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A NEW APPROACH TO NEUTROSOPHIC SOFT SET MINIMAL STRUCTURE SPACES

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ABSTRACT. This chapter is an introduction of neutrosophic soft set minimal structure space and addresses properties of neutrosophic soft set minimal structure space. Neutrosophic set has plenty of applications. Moreover, we introduced neutrosophic soft set minimal closed set , neutrosophic soft set minimal open set, neutrosophic soft set minimal interior of ($\widetilde{F}$, E), neutrosophic soft set minimal closure of ($\widetilde{F}$, E), $\mathrm{NSS}_m(\widetilde{F}$, E)-continuous, $\mathrm{NSS}_m(\widetilde{F}$, E)-$\mathrm{T}_0$, $\mathrm{NSS}_m(\widetilde{F}$, E)-$\mathrm{T}_1$, $\mathrm{NSS}_m(\widetilde{F}$, E)-$\mathrm{T}_2$, $\mathrm{NSS}_m(\widetilde{F}$, E)-compact, almost $\mathrm{NSS}_m(\widetilde{F}$, E)-compact. We also present some of their basic properties.

1. INTRODUCTION

The contribution of mathematics to the present-day technology in reaching to a fast trend cannot be ignored. The theories presented differently from classical methods in studies such as fuzzy set [54], intuitionistic set [21], soft set [41], neutrosophic set [50], etc., have great importance in this contribution of mathematics in recent years. Many works have been done on these sets by mathematicians in many areas of mathematics [8, 18, 20, 31, 32, 33, 34, 35, 45, 48]. R. Dhavaseelan et al. [6, 19, 26, 27, 28, 29, 30, 42] studied in various concept covered in neutrosophy. Neutrosophic set is described by three functions : a membership function, indeterminacy function and a nonmembership function that are independently related. The theories of neutrosophic set have achieved greater success in various areas such as medical diagnosis, database, topology, image processing and decision making problems. While the neutrosophic set is a powerful tool in dealing with indeterminate and inconsistent data, the theory of rough set is a powerful mathematical tool to deal with incompleteness. Neutrosophic sets and rough sets are two different topics, none conflicts the other. Later, P. K. Maji [38] has introduced a combined concept Neutrosophic soft set (NSS).Using this concept, several mathematicians have produced their research works in different mathematical structures for instance K. Atanassov [7], P. K. Maji et al. [39] I. Arockiarani et al. [4, 5], T. Bera and N. K. Mahapatra [10], I. Deli [23, 24], I. Deli and S. Broumi [22], P. K. Maji [40], S. Broumi and F. Smarandache [16],

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R. Saroja and A. Kalaichelvi [46], S. Broumi [17], Sahin et al. [44]. Later, this concept has been modified by I. Deli and S. Broumi [25]. Accordingly, T. Bera and N. K. Mahapatra [9, 11, 12, 13, 14, 15] have developed some algebraic structures over the neutrosophic soft set. F. Smarandache [52, 53] The extension of Soft Set to HyperSoft Set, by transforming the classical uni-argument function F into a multi-argument function. V. Popa & T. Noiri [43] introduced the notion of minimal structure which is a generalization of a topology on a given nonempty set. We introduced the concepts of $\mathcal{M}$-continuous maps.

The world of science and its related fields have accomplished such complicated processes for which consistent and complete information is not always conceivable. For the last few decades, a number of theories and postulates have been introduced by many researchers to handle indeterminate constituents in science and technologies. These theories include the theory of probability, interval mathematics, fuzzy set theory, intuitionistic fuzzy set theory, neutrosophic set theory, etc. Among all these theories, a powerful mathematical tool to deal with indeterminate and inconsistent data is the neutrosophic set theory introduced by Smarandache in 1998.

Neutrosophic soft set minimal structure is an essential tool for giving applications in mathematics and computer science, and it is also a technique for laying a foundation, the main objective of this study is to introduce a new hybrid intelligent structure called neutrosophic soft set minimal structure spaces. The significance of introducing hybrid structures is that the computational techniques, based on any one of these structures alone, will not always yield the best results but a fusion of two or more of them can often give better results.

The rest of this chapter is organized as follows. Some preliminary concepts required in our work are briefly recalled in Section 2. In Section 3, some properties of neutrosophic soft set minimal structure space are also investigated. In Section 4, the chapter with some properties on neutrosophic soft set minimal continuous, neutrosophic soft set minimal closed graph and neutrosophic soft set minimal compactness.

This chapter can be further developed into several possible such as Geographical Information Systems (GIS) field including remote sensing, object reconstruction from airborne laser scanner, real time tracking and routing applications.

2. Preliminaries

Definition 2.1. [43] A subfamily m_x of the power set $\wp$(X) of a nonempty set X is called a minimal structure (in short, m-structure) on X if $\emptyset \in m_x$ and $X \in m_x$. By (X, m_x), we denote a nonempty set X with a minimal structure m_x on X and call it an m-space. Each member of m_x is said to be m_x-open (or in short, m-open) and the complement of an m_x-open set is said to be m_x-closed (or in short, m-closed).

Definition 2.2. [49, 50, 51] Neutrosophic set (in short ns) K on a set $G \neq \emptyset$ is defined by K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ G$\}$, where P_K : G $\to$ [0,1], Q_K : G $\to$ [0,1] and R_K : G $\to$ [0,1] denotes the membership of an object, indeterminacy and non-membership of an object, for each element a $\in$ G to the set K, respectively and $0 \leq P_K(a) + Q_K(a) + R_K(a) \leq 3$ for each a $\in$ G.

Definition 2.3. [41] Let X be an initial universe, and E a set of all parameters. Let $\wp$(X) denote the power set of X. A pair (F,E) is called a soft set over X, where F is a mapping given by F : E $\to$ $\wp$(X).
In other words, the soft set is a parameterized family of subsets of the set X. For e $\in$ E,

F(e) may be considered as the set of e-elements of the soft set (F, E), or as the set of e-approximate elements of the soft set, i.e., (F, E) = $\{$(e, F(e)) : e $\in$ E, F : E $\to \wp$(X)$\}$.

