l_2)] \\
&= T^P_{(A_1 \bullet A_2)}(k_1, l_1) \wedge T^P_{(A_1 \bullet A_2)}(k_2, l_2)
\end{aligned}
$$

$$
\begin{aligned}
I^P_{(B_1 \bullet B_2)}((k_1 l_1)(k_2 l_2)) &= I^P_{B_1}(k_1 k_2) \wedge I^P_{B_2}(l_1 l_2) \\
&\leq [I^P_{A_1}(k_1) \wedge I^P_{A_1}(k_2)] \wedge [I^P_{A_2}(l_1) \wedge I^P_{A_2}(l_2)] \\
&= [I^P_{A_1}(k_1) \wedge I^P_{A_2}(l_1)] \wedge [I^P_{A_1}(k_2) \wedge I^P_{A_2}(l_2)] \\
&= I^P_{(A_1 \bullet A_2)}(k_1, l_1) \wedge I^P_{(A_1 \bullet A_2)}(k_2, l_2)
\end{aligned}
$$

$$
\begin{aligned}
F^P_{(B_1 \times B_2)}((k_1 l_1)(k_2 l_2)) &= F^P_{B_1}(k_1 k_2) \vee F^P_{B_2}(l_1 l_2) \\
&\leq [F^P_{A_1}(k_1) \vee F^P_{A_1}(k_2)] \vee [F^P_{A_2}(l_1) \vee F^P_{A_2}(l_2)] \\
&= [F^P_{A_1}(k_1) \vee F^P_{A_2}(l_1)] \vee [F^P_{A_1}(k_2) \vee F^P_{A_2}(l_2)] \\
&= F^P_{(A_1 \bullet A_2)}(k_1, l_1) \vee F^P_{(A_1 \bullet A_2)}(k_2, l_2)
\end{aligned}
$$

for all $k_1 l_1, k_2 l_2 \in R_1 \bullet R_2$. Similarly, we can prove the result for negative part also. $\square$

Definition 4.16. The strong product of two IBNGs G_1 and G_2 is denoted by the pair $G_1 \boxtimes G_2 = (R_1 \boxtimes R_2, S_1 \boxtimes S_2)$ and defined as

$$
\begin{aligned}
(i)T^P_{(A_1\boxtimes A_2)}(kl) &= T^P_{A_1}(k) \wedge T^P_{A_2}(l),\\
I^P_{(A_1\boxtimes A_2)}(kl) &= I^P_{A_1}(k) \wedge I^P_{A_2}(l),\\
F^P_{(A_1\boxtimes A_2)}(kl) &= F^P_{A_1}(k) \vee F^P_{A_2}(l),\\
T^N_{(A_1\boxtimes A_2)}(kl) &= T^N_{A_1}(k) \vee T^N_{A_2}(l),\\
I^N_{(A_1\boxtimes A_2)}(kl) &= I^N_{A_1}(k) \vee I^N_{A_2}(l),\\
F^N_{(A_1\boxtimes A_2)}(kl) &= F^N_{A_1}(k) \wedge F^N_{A_2}(l),
\end{aligned}
$$

for all $k, l \in R_1 \boxtimes R_2$

$$
\begin{aligned}
(ii)T^P_{(B_1\boxtimes B_2)}(kl_1)(kl_2) &= T^P_{A_1}(k) \wedge T^P_{B_2}(l_1l_2),\\
I^P_{(B_1\boxtimes B_2)}(kl_1)(kl_2) &= I^P_{A_1}(k) \wedge I^P_{B_2}(l_1l_2),\\
F^P_{(B_1\boxtimes B_2)}(kl_1)(kl_2) &= F^P_{A_1}(k) \vee F^P_{B_2}(l_1l_2),\\
T^N_{(B_1\boxtimes B_2)}(kl_1)(kl_2) &= T^N_{A_1}(k) \vee T^N_{B_2}(l_1l_2),\\
I^N_{(B_1\boxtimes B_2)}(kl_1)(kl_2) &= I^N_{A_1}(k) \vee I^N_{B_2}(l_1l_2),\\
F^N_{(B_1\boxtimes B_2)}(kl_1)(kl_2) &= F^N_{A_1}(k) \wedge F^N_{B_2}(l_1l_2),
\end{aligned}
$$

for all $k \in R_1, l_1l_2 \in S_2$.

