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ON Ⓢ-CLOSED SETS AND SEMI Ⓢ-CLOSED IN NANO TOPOLOGICAL SPACES

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ABSTRACT. In this article focuss on nano Ⓢ-closed sets and nano $Ⓢ_s$-closed sets are introduce and study. Also, we introduce and study nano Ⓢ-continuous functions and nano $Ⓢ_s$-continuous functions. Furthermore, we introduce the notions of nano topological spaces called nano Ⓢ-$T_{\frac{1}{2}}$ space and nano Ⓢ-T_s space.

1. INTRODUCTION AND PRELIMINARIES

Several idea of nano topology have been generalized by considering the concept of nano semi-open sets due to M. L. Thivagar (2013) instead of nano open sets. The study of nano generalized closed sets in a nano topological space was initiated by K. Bhuvaneshwari (2014) and introduced the class of nano semi-generalized closed (nsg-closed), nano generalized semi-closed (ngs-closed) sets are used them to obtain some properties.

In this article focuss on called nano Ⓢ-closed sets and nano $Ⓢ_s$-closed sets are introduce and study. Also, we introduce and study nano Ⓢ-continuous functions and nano $Ⓢ_s$-continuous functions. Furthermore, we introduce the notions of nano topological spaces called nano Ⓢ-$T_{\frac{1}{2}}$ space and nano Ⓢ-T_s space.

Definition 1.1. [9] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where $\tau_R(x)$ denotes the equivalence class determined by x.

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(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.
(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 1.2. [7] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$,
(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
(3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

Through out this paper, we denote a nano topological space by $(U, \mathcal{N})$, $(U', \mathcal{N}')$ where $\mathcal{N} = \tau_R(X)$. The nano- interior and nano-closure of a subset H of U are denoted by $I_n(H)$ and $C_n(H)$, respectively.

Definition 1.3. [7] A subset H of a space $(U, \mathcal{N})$ is called

(1) nano semi-open (resp. ns-open) if $H \subseteq C_n(I_n(H))$.
(2) nano α-open (resp. $n\alpha$-open) if $H \subseteq I_n(C_n(I_n(H)))$.
(3) nano regular-open (resp. nr-open) if $H = I_n(C_n(H))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 1.4. A subset H of a nano topological space $(U, \mathcal{N})$ is called

(1) nano generalized closed (resp. ng-closed) [1] if $C_n(H) \subseteq G$, whenever $H \subseteq G$ and G is n-open.
(2) nano semi generalized closed (resp. nsg-closed) [3] if $sC_n(H) \subseteq G$, whenever $H \subseteq G$ and G is ns-open.
(3) nano generalized semi closed (resp. ngs-closed) [3] if $sC_n(H) \subseteq G$, whenever $H \subseteq G$ and G is n-open.

Definition 1.5. A function $f : (U, \mathcal{N}) \to (U', \mathcal{N}')$ is said to be

(1) nano continuous (resp. n-continuous) [7] if $f^{-1}(H')$ is n-open in $(U, \mathcal{N})$ for every n-open set H' of $(U', \mathcal{N}')$.
(2) nano generalized-continuous (resp. ng-continuous) [2] if $f^{-1}(H')$ is ng-closed in $(U, \mathcal{N})$ for every n-closed set H' of $(U', \mathcal{N}')$.
(3) nano semi generalized-continuous (resp. nsg-continuous) [4] if $f^{-1}(H')$ is nsg-closed in $(U, \mathcal{N})$ for every n-closed set H' of $(U', \mathcal{N}')$.
(4) nano generalized semi-continuous (resp. ngs-continuous) [10] if $f^{-1}(H')$ is ngs-closed in $(U, \mathcal{N})$ for every n-closed set H' of $(U', \mathcal{N}')$.
(5) nano semi-continuous (resp. ns-continuous) [5] if $f^{-1}(H')$ is ns-open in $(U, \mathcal{N})$ for every n-open set H' of $(U', \mathcal{N}')$.
(6) nano contra-continuous (resp. nc-continuous) [8] if $f^{-1}(H')$ is n-closed in $(U, \mathcal{N})$ for every n-open set H' of $(U', \mathcal{N}')$.
(7) nano perfectly-continuous (resp. np-continuous) [8] if $f^{-1}(H')$ is both n-open and n-closed in $(U, \mathcal{N})$ for every n-open set H' of $(U', \mathcal{N}')$.

2. ON SOME Ⓢ-CLOSED SETS

Definition 2.1. Let $(U, \mathcal{N})$ be a nanotopological space and $H \subseteq U$ is said to be,

(1) nano Ⓢ-closed (resp. nⓈ-closed) if $H \subseteq G$, $G \in ns$-open $\Longrightarrow sC_n(H) \subseteq I_n(G)$.
(2) nano Ⓢ$_s$-closed (resp. nⓈ$_s$-closed) if $H \subseteq G$, $G \in ns$-open $\Longrightarrow sC_n(H) \subseteq I_n(C_n(G))$.