The neutrosophic soft set defined by Maji [38] and later this concept has been modified by Deli and Bromi [22] as given below:

Definition 2.4. [41] Let X be an initial universe set and E a set of parameters. Let $\wp$(X) denote the set of all neutrosophic sets of X. Then, a neutrosophic soft set ($\widetilde{F}$, E) over X is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F}$: E $\to \wp$(X) where $\widetilde{F}$ is called the approximate function of the neutrosophic soft set ($\widetilde{F}$, E). In other words, the neutrosophic soft set is a parameterized family of some elements of the set $\wp$(X) and therefore it can be written as a set of ordered pairs, ($\widetilde{F}$,E) = $\{$(e, $\prec$ x, $T_{\widetilde{F}(e)}$(x), $I_{\widetilde{F}(e)}$(x), $F_{\widetilde{F}(e)}$(x) $\succ$: x $\in$ X) : e $\in$ E$\}$, where $T_{\widetilde{F}(e)}$(x), $I_{\widetilde{F}(e)}$(x), $F_{\widetilde{F}(e)}$(x) $\in$ [0, 1], respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $\widetilde{F}$(e). Since supremum of each T, I, F is 1 so the inequality 0 $\leq T_{\widetilde{F}(e)}$(x) + $I_{\widetilde{F}(e)}$(x) + $F_{\widetilde{F}(e)}$(x) $\leq$ 3 is obvious.

Definition 2.5. [41] Let ($\widetilde{F}_1$, E) and ($\widetilde{F}_2$, E) be two neutrosophic soft sets over the universe set X. Then their union is denoted by ($\widetilde{F}_1$, E) $\cup$ ($\widetilde{F}_2$, E) = ($\widetilde{F}_3$, E) and is defined by ($\widetilde{F}_3$, E) = $\{$(e, $\prec$ x, $T_{\widetilde{F}_3(e)}$(x), $I_{\widetilde{F}_3(e)}$(x), $F_{\widetilde{F}_3(e)}$(x) $\succ$: x $\in$ X) : e $\in$ E$\}$, where
$T_{\widetilde{F}_3(e)}$(x) = max $\{T_{\widetilde{F}_1(e)}$(x), $T_{\widetilde{F}_2(e)}$(x)$\}$,
$I_{\widetilde{F}_3(e)}$(x) = max $\{I_{\widetilde{F}_1(e)}$(x), $I_{\widetilde{F}_2(e)}$(x)$\}$,
$F_{\widetilde{F}_3(e)}$(x) = min $\{F_{\widetilde{F}_1(e)}$(x), $F_{\widetilde{F}_2(e)}$(x)$\}$.

Definition 2.6. [41] Let ($\widetilde{F}_1$, E) and ($\widetilde{F}_2$, E) be two neutrosophic soft sets over the universe set X. Then their intersection is denoted by ($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E) = ($\widetilde{F}_3$, E) and is defined by ($\widetilde{F}_3$, E) = $\{$(e, $\prec$ x, $T_{\widetilde{F}_3(e)}$(x), $I_{\widetilde{F}_3(e)}$(x), $F_{\widetilde{F}_3(e)}$(x) $\succ$: x $\in$ X) : e $\in$ E$\}$, where
$T_{\widetilde{F}_3(e)}$(x) = min $\{T_{\widetilde{F}_1(e)}$(x), $T_{\widetilde{F}_2(e)}$(x)$\}$,
$I_{\widetilde{F}_3(e)}$(x) = min $\{I_{\widetilde{F}_1(e)}$(x), $I_{\widetilde{F}_2(e)}$(x)$\}$,
$F_{\widetilde{F}_3(e)}$(x) = max $\{F_{\widetilde{F}_1(e)}$(x), $F_{\widetilde{F}_2(e)}$(x)$\}$.

Definition 2.7. [41] Let ($\widetilde{F}_1$, E) and ($\widetilde{F}_2$, E) be two neutrosophic soft sets over the universe set X. Then ($\widetilde{F}_1$, E) difference ($\widetilde{F}_2$, E) operation on them is denoted by ($\widetilde{F}_1$, E) $\setminus$ ($\widetilde{F}_2$, E) = ($\widetilde{F}_3$, E) and is defined by ($\widetilde{F}_3$, E) = ($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E)^c as follows : ($\widetilde{F}_3$, E) = $\{$(e, $\prec$ x, $T_{\widetilde{F}_3(e)}$(x), $I_{\widetilde{F}_3(e)}$(x), $F_{\widetilde{F}_3(e)}$(x) $\succ$: x $\in$ X) : e $\in$ E$\}$, where
$T_{\widetilde{F}_3(e)}$(x) = min $\{T_{\widetilde{F}_1(e)}$(x), $F_{\widetilde{F}_2(e)}$(x)$\}$,
$I_{\widetilde{F}_3(e)}$(x) = min $\{I_{\widetilde{F}_1(e)}$(x), 1 $-I_{\widetilde{F}_2(e)}$(x)$\}$,
$F_{\widetilde{F}_3(e)}$(x) = max $\{F_{\widetilde{F}_1(e)}$(x), $T_{\widetilde{F}_2(e)}$(x)$\}$.

Definition 2.8. [41] Let ($\widetilde{F}$, E) be neutrosophic soft set over the universe set X. Then complement of ($\widetilde{F}$, E) is denoted by ($\widetilde{F}$, E)^c and is denoted by ($\widetilde{F}$, E)^c = $\{$(e, $\prec$ x, $F_{\widetilde{F}(e)}$(x), $1 - I_{\widetilde{F}_3(e)}$(x), $T_{\widetilde{F}_3(e)}$(x) $\succ$: x $\in$ X) : e $\in$ E$\}$.
Obvious that (($\widetilde{F}$, E)^c)^c = ($\widetilde{F}$, E)

Definition 2.9. [41] Let ($\widetilde{F}_1$, E) and ($\widetilde{F}_2$, E) be two neutrosophic soft sets over the universe set X. Then "AND" operation on them is denoted by ($\widetilde{F}_1$, E) $\wedge$ ($\widetilde{F}_2$, E) = ($\widetilde{F}_3$, E $\times$ E) and is defined by ($\widetilde{F}_3$, E $\times$ E) = $\{$((e_1, e_2), $\prec$ x, $T_{\widetilde{F}_3(e_1,e_2)}$(x), $I_{\widetilde{F}_3(e_1,e_2)}$(x), $F_{\widetilde{F}_3(e_1,e_2)}$(x) $\succ$: x

$\in$ X) : (e$_1$, e$_2$) $\in$ E $\times$ E$\}$, where
T$_{\widetilde{F}_3(e_1,e_2)}$(x) = min $\{$T$_{\widetilde{F}_1(e_1)}$(x), T$_{\widetilde{F}_2(e_2)}$(x)$\}$,
I$_{\widetilde{F}_3(e_1,e_2)}$(x) = min $\{$I$_{\widetilde{F}_1(e_1)}$(x), I$_{\widetilde{F}_2(e_2)}$(x)$\}$,
F$_{\widetilde{F}_3(e_1,e_2)}$(x) = max $\{$F$_{\widetilde{F}_1(e_1)}$(x), F$_{\widetilde{F}_2(e_2)}$(x)$\}$.