$$
\begin{aligned}
(iii)T^P_{B_1\boxtimes B_2}(k_1,l)(k_2,l) &= T^P_{A_2}(l) \wedge T^P_{B_2}(k_1k_2),\\
I^P_{B_1\boxtimes B_2}(k_1,l)(k_2,l) &= I^P_{A_2}(l) \wedge I^P_{B_2}(k_1k_2),\\
F^P_{B_1\boxtimes B_2}(k_1,l)(k_2,l) &= F^P_{A_2}(l) \vee F^P_{B_2}(k_1k_2),\\
T^N_{B_1\boxtimes B_2}(k_1,l)(k_2,l) &= T^N_{A_2}(l) \vee T^N_{B_2}(k_1k_2),\\
I^N_{B_1\boxtimes B_2}(k_1,l)(k_2,l) &= I^N_{A_2}(l) \vee I^N_{B_2}(k_1k_2),\\
F^N_{B_1\boxtimes B_2}(k_1,l)(k_2,l) &= F^N_{A_2}(l) \wedge F^N_{B_2}(k_1k_2),
\end{aligned}
$$

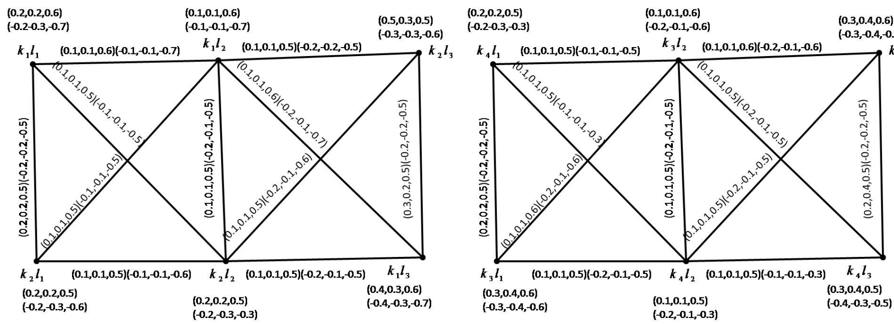
for all $k_1k_2 \in S_1, l \in R_2$.

$$
\begin{aligned}
(iv)T^P_{(B_1\boxtimes B_2)}(k_1l_1)(k_2l_2) &= T^P_{B_1}(k_1k_2) \wedge T^P_{B_2}(l_1l_2)\\
I^P_{(B_1\boxtimes B_2)}(k_1l_1)(k_2l_2) &= I^P_{B_1}(k_1k_2) \wedge I^P_{B_2}(l_1l_2)\\
F^P_{(B_1\boxtimes B_2)}(k_1l_1)(k_2l_2) &= F^P_{B_1}(k_1k_2) \vee F^P_{B_2}(l_1l_2)\\
T^N_{(B_1\boxtimes B_2)}(k_1l_1)(k_2l_2) &= T^N_{B_1}(k_1k_2) \vee T^N_{B_2}(l_1l_2)\\
I^N_{(B_1\boxtimes B_2)}(k_1l_1)(k_2l_2) &= I^N_{B_1}(k_1k_2) \vee I^N_{B_2}(l_1l_2)\\
F^N_{(B_1\boxtimes B_2)}(k_1l_1)(k_2l_2) &= F^N_{B_1}(k_1k_2) \wedge F^N_{B_2}(l_1l_2)
\end{aligned}
$$

for all $k_1k_2 \in S_1, l_1l_2 \in S_2$.

Example 4.17. Strong product of IBNG $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ shown in Figure 2 is defined as $G_1 \boxtimes G_2 = (R_1 \boxtimes R_2, S_1 \boxtimes S_2)$ and is represented in Figure 8.

Theorem 4.4. *Strong product $G_1 \boxtimes G_2 = (R_1 \boxtimes R_2, S_1 \boxtimes S_2)$ of two IBNG of G_1 and G_2 is an IBNG of $G_1 \boxtimes G_2$.*



$G_1 \boxtimes G_2$

$Figure\,8$: STRONG PRODUCT INTUITIONISTIC BIPOLAR NEUTROSOPHIC GRAPH

Proof. There are three cases:

Case 1: for $k \in R_1, l_1l_2 \in S_2$

$$\begin{aligned}
T^P_{(B_1 \boxtimes B_2)}((kl_1)(kl_2)) &= T^P_{A_1}(k) \wedge T^P_{B_2}(l_1l_2) \\
&\leq T^P_{A_1}(k) \wedge [T^P_{A_2}(l_1) \wedge T^P_{A_2}(l_2)] \\
&= [T^P_{A_1}(k) \wedge T^P_{A_2}(l_1)] \wedge [T^P_{A_1}(k) \wedge T^P_{A_2}(l_2)] \\
&= T^P_{(A_1 \boxtimes A_2)}(k, l_1) \wedge T^P_{(A_1 \boxtimes A_2)}(k, l_2)
\end{aligned}$$

$$\begin{aligned}
I^P_{(B_1 \boxtimes B_2)}((kl_1)(kl_2)) &= I^P_{A_1}(k) \wedge I^P_{B_2}(l_1l_2) \\
&\leq I^P_{A_1}(k) \wedge [I^P_{A_2}(l_1) \wedge I^P_{A_2}(l_2)] \\
&= [I^P_{A_1}(k) \wedge I^P_{A_2}(l_1)] \wedge [I^P_{A_1}(k) \wedge I^P_{A_2}(l_2)] \\
&= I^P_{(A_1 \boxtimes A_2)}(k, l_1) \wedge I^P_{(A_1 \boxtimes A_2)}(k, l_2)
\end{aligned}$$

$$\begin{aligned}
F^P_{(B_1 \boxtimes B_2)}((kl_1)(kl_2)) &= F^P_{A_1}(k) \vee F^P_{B_2}(l_1l_2) \\
&\leq F^P_{A_1}(k) \vee [F^P_{A_2}(l_1) \vee F^P_{A_2}(l_2)] \\
&= [F^P_{A_1}(k) \vee F^P_{A_2}(l_1)] \vee [F^P_{A_1}(k) \vee F^P_{A_2}(l_2)] \\
&= F^P_{(A_1 \boxtimes A_2)}(k, l_1) \vee F^P_{(A_1 \boxtimes A_2)}(k, l_2)
\end{aligned}$$

for all $kl_1, kl_2 \in R_1 \boxtimes R_2$.

Case 2: for $k \in R_2, l_1l_2 \in S_1$

$$\begin{aligned}
T^P_{(B_1 \boxtimes B_2)}((l_1k)(l_2k)) &= T^P_{A_2}(k) \wedge T^P_{B_1}(l_1l_2) \\
&\leq T^P_{A_2}(k) \wedge [T^P_{A_1}(l_1) \wedge T^P_{A_1}(l_2)] \\
&= [T^P_{A_2}(k) \wedge T^P_{A_1}(l_1)] \wedge [T^P_{A_2}(k) \wedge T^P_{A_1}(l_2)] \\
&= T^P_{(A_1 \boxtimes A_2)}(l_1, k) \wedge T^P_{(A_1 \boxtimes A_2)}(l_2, k)
\end{aligned}$$

$$\begin{aligned}
I^P_{(B_1\boxtimes B_2)}((l_1k)(l_2k)) &= I^P_{A_2}(k)\wedge I^P_{B_1}(l_1l_2)\\
&\le I^P_{A_2}(k)\wedge [I^P_{A_1}(l_1)\wedge I^P_{A_1}(l_2)]\\
&= [I^P_{A_2}(k)\wedge I^P_{A_1}(l_1)]\wedge [I^P_{A_2}(k)\wedge I^P_{A_1}(l_2)]\\
&= I^P_{(A_1\boxtimes A_2)}(l_1,k)\wedge I^P_{(A_1\boxtimes A_2)}(l_2,k)
\end{aligned}$$

$$\begin{aligned}
F^P_{(B_1\boxtimes B_2)}((l_1k)(l_2k)) &= F^P_{A_2}(k)\vee F^P_{B_1}(l_1l_2)\\
&\le F^P_{A_2}(k)\vee [F^P_{A_1}(l_1)\vee F^P_{A_1}(l_2)]\\
&= [F^P_{A_2}(k)\vee F^P_{A_1}(l_1)]\vee [F^P_{A_2}(k)\vee F^P_{A_1}(l_2)]\\
&= F^P_{(A_1\boxtimes A_2)}(l_1,k)\vee F^P_{(A_1\boxtimes A_2)}(l_2,k)
\end{aligned}$$