Proposition 2.1. *A nanotopological space $(U, \mathcal{N})$, if*

(1) H is n-open and ns-closed $\Longrightarrow H$ is nⓈ-closed
(2) H is nⓈ-closed set $\Longrightarrow H$ is nⓈ$_s$-closed and ngs-closed.

Proof. (1) Let H be a n-open and ns-closed and $H \subseteq G$, where G is a ns-open. Then, $sC_n(H) = H = I_n(H) \subseteq I_n(H)$. Hence, H is nⓈ-closed.
(2) Let H be a nⓈ-closed and $H \subseteq G$, where G is ns-open. Then, $sC_n(G) \subseteq I_n(G) \subseteq C_n(I_n(G))$. Hence, H is a Ⓢ$_s$-closed. To prove the second part, let H be a nⓈ-closed and $H \subseteq G$, where G is a n-open. Then, $sC_n(H) \subseteq I_n(G) \subseteq G$. Hence, H is ngs-closed.

□

Remark. In Proposition 2.1, the converses are not necessarily true.

(1) Not every nⓈ-closed set is ns-closed.
(2) Not every nⓈ-closed set is nⓈ$_s$-closed.
(3) Not every nⓈ-closed set is ngs-closed.

Example 2.2. (1) Let $U = \{Y_1, Y_2, Y_3, Y_4\}$, $\frac{U}{R} = \{\{Y_1, Y_2\}, \{Y_3, Y_4\}\}$, $X = \{Y_3, Y_4\}$ and $\mathcal{N} = \{\phi, \{Y_3, Y_4\}, U\}$. The subset $\{Y_1, Y_2, Y_3\}$ is nⓈ-closed but not ns-closed.
(2) Let $U = \{Y_1, Y_2, Y_3\}$, $\frac{U}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X = \{Y_1\}$ and $\mathcal{N} = \{\phi, \{Y_1\}, U\}$. The subset $\{Y_1\}$ is nⓈ$_s$-closed but it is neither nⓈ-closed nor ngs-closed. Therefore $\{Y_1\}$ is not ng-closed.
(3) Let $U = \{Y_1, Y_2, Y_3\}$, $\frac{U}{R} = \{\{Y_2\}, \{Y_1, Y_3\}\}$, $X = \{Y_2, Y_3\}$ and $\mathcal{N} = \{\phi, \{Y_2\}, \{Y_3\}, \{Y_2, Y_3\}, U\}$. The subset $\{Y_1\}$ is ng-closed and ngs-closed but it is neither nⓈ$_s$-closed nor nⓈ-closed.

Remark. In nano topological spaces,

(1) ng-closed and nⓈ-closed are independent.
(2) ngs-closed and nⓈ$_s$-closed are independent.
(3) ng-closed and nⓈ$_s$-closed are independent.

Example 2.3. By Example 2.2(3),

(1) the subset $\{Y_1\}$ is ng-closed but nⓈ-closed.
(2) the subset $\{Y_2\}$ is nⓈ-closed but not ng-closed.
(3) the subset $\{Y_1, Y_2\}$ is ng-closed but not nⓈ$_s$-closed.
(4) the subset $\{Y_2, Y_3\}$ is nⓈ$_s$-closed but not ng-closed.

Example 2.4. Let $U = \{Y_1, Y_2, Y_3\}$, $\frac{U}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X = \{Y_1\}$ then $\mathcal{N} = \{\phi, \{Y_1\}, U\}$.

(1) the subset $\{Y_2\}$ is ngs-closed but nⓈ$_s$-closed.
(2) the subset $\{Y_1\}$ is nⓈ$_s$-closed but not ngs-closed.

Remark. We have the following relationship between $n\circledS$-closed sets, $n\circledS_s$-closed sets and related sets.

Diagram -I

n**-clopen**	$\rightarrow$	n**-closed**	$\rightarrow$	ns**-closed**
$\downarrow$		$\downarrow$		$\downarrow$
n**-open and** ns**-closed**	$\not\leftrightarrow$	ng**-closed**	$\not\leftrightarrow$	nsg**-closed**
$\downarrow$				$\downarrow$
$n\circledS$**-closed**	$\not\leftrightarrow$	$n\circledS_s$**-closed**	$\not\leftrightarrow$	ngs**-closed**

Diagram -II

$n\circledS_s$**-closed set**	$\not\leftrightarrow$	ng**-closed set**	$\not\leftrightarrow$	$n\circledS$**-closed set**
				$\downarrow$
				ngs**-closed set**

Proposition 2.2. *If a subset H is $n\circledS$-closed, then $sC_n(H) - H$ does not contain a non empty ns-closed set.*

Proof.
Let G be a ns-closed set such that $G \subseteq sC_n(H) - H$. Then $G \subseteq sC_n(H)$ and $H \subseteq U - G$. Since H is $n\circledS$-closed, then $sC_n(H) \subseteq I_n(U - G) = U - C_n(G)$. Therefore, $G \subseteq C_n(G) \subseteq U - sC_n(H)$. Hence, $G \subseteq (U - sC_n(H)) \cap sC_n(H) = \phi$. □