Definition 2.10. [41] Let ($\widetilde{F}_1$, E) and ($\widetilde{F}_2$, E) be two neutrosophic soft sets over the universe set X. Then "OR" operation on them is denoted by ($\widetilde{F}_1$, E) $\vee$ ($\widetilde{F}_2$, E) = ($\widetilde{F}_3$, E $\times$ E) and is defined by ($\widetilde{F}_3$, E $\times$ E) = $\{$((e$_1$, e$_2$), $\prec$ x, T$_{\widetilde{F}_3(e_1,e_2)}$(x), I$_{\widetilde{F}_3(e_1,e_2)}$(x), F$_{\widetilde{F}_3(e_1,e_2)}$(x) $\succ$: x $\in$ X) : (e$_1$, e$_2$) $\in$ E $\times$ E$\}$, where
T$_{\widetilde{F}_3(e_1,e_2)}$(x) = max $\{$T$_{\widetilde{F}_1(e_1)}$(x), T$_{\widetilde{F}_2(e_2)}$(x)$\}$,
I$_{\widetilde{F}_3(e_1,e_2)}$(x) = max $\{$I$_{\widetilde{F}_1(e_1)}$(x), I$_{\widetilde{F}_2(e_2)}$(x)$\}$,
F$_{\widetilde{F}_3(e_1,e_2)}$(x) = min $\{$F$_{\widetilde{F}_1(e_1)}$(x), F$_{\widetilde{F}_2(e_2)}$(x)$\}$.

Definition 2.11. [41] (1) A neutrosophic soft set ($\widetilde{F}$, E) over the universe set X is said to be null neutrosophic soft set if T$_{\widetilde{F}(e)}$(x) = 0, I$_{\widetilde{F}(e)}$(x) = 0, F$_{\widetilde{F}(e)}$(x) = 1 for all e $\in$ E, for all x $\in$ X. It is denoted by $0_{(X,E)}$.
(2) A neutrosophic soft set ($\widetilde{F}$, E) over the universe set X is said to be absolute neutrosophic soft set if T$_{\widetilde{F}(e)}$(x) = 1, I$_{\widetilde{F}(e)}$(x) = 1, F$_{\widetilde{F}(e)}$(x) = 0 for all e $\in$ E, for all x $\in$ X. It is denoted by $1_{(X,E)}$.
Clearly, $0^c_{(X,E)}$ = $1_{(X,E)}$ and $1^c_{(X,E)}$ = $0_{(X,E)}$.

Definition 2.12. [41] Let NSS(X, E) be the family of all neutrosophic soft sets over the universe set X and τ^{NSS} $\subset$ NSS(X, E). Then τ^{NSS} is said to be a neutrosophic soft topology on X if

(1) $0_{(X,E)}$ and $1_{(X,E)}$ belongs to τ^{NSS}.
(2) the union of any number of neutrosophic soft sets in τ^{NSS} belongs to τ^{NSS}. the intersection of finite number of neutrosophic soft sets in τ^{NSS} belongs to τ^{NSS}.

Then (X, τ^{NSS}, E) is said to be a neutrosophic soft topological space over X. Each members of τ^{NSS} is said to be neutrosophic soft open set.

Definition 2.13. [41] Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X and ($\widetilde{F}$, E) be a neutrosophic soft set over X. Then ($\widetilde{F}$, E) is said to be neutrosophic soft closed set iff its complement is a neutrosophic soft open set.

Definition 2.14. [41] Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X and ($\widetilde{F}$, E) $\in$ NSS(X, E) be a neutrosophic soft set. Then, the neutrosophic soft interior of ($\widetilde{F}$, E), denoted by ($\widetilde{F}$, E)$^\circ$, is defined as the neutrosophic soft union of all neutrosophic soft open subsets of ($\widetilde{F}$, E).
Clearly, ($\widetilde{F}$, E)$^\circ$ is the biggest neutrosophic soft open set that is contained by ($\widetilde{F}$, E).

Definition 2.15. [41] Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X and ($\widetilde{F}$, E) $\in$ NSS(X, E) be a neutrosophic soft set. Then, the neutrosophic soft closure of ($\widetilde{F}$, E), denoted by $\overline{(\widetilde{F}, E)}$, is defined as the neutrosophic soft intersection of all neutrosophic soft closed supersets of ($\widetilde{F}$, E).
Clearly, $\overline{(\widetilde{F}, E)}$ is the smallest neutrosophic soft closed set that containing ($\widetilde{F}$, E).

3. NEUTROSOPHIC SOFT SET MINIMAL STRUCTURE SPACES

Definition 3.1. Let the neutrosophic soft set minimal space over the universe set X be denoted by m^{NSS}. m^{NSS} is said to be neutrosophic soft set minimal structure space (in

short, nssmss) over X if it satisfies the following axiom: $0_{\sim(X,E)}$ and $1_{\sim(X,E)}$ belong to m^{NSS}.
A family of neutrosophic soft set minimal structure space is denoted by (X, m_x^{NSS}, E).
Note that neutrosophic soft empty set and neutrosophic soft universal set can form a topology and it is known as neutrosophic soft set minimal structure space.

Definition 3.2. Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal structure space over X and NSS($\widetilde{F}$, E) be a neutrosophic soft set over X. Then NSS$_m$($\widetilde{F}$, E) is said to be neutrosophic soft set minimal closed set iff its complement is a neutrosophic soft set minimal open set.

Definition 3.3. Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal space over X and ($\widetilde{F}$, E) $\in$ NSS(X, E) be a neutrosophic soft set. Then, the neutrosophic soft set minimal interior of ($\widetilde{F}$, E), denoted by NSS$_m(\widetilde{F}, E)^\circ$, is defined as the neutrosophic soft set minimal union of all neutrosophic soft set minimal open subsets of $(\widetilde{F}, E)$.
$Clearly, NSS_m(\widetilde{F}, E)^\circ$ is the biggest neutrosophic soft set minimal open set that is contained by $(\widetilde{F}, E)$.