for all $l_1k, l_2k \in R_1 \boxtimes R_2$. □

Case 3: For all $k_1k_2 \in S_1$,$l_1l_2 \in S_2$,

$$\begin{aligned}
T^P_{(B_1\boxtimes B_2)}((k_1l_1)(k_2l_2)) &= T^P_{B_1}(k_1k_2)\wedge T^P_{B_2}(l_1l_2)\\
&\le [T^P_{A_1}(k_1)\wedge T^P_{A_1}(k_2)]\wedge [T^P_{A_2}(l_1)\wedge T^P_{A_2}(l_2)]\\
&= [T^P_{A_1}(k_1)\wedge T^P_{A_2}(l_1)]\wedge [T^P_{A_1}(k_2)\wedge T^P_{A_2}(l_2)]\\
&= T^P_{(A_1\boxtimes A_2)}(k_1,l_1)\wedge T^P_{(A_1\boxtimes A_2)}(k_2,l_2)
\end{aligned}$$

$$\begin{aligned}
I^P_{(B_1\boxtimes B_2)}((k_1l_1)(k_2l_2)) &= I^P_{B_1}(k_1k_2)\wedge I^P_{B_2}(l_1l_2)\\
&\le [I^P_{A_1}(k_1)\wedge I^P_{A_1}(k_2)]\wedge [I^P_{A_2}(l_1)\wedge I^P_{A_2}(l_2)]\\
&= [I^P_{A_1}(k_1)\wedge I^P_{A_2}(l_1)]\wedge [I^P_{A_1}(k_2)\wedge I^P_{A_2}(l_2)]\\
&= I^P_{(A_1\boxtimes A_2)}(k_1,l_1)\wedge I^P_{(A_1\boxtimes A_2)}(k_2,l_2)
\end{aligned}$$

$$\begin{aligned}
F^P_{(B_1\boxtimes B_2)}((k_1l_1)(k_2l_2)) &= F^P_{B_1}(k_1k_2)\vee F^P_{B_2}(l_1l_2)\\
&\le [F^P_{A_1}(k_1)\vee F^P_{A_1}(k_2)]\vee [F^P_{A_2}(l_1)\vee F^P_{A_2}(l_2)]\\
&= [F^P_{A_1}(k_1)\vee F^P_{A_2}(l_1)]\vee [F^P_{A_1}(k_2)\vee F^P_{A_2}(l_2)]\\
&= F^P_{(A_1\boxtimes A_2)}(k_1,l_1)\vee F^P_{(A_1\boxtimes A_2)}(k_2,l_2)
\end{aligned}$$

for all $k_1l_1, k_2l_2 \in R_1 \boxtimes R_2$. Similarly, we can prove the result for negative part also.

Definition 4.18. The composition of two IBNGs G_1 and G_2 is denoted by the pair $G_1 \circ G_2 = (R_1 \circ R_2, S_1 \circ S_2)$ and defined as

$$\begin{aligned}
(i)T^P_{(A_1\circ A_2)}(kl) &= T^P_{A_1}(k)\wedge T^P_{A_2}(l)\\
I^P_{(A_1\circ A_2)}(kl) &= I^P_{A_1}(k)\wedge I^P_{A_2}(l)\\
F^P_{(A_1\circ A_2)}(kl) &= F^P_{A_1}(k)\vee F^P_{A_2}(l)\\
T^N_{(A_1\circ A_2)}(kl) &= T^N_{A_1}(k)\vee T^N_{A_2}(l)\\
I^N_{(A_1\circ A_2)}(kl) &= I^N_{A_1}(k)\vee I^N_{A_2}(l)\\
F^N_{(A_1\circ A_2)}(kl) &= F^N_{A_1}(k)\wedge F^N_{A_2}(l),
\end{aligned}$$

for all $k, l \in R_1 \circ R_2$

$$\begin{aligned}(ii) T^P_{(B_1\circ B_2)}(kl_1)(kl_2) &= T^P_{A_1}(k) \wedge T^P_{B_2}(l_1l_2)\\ I^P_{(B_1\circ B_2)}(kl_1)(kl_2) &= I^P_{A_1}(k) \wedge I^P_{B_2}(l_1l_2)\\ F^P_{(B_1\circ B_2)}(kl_1)(kl_2) &= F^P_{A_1}(k) \vee F^P_{B_2}(l_1l_2)\\ T^N_{(B_1\circ B_2)}(kl_1)(kl_2) &= T^N_{A_1}(k) \vee T^N_{B_2}(l_1l_2)\\ I^N_{(B_1\circ B_2)}(kl_1)(kl_2) &= I^N_{A_1}(k) \vee I^N_{B_2}(l_1l_2)\\ F^N_{(B_1\circ B_2)}(kl_1)(kl_2) &= F^N_{A_1}(k) \wedge F^N_{B_2}(l_1l_2),\end{aligned}$$

for all $k \in R_1, l_1l_2 \in S_2$.

$$\begin{aligned}(iii) T^P_{B_1\circ B_2}(k_1l)(k_2l) &= T^P_{A_2}(l) \wedge T^P_{B_2}(k_1k_2)\\ I^P_{B_1\circ B_2}(k_1l)(k_2l) &= I^P_{A_2}(l) \wedge I^P_{B_2}(k_1k_2)\\ F^P_{B_1\circ B_2}(k_1l)(k_2l) &= F^P_{A_2}(l) \vee F^P_{B_2}(k_1k_2)\\ T^N_{B_1\circ B_2}(k_1l)(k_2l) &= T^N_{A_2}(l) \vee T^N_{B_2}(k_1k_2)\\ I^N_{B_1\circ B_2}(k_1l)(k_2l) &= I^N_{A_2}(l) \vee I^N_{B_2}(k_1k_2)\\ F^N_{B_1\circ B_2}(k_1l)(k_2l) &= F^N_{A_2}(l) \wedge F^N_{B_2}(k_1k_2),\end{aligned}$$

for all $k_1k_2 \in S_1, l \in R_2$.

$$\begin{aligned}(iv) T^P_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) &= T^P_{B_1}(k_1k_2) \wedge T^P_{A_2}(l_1) \wedge T^P_{A_2}(l_2)\\ I^P_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) &= I^P_{B_1}(k_1k_2) \wedge I^P_{A_2}(l_1) \wedge I^P_{A_2}(l_2)\\ F^P_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) &= F^P_{B_1}(k_1k_2) \vee F^P_{A_2}(l_1) \vee F^P_{A_2}(l_2)\end{aligned}$$

$$\begin{aligned}T^N_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) &= T^N_{B_1}(k_1k_2) \vee T^N_{A_2}(l_1) \vee T^N_{A_2}(l_2)\\ I^N_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) &= I^N_{B_1}(k_1k_2) \vee I^N_{A_2}(l_1) \vee I^N_{A_2}(l_2)\\ F^N_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) &= F^N_{B_1}(k_1k_2) \wedge F^N_{A_2}(l_1) \wedge F^N_{A_2}(l_2),\end{aligned}$$

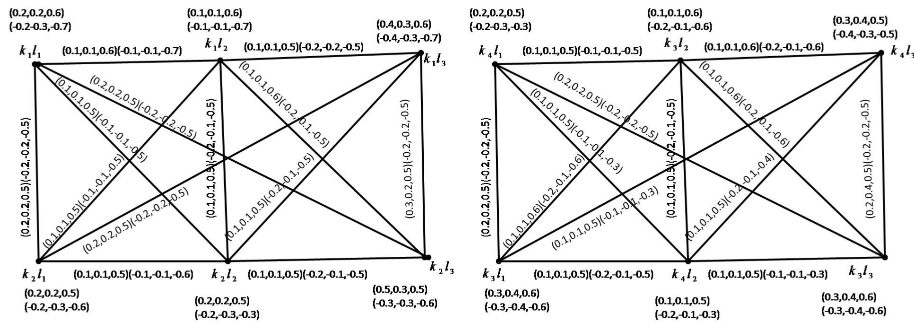
for all $k_1k_2 \in S_1, l_1l_2 \in S_2$ such that $l_1 \neq l_2$

Example 4.19. Composition of IBNG $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ shown in Figure 2 is defined as $G_1 \circ G_2 = (R_1 \circ R_2, S_1 \circ S_2)$ and is represented in Figure 9.

Theorem 4.5. *Composition $G_1 \circ G_2 = (R_1 \circ R_2, S_1 \circ S_2)$ of two IBNG of G_1 and G_2 is an IBNG of $G_1 \circ G_2$.*

Proof. There are three cases:
Case 1: for $k \in R_1, l_1l_2 \in S_2$

$$\begin{aligned}T^P_{(B_1\circ B_2)}((kl_1)(kl_2)) &= T^P_{A_1}(k) \wedge T^P_{B_2}(l_1l_2)\\ &\leq T^P_{A_1}(k) \wedge [T^P_{A_2}(l_1) \wedge T^P_{A_2}(l_2)]\\ &= [T^P_{A_1}(k) \wedge T^P_{A_2}(l_1)] \wedge [T^P_{A_1}(k) \wedge T^P_{A_2}(l_2)]\\ &= T^P_{(A_1\circ A_2)}(k, l_1) \wedge T^P_{(A_1\circ A_2)}(k, l_2)\end{aligned}$$