Example 3.4. Let X = $\{$x$_1$, x$_2$, x$_3\}$ be an initial universe set, E = $\{$e$_1$, e$_2\}$ be a set of parameters and m^{NSS} = $\{0_{\sim(X,E)}$, ($\widetilde{F}_1$, E), ($\widetilde{F}_2$, E), ($\widetilde{F}_3$, E), $1_{\sim(X,E)}\}$; $(m^{NSS})^c = \{1^c_{\sim(X,E)}$, ($\widetilde{F}_1$, E)^c, ($\widetilde{F}_2$, E)^c, ($\widetilde{F}_3$, E)^c,$0^c_{\sim(X,E)}\}$ be neutrosophic soft set minimal spaces over X. Here, the neutrosophic soft sets ($\widetilde{F}_1$, E), ($\widetilde{F}_2$, E) and ($\widetilde{F}_3$, E) over X are defined as following:
($\widetilde{F}_1$, E) = $\{e_1 = \{\prec x_1, 0.9, 0.5, 0.4\succ, \prec x_2, 0.6, 0.7, 0.4\succ, \prec x_3, 0.5, 0.6, 0.4\succ\}$; $e_2 = \{\prec x_1, 0.8, 0.4, 0.5\succ, \prec x_2, 0.7, 0.7, 0.3\succ, \prec x_3, 0.7, 0.5, 0.6\succ\}\}$,
($\widetilde{F}_2$, E) = $\{e_1 = \{\prec x_1, 0.8, 0.5, 0.6\succ, \prec x_2, 0.5, 0.6, 0.6\succ, \prec x_3, 0.4, 0.4, 0.5\succ\}$; $e_2 = \{\prec x_1, 0.7, 0.3, 0.5\succ, \prec x_2, 0.6, 0.5, 0.4\succ, \prec x_3, 0.5, 0.2, 0.6\succ\}\}$,
($\widetilde{F}_3$, E) = $\{e_1 = \{\prec x_1, 0.6, 0.4, 0.7\succ, \prec x_2, 0.4, 0.5, 0.8\succ, \prec x_3, 0.3, 0.3, 0.6\succ\}$; $e_2 = \{\prec x_1, 0.5, 0.2, 0.6\succ, \prec x_2, 0.5, 0.4, 0.5\succ, \prec x_3, 0.2, 0.2, 0.7\succ\}\}$.

We know that $0_{\sim(X,E)} = \{\prec x, 0, 0, 1 \succ : x \in X\}$, $1_{\sim(X,E)} = \{\prec x, 1, 1, 0 \succ : x \in X\}$ and $0^c_{\sim(X,E)} = \{\prec x, 1, 1, 0 \succ : x \in X\}$, $1^c_{\sim(X,E)} = \{\prec x, 0, 0, 1 \succ : x \in X\}$.
($\widetilde{F}_1$, E)$^c = \{e_1 = \{\prec x_1, 0.4, 0.5, 0.9\succ, \prec x_2, 0.4, 0.3, 0.6\succ, \prec x_3, 0.4, 0.4, 0.5\succ\}$; $e_2 = \{\prec x_1, 0.5, 0.6, 0.8\succ, \prec x_2, 0.3, 0.3, 0.7\succ, \prec x_3, 0.6, 0.5, 0.7\succ\}\}$,
($\widetilde{F}_2$, E)$^c = \{e_1 = \{\prec x_1, 0.6, 0.5, 0.8\succ, \prec x_2, 0.6, 0.4, 0.5\succ, \prec x_3, 0.5, 0.6, 0.4\succ\}$; $e_2 = \{\prec x_1, 0.5, 0.7, 0.7\succ, \prec x_2, 0.4, 0.5, 0.6\succ, \prec x_3, 0.6, 0.8, 0.5\succ\}\}$,
($\widetilde{F}_3$, E)$^c = \{e_1 = \{\prec x_1, 0.7, 0.6, 0.6\succ, \prec x_2, 0.8, 0.5, 0.4\succ, \prec x_3, 0.6, 0.7, 0.3\succ\}$; $e_2 = \{\prec x_1, 0.6, 0.8, 0.5\succ, \prec x_2, 0.5, 0.6, 0.5\succ, \prec x_3, 0.7, 0.8, 0.2\succ\}\}$. Suppose that an any ($\widetilde{F}$, E) $\in$ NSS(X, E) is defined as follows :
($\widetilde{F}$, E) = $\{e_1 = \{\prec x_1, 0.9, 0.6, 0.3\succ, \prec x_2, 0.6, 0.7, 0.4\succ, \prec x_3, 0.5, 0.5, 0.4\succ\}$; $e_2 = \{\prec x_1, 0.9, 0.5, 0.2\succ, \prec x_2, 0.8, 0.7, 0.3\succ, \prec x_3, 0.9, 0.5, 0.5\succ\}\}$. Then $0_{\sim(X,E)}$, ($\widetilde{F}_2$, E), ($\widetilde{F}_3$, E) $\subseteq$ ($\widetilde{F}$, E). Therefore NSS$_m$($\widetilde{F}$, E)$^\circ$ = $0_{\sim(X,E)}$ $\cup$ ($\widetilde{F}_2$, E) $\cup$ ($\widetilde{F}_3$, E) = ($\widetilde{F}_2$, E).

Theorem 3.1. *Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal space over X and ($\widetilde{F}$, E)$\in$ NSS(X, E). ($\widetilde{F}$, E) is a neutrosophic soft set minimal open set iff ($\widetilde{F}$, E) = NSS$_m$($\widetilde{F}$, E)$^\circ$.*

Proof. Let ($\widetilde{F}$, E) be a neutrosophic soft set minimal open set. Then the biggest neutrosophic soft set minimal open set that is contained by ($\widetilde{F}$, E) is equal to ($\widetilde{F}$, E). Hence, ($\widetilde{F}$,

E) = NSS$_m(\widetilde{F}$, E)$^\circ$.
Conversely, it is known that NSS$_m(\widetilde{F}$, E)$^\circ$ is a neutrosophic soft set minimal open set and if ($\widetilde{F}$, E) = NSS$_m(\widetilde{F}$, E)$^\circ$, then ($\widetilde{F}$, E) is a neutrosophic soft set minimal open set. □

Theorem 3.2. *Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal space over X and ($\widetilde{F}_1$, E), ($\widetilde{F}_2$, E) $\in$ NSS(X, E). Then*

(1) NSS$_m((NSS_m(\widetilde{F}_1$, E)$^\circ)^\circ$) = NSS$_m(\widetilde{F}_1$, E)$^\circ$.
(2) $(0_{\sim(X,E)})^\circ = 0_{\sim(X,E)}$ and $(1_{\sim(X,E)})^\circ = 1_{\sim(X,E)}$.
(3) ($\widetilde{F}_1$, E) $\subseteq$ ($\widetilde{F}_2$, E) $\Rightarrow$ NSS$_m(\widetilde{F}_1$, E)$^\circ \subseteq$ NSS$_m(\widetilde{F}_2$, E)$^\circ$.
(4) NSS$_m$((($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E))$^\circ$) = NSS$_m(\widetilde{F}_1$, E)$^\circ \cap$ NSS$_m(\widetilde{F}_2$, E)$^\circ$.
(5) NSS$_m(\widetilde{F}_1$, E)$^\circ \cup$ NSS$_m(\widetilde{F}_2$, E)$^\circ \subseteq$ NSS$_m$((($\widetilde{F}_1$, E) $\cup$ ($\widetilde{F}_2$, E))$^\circ$).