$G_1 \circ G_2$

$Figure\,9$: COMPOSITION INTUITIONISTIC BIPOLAR NEUTROSOPHIC GRAPH

$$\begin{aligned}
I^P_{(B_1 \circ B_2)}((kl_1)(kl_2)) &= I^P_{A_1}(k) \wedge I^P_{B_2}(l_1 l_2) \\
&\leq I^P_{A_1}(k) \wedge [I^P_{A_2}(l_1) \wedge I^P_{A_2}(l_2)] \\
&= [I^P_{A_1}(k) \wedge I^P_{A_2}(l_1)] \wedge [I^P_{A_1}(k) \wedge I^P_{A_2}(l_2)] \\
&= I^P_{(A_1 \circ A_2)}(k, l_1) \wedge I^P_{(A_1 \circ A_2)}(k, l_2)
\end{aligned}$$

$$\begin{aligned}
F^P_{(B_1 \circ B_2)}((kl_1)(kl_2)) &= F^P_{A_1}(k) \vee F^P_{B_2}(l_1 l_2) \\
&\leq F^P_{A_1}(k) \vee [F^P_{A_2}(l_1) \vee F^P_{A_2}(l_2)] \\
&= [F^P_{A_1}(k) \vee F^P_{A_2}(l_1)] \vee [F^P_{A_1}(k) \vee F^P_{A_2}(l_2)] \\
&= F^P_{(A_1 \circ A_2)}(k, l_1) \vee F^P_{(A_1 \circ A_2)}(k, l_2)
\end{aligned}$$

for all $kl_1, kl_2 \in R_1 \circ R_2$.
Case 2: for $k \in R_2, l_1 l_2 \in S_1$

$$\begin{aligned}
T^P_{(B_1 \circ B_2)}((l_1 k)(l_2 k)) &= T^P_{A_2}(k) \wedge T^P_{B_1}(l_1 l_2) \\
&\leq T^P_{A_2}(k) \wedge [T^P_{A_1}(l_1) \wedge T^P_{A_1}(l_2)] \\
&= [T^P_{A_2}(k) \wedge T^P_{A_1}(l_1)] \wedge [T^P_{A_2}(k) \wedge T^P_{A_1}(l_2)] \\
&= T^P_{(A_1 \circ A_2)}(l_1, k) \wedge T^P_{(A_1 \circ A_2)}(l_2, k)
\end{aligned}$$

$$\begin{aligned}
I^P_{(B_1 \circ B_2)}((l_1 k)(l_2 k)) &= I^P_{A_2}(k) \wedge I^P_{B_1}(l_1 l_2) \\
&\leq I^P_{A_2}(k) \wedge [I^P_{A_1}(l_1) \wedge I^P_{A_1}(l_2)] \\
&= [I^P_{A_2}(k) \wedge I^P_{A_1}(l_1)] \wedge [I^P_{A_2}(k) \wedge I^P_{A_1}(l_2)] \\
&= I^P_{(A_1 \circ A_2)}(l_1, k) \wedge I^P_{(A_1 \circ A_2)}(l_2, k)
\end{aligned}$$

$$\begin{aligned}
F^P_{(B_1 \circ B_2)}((l_1 k)(l_2 k)) &= F^P_{A_2}(k) \vee F^P_{B_1}(l_1 l_2) \\
&\leq F^P_{A_2}(k) \vee [F^P_{A_1}(l_1) \vee F^P_{A_1}(l_2)] \\
&= [F^P_{A_2}(k) \vee F^P_{A_1}(l_1)] \vee [F^P_{A_2}(k) \vee F^P_{A_1}(l_2)] \\
&= F^P_{(A_1 \circ A_2)}(l_1, k) \vee F^P_{(A_1 \circ A_2)}(l_2, k)
\end{aligned}$$

for all $l_1k, l_2k \in R_1 \circ R_2$.
Case 3: For $k_1k_2 \in S_1, l_1, l_2 \in R_2$ such that $l_1 \neq l_2$

$$\begin{aligned}
T^P_{(B_1 \circ B_2)}((k_1l_1)(k_2l_2)) &= T^P_{B_1}(k_1, k_2) \wedge T^P_{A_2}(l_1) \wedge T^P_{A_2}(l_2)\\
&\leq [T^P_{A_1}(k_1) \wedge T^P_{A_1}(k_2)] \wedge [T^P_{A_2}(l_1) \wedge T^P_{A_2}(l_2)]\\
&= [T^P_{A_1}(k_1) \wedge T^P_{A_2}(l_1)] \wedge [T^P_{A_1}(k_2) \wedge T^P_{A_2}(l_2)]\\
&= T^P_{(A_1 \circ A_2)}(k_1l_1) \wedge T^P_{(A_1 \circ A_2)}(k_2l_2)
\end{aligned}$$

$$\begin{aligned}
I^P_{(B_1 \circ B_2)}((k_1, l_1)(k_2, l_2)) &= I^P_{B_1}(k_1, k_2) \wedge I^P_{A_2}(l_1) \wedge I^P_{A_2}(l_2)\\
&\leq [I^P_{A_1}(k_1) \wedge I^P_{A_1}(k_2)] \wedge [I^P_{A_2}(l_1) \wedge I^P_{A_2}(l_2)]\\
&= [I^P_{A_1}(k_1) \wedge I^P_{A_2}(l_1)] \wedge [I^P_{A_1}(k_2) \wedge I^P_{A_2}(l_2)]\\
&= I^P_{(A_1 \circ A_2)}(k_1l_1) \wedge I^P_{(A_1 \circ A_2)}(k_2l_2)
\end{aligned}$$

$$\begin{aligned}
F^P_{(B_1 \circ B_2)}((k_1, l_1)(k_2, l_2)) &= F^P_{B_1}(k_1, k_2) \vee F^P_{A_2}(l_1) \vee F^P_{A_2}(l_2)\\
&\leq [F^P_{A_1}(k_1) \vee F^P_{A_1}(k_2)] \vee [F^P_{A_2}(l_1) \vee F^P_{A_2}(l_2)]\\
&= [F^P_{A_1}(k_1) \vee F^P_{A_2}(l_1)] \vee [F^P_{A_1}(k_2) \vee F^P_{A_2}(l_2)]\\
&= F^P_{(A_1 \circ A_2)}(k_1l_1) \vee F^P_{(A_1 \circ A_2)}(k_2l_2)
\end{aligned}$$

for all $k_1l_1, k_2l_2 \in R_1 \circ R_2$. Similarly, we can prove the result for negative part also. □

5. CONCLUSIONS

In this work, a new concept of intuitionistic bipolar neutrosophic set with the operations like union, intersection and complement have been developed. Also, an application to intuitionistic bipolar neutrosophic graph with examples have established. Further, we presented the Cartesian product, cross product, lexicographic product and strong product with suitable examples. In future, isomorphic properties will be investigated.

6. ACKNOWLEDGEMENTS

The authors appreciate both the anonymous editor and reviewer for improve the quality of this manuscript.

REFERENCES

[1] A. Al-Masarwah and A.G. Ahmad, A new form of generalized m-PF ideals in BCK/BCI-algebras, Annals of Communications in Mathematics 2(1) (2019), 11-16.
[2] A. Al-Masarwah and A.G. Ahmad, On (complete) normality of m-pF subalgebras in BCK/BCI- algebras, AIMS Mathematics 4(3) (2019), 740-750.
[3] M. Akram and M. Sitara, Bipolar neutrosophic graph structures, J. Indones. Math. Soc., 23(1)(2017), 55-76.
[4] M. Akram and M. Sarwar, Novel multiple criteria decision making methods based on bipolar neutrosophic sets and bipolar neutrosophic graphs, Ital. J. Pure Appl. Math., 38 (2017), 368-389
[5] M. Akram and S. Shahzadi, Representation of Graphs using Intuitionistic Neutrosophic Soft Sets, J. Math. Anal., 7(6) (2016), 31-53.
[6] M. Akram and M. Sitara, Application of intuitionistic neutrosophic graph structures in decision-making, Annals of Fuzzy Mathematics and Informatics, 14(1) (2017), 1-27
[7] M. Akram and M. Sitara, Certain concepts in intuitionistic neutrosophic graph structures, Informat., 8(4) (2017), 154.
[8] R. A. Borzooei and H. Rashmanlou, Domination in vague graphs and its applications, J. Intell. Fuzzy Syst., 29 (2015), 1933-1940.