Proof. (1) Let NSS$_m(\widetilde{F}_1$, E)$^\circ$ = ($\widetilde{F}_1$, E). Then ($\widetilde{F}_1$, E) is a neutrosophic soft set minimal open set iff ($\widetilde{F}_1$, E) = NSS$_m(\widetilde{F}_1$, E)$^\circ$. So, NSS$_m((NSS_m(\widetilde{F}_1$, E)$^\circ)^\circ$) = NSS$_m(\widetilde{F}_1$, E)$^\circ$.
(2) Straightforward.
(3) It is known that NSS$_m(\widetilde{F}_1$, E)$^\circ \subseteq$ ($\widetilde{F}_1$, E) $\subseteq$ ($\widetilde{F}_2$, E) and NSS$_m(\widetilde{F}_2$, E)$^\circ \subseteq$ ($\widetilde{F}_2$, E). Since NSS$_m(\widetilde{F}_2$, E)$^\circ$ is the biggest neutrosophic soft set minimal open set contained in ($\widetilde{F}_2$, E) and so, NSS$_m(\widetilde{F}_1$, E)$^\circ$ = NSS$_m(\widetilde{F}_2$, E)$^\circ$.
(4) Since ($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E) $\subseteq$ ($\widetilde{F}_1$, E) and ($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E) $\subseteq$ ($\widetilde{F}_2$, E), then NSS$_m$((($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E))$^\circ$) = NSS$_m(\widetilde{F}_1$, E)$^\circ$ and NSS$_m$((($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E))$^\circ$) = NSS$_m(\widetilde{F}_2$, E)$^\circ$ and so, NSS$_m$((($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E))$^\circ$) = NSS$_m(\widetilde{F}_1$, E)$^\circ \cap$ NSS$_m(\widetilde{F}_2$, E)$^\circ$.

On the other hand, since NSS$_m(\widetilde{F}_1$, E)$^\circ \subseteq$ ($\widetilde{F}_1$, E) and NSS$_m(\widetilde{F}_2$, E)$^\circ \subseteq$ ($\widetilde{F}_2$, E), then NSS$_m(\widetilde{F}_1$, E)$^\circ \cap$ NSS$_m(\widetilde{F}_2$, E)$^\circ \subseteq$ ($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E). Besides, NSS$_m$((($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E))$^\circ$) $\subseteq$ ($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E) and it is the biggest neutrosophic soft set minimal open set. Therefore, NSS$_m(\widetilde{F}_1$, E)$^\circ \cap$ NSS$_m(\widetilde{F}_2$, E)$^\circ \subseteq$ NSS$_m$((($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E))$^\circ$). Thus, NSS$_m$((($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E))$^\circ$) = NSS$_m(\widetilde{F}_1$, E)$^\circ \cap$ NSS$_m(\widetilde{F}_2$, E)$^\circ$.

(5) Since ($\widetilde{F}_1$, E) $\subseteq$ ($\widetilde{F}_1$, E) $\cup$ ($\widetilde{F}_2$, E) and ($\widetilde{F}_2$, E) $\subseteq$ ($\widetilde{F}_1$, E) $\cup$ ($\widetilde{F}_2$, E), then NSS$_m(\widetilde{F}_1$, E)$^\circ \subseteq$ NSS$_m$((($\widetilde{F}_1$, E) $\cup$ ($\widetilde{F}_2$, E))$^\circ$) and NSS$_m(\widetilde{F}_2$, E)$^\circ \subseteq$ NSS$_m$((($\widetilde{F}_1$, E) $\cup$ ($\widetilde{F}_2$, E))$^\circ$). Therefore, NSS$_m(\widetilde{F}_1$, E)$^\circ \cup$ NSS$_m(\widetilde{F}_2$, E)$^\circ \subseteq$ NSS$_m$((($\widetilde{F}_1$, E) $\cup$ ($\widetilde{F}_2$, E))$^\circ$). □

Definition 3.5. Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal space over X and ($\widetilde{F}$, E) $\in$ NSS(X, E) be a neutrosophic soft set. Then, the neutrosophic soft set minimal closure of ($\widetilde{F}$, E), denoted by NSS$_m\overline{(\widetilde{F}, E)}$, is defined as the neutrosophic soft set minimal intersection of all neutrosophic soft set minimal closed supersets of ($\widetilde{F}$, E).
Clearly, NSS$_m\overline{(\widetilde{F}, E)}$ is the smallest neutrosophic soft set minimal closed set containing ($\widetilde{F}$, E).

Example 3.6. Let us consider the neutrosophic soft set minimal spaces m^{NSS} given in Example 3.4. Suppose that ($\widetilde{F}$, E) $\in$ NSS(X, E) is defined as follows :
($\widetilde{F}$, E) = $\{e_1 = \{\prec$ x$_1, 0.1, 0.5, 0.9\succ$, $\prec$ x$_2, 0.4, 0.2, 0.8\succ$, $\prec$ x$_3, 0.3, 0.4, 0.7\succ\}$; e$_2$ = $\{\prec$ x$_1, 0.2, 0.5, 0.9\succ$, $\prec$ x$_2, 0.2, 0.2, 0.8\succ$, $\prec$ x$_3, 0.4, 0.8, 0.9\succ\}\}$. Then $0^c_{\sim(X,E)}$, ($\widetilde{F}_1$, E)^c, ($\widetilde{F}_2$, E)^c, ($\widetilde{F}_3$, E)$^c \supseteq$ ($\widetilde{F}$, E). Therefore NSS$_m\overline{(\widetilde{F}, E)}$ = $0^c_{\sim(X,E)} \cap$ ($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E) $\cap$ ($\widetilde{F}_3$, E) = ($\widetilde{F}_1$, E)^c.

Theorem 3.3. *Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal space over X and ($\widetilde{F}$, E) $\in$ NSS(X, E). ($\widetilde{F}$, E) is a neutrosophic soft set minimal closed set iff ($\widetilde{F}$, E) = $NSS_m\overline{(\widetilde{F}, E)}$.*

Proof. Let ($\widetilde{F}$, E) be a neutrosophic soft set minimal closed set. Then the smallest neutrosophic soft set minimal closed set which is contained by ($\widetilde{F}$, E) is equal to ($\widetilde{F}$, E). Hence, ($\widetilde{F}$, E) = $\mathrm{NSS}_m\overline{(\widetilde{F}, E)}$.
Conversely, it is known that $\mathrm{NSS}_m\overline{(\widetilde{F}, E)}$ is a neutrosophic soft set minimal closed set and if ($\widetilde{F}$, E) = $\mathrm{NSS}_m\overline{(\widetilde{F}, E)}$, then ($\widetilde{F}$, E) is a neutrosophic soft set minimal closed set. □

Theorem 3.4. *Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal space over X and ($\widetilde{F}_1$, E), ($\widetilde{F}_2$, E) $\in$ nsms(X, E). Then*

(1) $NSS_m(((\overline{\overline{NSS_m(\widetilde{F}_1, E)}}))) = NSS_m\overline{(\widetilde{F}_1, E)}$.
(2) $\overline{(0_\sim(X, E))} = 0_{\sim(X,E)}$ and $\overline{(1_\sim(X, E))} = 1_{\sim(X,E)}$.
(3) $(\widetilde{F}_1, E) \subseteq (\widetilde{F}_2, E) \Rightarrow NSS_m\overline{(\widetilde{F}_1, E)} \subseteq NSS_m\overline{(\widetilde{F}_2, E)}$.
(4) $NSS_m\overline{(\widetilde{F}_1, E) \cup (\widetilde{F}_2, E)} = NSS_m\overline{(\widetilde{F}_1, E)} \cup NSS_m\overline{(\widetilde{F}_1, E)}$.
(5) $NSS_m\overline{(\widetilde{F}_1, E) \cap (\widetilde{F}_2, E)} \subseteq NSS_m\overline{(\widetilde{F}_1, E)} \cap NSS_m\overline{(\widetilde{F}_2, E)}$.

Proof. (1) Let $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)}$ = ($\widetilde{F}_1$, E). Then ($\widetilde{F}_1$, E) is a neutrosophic soft set minimal closed set. Hence $\mathrm{NSS}_m(\widetilde{F}_1$, E) = $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)}$. So, $\mathrm{NSS}_m(((\overline{\overline{NSS_m(\widetilde{F}_1, E)}})))$ = $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)}$.
(2) Straightforward.
(3) It is known that ($\widetilde{F}_1$, E) $\subseteq$ $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)}$ and ($\widetilde{F}_2$, E) $\subseteq$ $\mathrm{NSS}_m\overline{(\widetilde{F}_2, E)}$ and so ($\widetilde{F}_1$, E) $\subseteq$ ($\widetilde{F}_2$, E) $\subseteq$ $\mathrm{NSS}_m\overline{(\widetilde{F}_2, E)}$. Since $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)}$ is the smallest neutrosophic soft set minimal closed set containing ($\widetilde{F}_1$, E) then $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)} \subseteq \mathrm{NSS}_m\overline{(\widetilde{F}_2, E)}$.
(4) Since ($\widetilde{F}_1$, E) $\subseteq$ ($\widetilde{F}_1$, E) $\cup$ ($\widetilde{F}_2$, E) and $(\widetilde{F}_2, E) \subseteq$ ($\widetilde{F}_1$, E) $\cup$($\widetilde{F}_2$, E), then $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)} \subseteq \mathrm{NSS}_m\overline{(\widetilde{F}_1, E) \cup (\widetilde{F}_2, E)}$ and $\mathrm{NSS}_m\overline{(\widetilde{F}_2, E)} \subseteq \mathrm{NSS}_m\overline{(\widetilde{F}_1, E) \cup (\widetilde{F}_2, E)}$ and so $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)}$ $\cup$ $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)} \subseteq \mathrm{NSS}_m\overline{(\widetilde{F}_1, E) \cup (\widetilde{F}_2, E)}$.
Conversely, since ($\widetilde{F}_1$, E) $\subseteq$ $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)}$ and ($\widetilde{F}_2$, E) $\subseteq$ $\mathrm{NSS}_m\overline{(\widetilde{F}_2, E)}$ then ($\widetilde{F}_1$, E) $\cup$ ($\widetilde{F}_2$, E) $\subseteq$ $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)} \cup \mathrm{NSS}_m\overline{(\widetilde{F}_2, E)}$. Besides, $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E) \cup (\widetilde{F}_2, E)}$ is the smallest neutrosophic soft set minimal closed set that containing ($\widetilde{F}_1$, E) $\cup$ ($\widetilde{F}_2$, E). Therefore, $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E) \cup (\widetilde{F}_2, E)} \subseteq \mathrm{NSS}_m\overline{(\widetilde{F}_1, E)} \cup \mathrm{NSS}_m\overline{(\widetilde{F}_2, E)}$. Thus, $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E) \cup (\widetilde{F}_2, E)}$ = $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)} \cup \mathrm{NSS}_m\overline{(\widetilde{F}_1, E)}$.
(5) Since ($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E) $\subseteq$ $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)} \cap \mathrm{NSS}_m\overline{(\widetilde{F}_2, E)}$ and $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E) \cap (\widetilde{F}_2, E)}$ is the smallest neutrosophic soft closed set containing ($\widetilde{F}_1$, E) $\cap$ ($\widetilde{F}_2$, E), then $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E) \cap (\widetilde{F}_2, E)}$ $\subseteq$ $\mathrm{NSS}_m\overline{(\widetilde{F}_1, E)} \cap \mathrm{NSS}_m\overline{(\widetilde{F}_2, E)}$. □

Theorem 3.5. *Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal space over X and ($\widetilde{F}_1$, E), ($\widetilde{F}_2$, E) $\in$ NSS(X, E). Then*

(1) $(NSS_m\overline{(\widetilde{F}, E)})^c = (NSS_m((\widetilde{F}, E)^c))^\circ$.
(2) $(NSS_m(\widetilde{F}, E)^\circ)^c = (NSS_m\overline{((\widetilde{F}, E)^c)})$.

Proof. (1) NSS$_m\overline{(\widetilde{F},E)}$ = $\cap\{(\widetilde{G},E) \in (\text{m}_x^{NSS})^c : (\widetilde{G},E) \supseteq (\widetilde{F},E)\} \Rightarrow$ (NSS$_m\overline{(\widetilde{F},E)})^c$ = $(\cap\{(\widetilde{G},E) \in (\text{m}_x^{NSS})^c : (\widetilde{G},E) \supseteq (\widetilde{F},E)\})^c$ = $\cup\{(\widetilde{G},E)^c \in \tau_m^{NSS} : (\widetilde{G},E)^c \supseteq (\widetilde{F},E)^c\}$ = (NSS$_m((\widetilde{F}$, E$)^c))^\circ$.
(2) $(\widetilde{F}$, E$)^\circ$ = $\cup\{(\widetilde{G},E) \in \tau_m^{NSS} : (\widetilde{G},E) \subseteq (\widetilde{F},E)\} \Rightarrow$ (NSS$_m(\widetilde{F}$, E$)^\circ)^c$ = $(\cup\{\overline{(\widetilde{G},E)} \in \text{m}_x^{NSS} : (\widetilde{G},E) \subseteq (\widetilde{F},E)\})^c$ = $\cap\{(\widetilde{G},E)^c \in (\text{m}_x^{NSS})^c : (\widetilde{G},E)^c \supseteq (\widetilde{F},E)^c\}$ = (NSS$_m\overline{((\widetilde{F},E)^c)}$). □

4. Some properties of NSS$_m(\widetilde{F}$, E)-continuous map

Definition 4.1. A map f: (X, m_x^{NSS}, E) $\rightarrow$ (Y, m_y^{NSS}, E) is called neutrosophic soft set minimal continuous map (in short, NSS$_m(\widetilde{F}$, E)-continuous) if and only if f^{-1}(V) is a NSS$_m(\widetilde{F}$, E)-closed sets of (X, m_x^{NSS}, E), for every NSS$_m(\widetilde{F}$, E)-closed set V of (Y, m_y^{NSS}, E).