[9] R. A. Borzooei, H. Rashmanlou, S. Samanta and M. Pal, Regularity of vague graphs, J. Intell. Fuzzy Syst., 30 (2016), 3681-3689.
[10] S. Broumi and F. Smarandache, Intuitionistic neutrosophic soft set, Infinite Study., (2013), 130-140.
[11] S. Broumi and F. Smarandache, Intuitionistic neutrosophic soft set, J. Inf. Comp. Sci., 8(2) (2013), 130-140.
[12] S. Broumi, F. Smarandache, M. Talea and A. Bakali, Single valued neutrosophic graphs: Degree, Order and Size, 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), 2016.
[13] S. Broumi, M. Talea, A. Bakali and F. Smarandache, Single valued neutrosophic graphs, J. New Theory, 10 (2016), 861-1001
[14] M. Bhowmik and M. Pal, Intuitionistic neutrosophic set, J. Inf. Comp. Sci., 4(2) (2009), 142-152.
[15] M Bhowmik and M Pal, Intuitionistic neutrosophic set, J. Inf. Comp. Sci., 5(3) (2010), 183-192.
[16] I. Deli, M. Ali and F. Smarandache, Bipolar neutrosophic sets and their application based on multi-criteria decision making problems, 2015 International Conference on Advanced Mechatronic Systems (ICAMechS), (2015) 249-254. DOI:10.1109/icamechs.2015.7287068
[17] R. Hashim, M. Gulistan and F. Smarandache, Applications of neutrosophic bipolar fuzzy sets in HOPE foundation for planning to build a children hospital with different types of similarity measures, Symmetry, 10(8) (2018) 331.
[18] G. Muhiuddin, Bipolar fuzzy KU-subalgebras/ideals of KU-algebras, Annals of Fuzzy Mathematics and Informatics, 8(3) (2014), 409-418.
[19] S. S. Hussain, R. J. Hussain, Y. B. Jun and F. Smarandache, Neutrosophic bipolar vague set and its application to neutrosophic bipolar vague graphs, Neutrosophic Sets & Systems, 28 (2019), 68-86.
[20] S. S. Hussain, R. J. Hussain and F. Smarandache, On neutrosophic vague graphs, Neutrosophic Sets & Systems, 28(2019) 245-258.
[21] Y. B. Jun, F. Smarandache, S. Z. Song and M. Khan, Neutrosophic positive implicative N-ideals in BCK-algebras, Axioms, 7(1) (2018), 3.
[22] Y. B. Jun, S. Kim and F. Smarandache, Interval neutrosophic sets with applications in BCK/BCI-algebra. Axioms, 7(2) (2018), 23.
[23] Y. B. Jun and K. Hur, Bipolar-valued fuzzy subalgebras based on bipolar-valued fuzzy points, Annals of Fuzzy Mathematics and Informatics, 12(6) (2016), 893-902.
[24] Y.B. Jun, M.S. Kang, H.S. Kim, Bipolar fuzzy hyper BCK-ideals in hyper BCK-algebras, Iran. J. Fuzzy Systems 8(2), (2011) 105-120.
[25] F. Smarandache, A Unifying Field in Logics, Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press, 1999.
[26] V. Ulucay, I. Deli and M. Sahin, Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making, Neural Comput. Appl., 29(3)(2018), 739-748.

S. SATHAM HUSSAIN
PG AND RESEARCH DEPARTMENT OF MATHEMATICS, JAMAL MOHAMED COLLEGE, TRICHY - 620 020, TAMIL NADU, INDIA.
Email address: sathamhussain5592@gmail.com

SAID BROUMI
LABORATORY OF INFORMATION PROCESSING, FACULTY OF SCIENCE BEN MSIK, UNIVERSITY HASSAN II, B.P 7955, SIDI OTHMAN, CASABLANCA, MOROCCO.
Email address: broumisaid78@gmail.com

YOUNG BAE JUN
DEPARTMENT OF MATHEMATICS EDUCATION, GYEONGSANG NATIONAL UNIVERSITY, JINJU 52828, KOREA.
Email address: skywine@gmail.com

DURGA NAGARAJAN
DEPARTMENT OF MATHEMATICS, THE GANDHIGRAM RURAL INSTITUTE (DEEMED TO BE UNIVERSITY), GANDHIGRAM - 624 302, TAMIL NADU, INDIA.
Email address: durga1992mdu@gmail.com