Definition 4.2. Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal space over X.

(1) Arbitrary union of neutrosophic soft set minimal open sets in (X, m_x^{NSS}, E) is neutrosophic soft set minimal open. (Union Property).
(2) Finite intersection of neutrosophic soft minimal open sets in (X, m_x^{NSS}, E) is neutrosophic soft set minimal open. (Intersection Property).

Theorem 4.1. *Let f : X $\rightarrow$ Y be a map on two neutrosophic soft set minimal spaces (X, m_x^{NSS}, E) and (Y, m_y^{NSS}, E). Then the following statements are equivalent:*

(1) Identity map from (X, m_x^{NSS}, E) to (Y, m_y^{NSS}, E) is a neutrosophic soft set minimal continuous map.
(2) Any constant map which map from (X, m_x^{NSS}, E) to (Y, m_y^{NSS}, E) is a neutrosophic soft set minimal continuous map.

Proof. The proof is obvious. □

Definition 4.3. Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal spaces. Then X is said to be

(1) NSS$_m(\widetilde{F}$, E)-T$_0$ if for each pair of distinct points x and y in X, there exist a NSS$_m(\widetilde{F}$, E)-open set U such that either x $\in$ U and y $\notin$ U or x $\notin$ U and y $\in$ U.
(2) NSS$_m(\widetilde{F}$, E)-T$_1$ if for each pair of distinct points x and y in X, there exist two NSS$_m(\widetilde{F}$, E)-open sets U and V such that either x $\in$ U but y $\notin$ U and y $\in$ V but x $\notin$ V.
(3) NSS$_m(\widetilde{F}$, E)-T$_2$ if for each distinct points x and y of X, there exist two disjoint NSS$_m(\widetilde{F}$, E)-open sets U, V such that x $\in$ U and y $\in$ V.

Theorem 4.2. *Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal spaces. Then, X is NSS$_m(\widetilde{F}$, E)-T$_0$ if and only if for each pair of distinct points x, y of X, NSS$_m\overline{(\widetilde{F},E)}(\{x\})$ $\neq$ NSS$_m\overline{(\widetilde{F},E)}(\{y\})$.*

Proof. Necessity. Let X be NSS$_m(\widetilde{F}$, E)-T$_0$ and x, y be any two distinct points of X. Then, there exists a NSS$_m(\widetilde{F}$, E)-open set U containing x or y, say x but not y. Then, X $\setminus$ U is a NSS$_m(\widetilde{F}$, E)-closed set which does not contain x but contains y. Since NSS$_m\overline{(\widetilde{F},E)}$({y}) is the smallest NSS$_m(\widetilde{F}$, E)-closed set containing y, then NSS$_m\overline{(\widetilde{F},E)}$({y}) $\leq$ X $\setminus$ U and therefore x $\notin$ NSS$_m\overline{(\widetilde{F},E)}$({y}). Consequently NSS$_m\overline{(\widetilde{F},E)}$({x}) $\neq$ NSS$_m\overline{(\widetilde{F},E)}$({y}).

Sufficiency. Suppose that x, y ∈ X, x ≠ y and $NSS_m\overline{(\widetilde{F},E)}(\{x\}) \neq NSS_m\overline{(\widetilde{F},E)}(\{y\})$. Let z be a point of X such that z ∈ $NSS_m\overline{(\widetilde{F},E)}(\{x\})$ but z ∉ $NSS_m\overline{(\widetilde{F},E)}(\{y\})$. We claim that x ∉ $NSS_m\overline{(\widetilde{F},E)}(\{y\})$. For, if x ∈ $NSS_m\overline{(\widetilde{F},E)}(\{y\})$ then $NSS_m\overline{(\widetilde{F},E)}(\{x\})$ ≤ $NSS_m\overline{(\widetilde{F},E)}(\{y\})$. This contradicts the fact that z ∉ $NSS_m\overline{(\widetilde{F},E)}(\{y\})$. Consequently x belongs to the $NSS_m(\widetilde{F}$, E)-open set X \ $NSS_m\overline{(\widetilde{F},E)}(\{y\})$ to which y does not belong. □

Theorem 4.3. *Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal spaces. Then, X is $NSS_m(\widetilde{F}$, E)-T_1 if and only if the singletons are $NSS_m(\widetilde{F}$, E)-closed sets.*

Proof. Let X be $NSS_m(\widetilde{F}$, E)-T_1 and x any point of X. Suppose y ∈ X \ {x}, then x ≠ y and so there exists a $NSS_m(\widetilde{F}$, E)-open set U such that y ∈ U but x ∉ U. Consequently y ∈ U ≤ X \ {x}, that is X \ {x} = ∪ {U : y ∈ X \ {x}} which is $NSS_m(\widetilde{F}$, E)-open.
Conversely, suppose {r} is $NSS_m(\widetilde{F}$, E)-closed for every r ∈ X. Let x, y ∈ X with x ≠ y. Now, x ≠ y implies y ∈ X \ {x}. Hence, X \ {x} is a $NSS_m(\widetilde{F}$, E)-open set contains y but not x. Similarly X \ {y} is a $NSS_m(\widetilde{F}$, E)-open set contains x but not y. Accordingly X is $NSS_m(\widetilde{F}$, E)-T_1. □

Theorem 4.4. *Let (X, m_x^{NSS}, E) be a neutrosophic soft set minimal spaces. Then, the following statements are equivalent:*

(1) X is $NSS_m(\widetilde{F}$, E)-T_2.
(2) Let x ∈ X. For each y ≠ x, there exists a $NSS_m(\widetilde{F}$, E)-open set U containing x such that y ∉ $NSS_m\overline{(\widetilde{F},E)}$(U).
(3) For each x ∈ X, ∪ {$NSS_m\overline{(\widetilde{F},E)}$(U) : U ∈ $NSS_m(\widetilde{F}$, E) and x ∈ U} = {x}.

Proof. (1) ⇒ (2): Since X is $NSS_m(\widetilde{F}$, E)-T_2, then there exist disjoint $NSS_m(\widetilde{F}$, E)-open sets U and V containing x and y respectively. So, U ≤ X \ V. Therefore, $NSS_m\overline{(\widetilde{F},E)}$(U) ≤ X \ V. So, y ∉ $NSS_m\overline{(\widetilde{F},E)}$(U).
(2) ⇒ (3): If possible for some y ≠ x, we have y ∈ $NSS_m\overline{(\widetilde{F},E)}$(U) for every $NSS_m(\widetilde{F}$, E)-open set U containing x, which then contradicts (2).
(3) ⇒ (1): Let x, y ∈ X and x ≠ y. Then, there exists a $NSS_m(\widetilde{F}$, E)-open set U containing x such that y ∉ $NSS_m\overline{(\widetilde{F},E)}$(U). Let V = X \ $NSS_m\overline{(\widetilde{F},E)}$(U), then y ∈V, x ∈ U and U ∩ V = $0_{\sim(X,E)}$. Thus, X is $NSS_m(\widetilde{F}$, E)-T_2. □

Definition 4.4. Let f : X → Y be a map on two neutrosophic soft set minimal spaces (X, m_x^{NSS}, E) and (Y, m_y^{NSS}, E). Then f has an $NSS_m(\widetilde{F}$, E)-closed graph if for each (x, y) ∈ (X × Y) − G(f), there exist an $NSS_m(\widetilde{F}$, E)-open set U containing x and an $NSS_m(\widetilde{F}$, E)-open set V containing y such that (U × V) ∩ G(f) = $0_{\sim(X,E)}$.

Lemma 4.5. *Let f : X → Y be a map on two neutrosophic soft set minimal spaces (X, m_x^{NSS}, E) and (Y, m_y^{NSS}, E). Then f has an $NSS_m(\widetilde{F}$, E)-closed graph if and only if for each (x, y) ∈ (X × Y) − G(f), there exist an $NSS_m(\widetilde{F}$, E)-open set U containing x and an $NSS_m(\widetilde{F}$, E)-open set V containing y such that f(U) ∩ V = $0_{\sim(X,E)}$.*

Theorem 4.6. *Let f : X → Y be a map on two neutrosophic soft set minimal spaces (X, m_x^{NSS}, E) and (Y, m_y^{NSS}, E). If f is $NSS_m(\widetilde{F}$, E)-continuous and (Y, m_y^{NSS}, E) is $NSS_m(\widetilde{F}$, E)-T_2, then G(f) is an $NSS_m(\widetilde{F}$, E)-closed graph.*

Proof. Let (x, y) $\in$ (X $\times$ Y) $-$ G(f) then f(x) $\neq$ y. Since Y is NSS$_m(\widetilde{F}$, E)-T$_2$, there are disjoint NSS$_m(\widetilde{F}$, E)-open sets U, V such that f(x) $\in$ U, y $\in$ V . Then for f(x) $\in$ U, by NSS$_m(\widetilde{F}$, E)-continuity, there exists an NSS$_m(\widetilde{F}$, E)-open set G containing x such that f(G) $\leq$ U. Consequently, there exist an NSS$_m(\widetilde{F}$, E)-open set V and NSS$_m(\widetilde{F}$, E)-open set G containing y, x respectively, such that f(G) $\cap$V =$0_{\sim(X,E)}$. Therefore, by Lemma 4.5, G(f) is NSS$_m(\widetilde{F}$, E)-closed. □

Theorem 4.7. *Let f : X $\rightarrow$ Y be a map on two neutrosophic soft set minimal spaces (X, m_x^{NSS}, E) and (Y, m_y^{NSS}, E). If f is an injective and NSS$_m(\widetilde{F}$, E)-continuous map with an NSS$_m(\widetilde{F}$, E)-closed graph, then X is NSS$_m(\widetilde{F}$, E)-T$_2$.*

Proof. Let x$_1$ and x$_2$ be any distinct points of X. Then f(x$_1$) $\neq$ f(x$_2$), so (x$_1$, f(x$_2$)) $\in$ (X $\times$ Y) $-$ G(f). Since the graph G(f) is NSS$_m(\widetilde{F}$, E)-closed, there exist an NSS$_m(\widetilde{F}$, E)-open set U containing x$_1$ and V $\in N_{mY}$ containing f(x$_2$) such that f(U) $\cap$ V = $0_{\sim(X,E)}$. Since f is NSS$_m(\widetilde{F}$, E)-continuous, f^{-1}(V) is an NSS$_m(\widetilde{F}$, E)-open set containing x$_2$ such that U $\cap$ f^{-1}(V) = $0_{\sim(X,E)}$. Hence X is NSS$_m(\widetilde{F}$, E)-T$_2$. □

Definition 4.5. (X, m_x^{NSS}, E) be a neutrosophic soft set minimal spaces and A $\subseteq$ X, A is said to be NSS$_m(\widetilde{F}$, E)-compact (resp. almost NSS$_m(\widetilde{F}$, E)-compact) relative to A if every collection $\{$U$_i$: i $\in \Delta\}$ of NSS$_m(\widetilde{F}$, E)-open subsets of X such that A $\subseteq \cup$ $\{$U$_i$: i $\in \Delta\}$, there exists a finite subset Δ_0 of Δ such that A $\subseteq \cup$ $\{$U$_j$: j $\in \Delta_0\}$ (resp. A $\subseteq \cup$ NSS$_m\overline{(U_j)}$: j $\in \Delta_0\}$). A subset A of a neutrosophic soft set minimal spaces (X, m_x^{NSS}, E) is said to be NSS$_m(\widetilde{F}$, E)-compact (resp. almost NSS$_m(\widetilde{F}$, E)-compact) if A is NSS$_m(\widetilde{F}$, E)-compact (resp. almost NSS$_m(\widetilde{F}$, E)-compact) as a subspace of X.

Theorem 4.8. *Let f : X $\rightarrow$ Y be a map on two neutrosophic soft set minimal spaces (X, m_x^{NSS}, E) and (Y, m_y^{NSS}, E). If A is an NSS$_m(\widetilde{F}$, E)-compact set, then f(A) is NSS$_m(\widetilde{F}$, E)-compact.*

Proof. Obvious. □

5. Conclusions

Neutrosophic set is a general formal framework, which generalizes the concept of classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and interval intuitionistic fuzzy set. Since the world is full of indeterminacy, the neutrosophic soft set minimal spaces found its place into contemporary research world. This chapter can be further developed into several possible such as Geographical Information Systems (GIS) field including remote sensing, object reconstruction from airborne laser scanner, real time tracking, routing applications and modeling cognitive agents. In GIS there is a need to model spatial regions with indeterminate boundary and under indeterminacy. Hence this neutrosophic soft set minimal spaces can also be extended to a neutrosophic spatial region.In future we will research neutrosophic HyperSoft set minimal structure spaces.. The results of this study may be help in many researches.

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