Proposition 2.3. *If a subset H is $n\circledS_s$-closed, then $sC_n(H) - H$ does not contain a non empty ns-clopen set.*

Proof. Let G be ns-clopen such that $G \subseteq sC_n(H) - H$. Then we have that $H \subseteq U - G$ and $sC_n(H) \subseteq I_n(C_n(U - G)) = U - C_n(I_n(G))$. Thus we obtain $G \subseteq C_n(I_n(G)) \subseteq U - sC_n(H)$. Therefore, $G \subseteq (U - sC_n(H)) \cap sC_n(H) = \phi$. □

Proposition 2.4. *If a subset H of $(U, \mathcal{N})$ is ns-open and $n\circledS$-closed, then it is ns-closed.*

Proof. Since H is ns-open and $n\circledS$-closed, then $sC_n(H) \subseteq I_n(H) \subseteq H$. Hence, $sC_n(H) = H$ and H is ns-closed. □

Theorem 2.5. *A subset H of nono topological spaces is nr-open $\iff$ H is $n\alpha$-open and $n\circledS$-closed.*

Proof. Suppose H is $n\alpha$-open and $n\circledS$-closed set. Then H is ns-open and $n\circledS$-closed and by Proposition 2.4, H is ns-closed. So, $I_n(C_n(H)) \subseteq H$. Since H is $n\alpha$-open, then $H \subseteq I_n(C_n(I_n(H))) \subseteq I_n(C_n(H))$. Thus, $H = I_n(C_n(H))$ and H is nr-open.

Conversely, let H be nr-open, then H is $n\alpha$-open. Since H is nr-open, n-open and ns-closed. By Proposition 2.1, H is $n\circledS$-closed. □

Theorem 2.6. *Every n-open set is ngs-closed $\iff$ $n\circledS$-closed.*

Proof. Let H be n-open and ngs-closed set. Assume that $H \subseteq G$, where G is a ns-open set. Thus $H = I_n(H) \subseteq I_n(G)$. Since $I_n(G)$ is n-open in U and H is ngs-closed, then $sC_n(H) \subseteq I_n(G)$ and H is $n\circledS$-closed set.

Conversely, it is obvious that every $n\circledS$-closed set is ngs-closed. □

3. ON NANO $\circledS$-CONTINUITY AND NANO $\circledS_s$-CONTINUITY

Definition 3.1. A function $f : (U,\mathcal{N}) \to (U',\mathcal{N}')$ is said to be

(1) nano $\circledS$-continuous (resp. $n\circledS$-continuous) if $f^{-1}(H')$ is $n\circledS$-closed in $(U,\mathcal{N})$ for every n-closed set H' of $(U',\mathcal{N}')$.
(2) nano $\circledS_s$-continuous (resp. $n\circledS_s$-continuous) if $f^{-1}(H')$ is $n\circledS_s$-closed in $(U,\mathcal{N})$ for every n-closed set H' of $(U',\mathcal{N}')$.
(3) nano $\circledS$-irresolute (resp. $n\circledS$-irresolute) if $f^{-1}(H')$ is $n\circledS$-closed in $(U,\mathcal{N})$ for every $n\circledS$-closed set H' of $(U',\mathcal{N}')$.
(4) nano $\circledS_S$-irresolute (resp. $n\circledS_s$-irresolute) if $f^{-1}(H')$ is $n\circledS_s$-closed in $(U,\mathcal{N})$ for every $n\circledS_s$-closed set H' of $(U',\mathcal{N}')$.

Proposition 3.1. *In a nano topological spaces,*

(1) Every $n\circledS$-continuous function is ngs-continuous.
(2) Every $n\circledS$-continuous function is $n\circledS_s$-continuous.
(3) Every nc-continuous and ns-continuous function is $n\circledS$-continuous.

Remark. We have the following relationship between $n\circledS$-closed sets, $n\circledS_s$-closed sets and related sets.

Diagram -III

$$\begin{array}{ccccc}
np\textbf{-cts} & \to & n\textbf{-cts} & \to & ns\textbf{-cts} \\
\downarrow & & \downarrow & & \downarrow \\
n\textbf{-contra cts and } ns\textbf{-cts} & \not\leftrightarrow & ng\textbf{-cts} & \leftrightarrow & nsg\textbf{-cts} \\
\downarrow & & & & \downarrow \\
n\circledS\textbf{-cts} & \not\leftrightarrow & n\circledS_s\textbf{-cts} & \not\leftrightarrow & ngs\textbf{-cts}
\end{array}$$

Diagram -IV

$$\begin{array}{ccccc}
n\circledS_s\textbf{-cts} & \not\leftrightarrow & ns\textbf{-cts} & \not\leftrightarrow & n\circledS\textbf{-cts} \\
 & & & & \downarrow \\
 & & & & nsg\textbf{-cts}
\end{array}$$

Remark. In Proposition 3.1, the converses are not necessarily true.

(1) Not every ngs-continuous function is $n\circledS$-continuous.
(2) $n\circledS_s$-continuous function need not be $n\circledS$-continuous.
(3) $n\circledS$-continuous function is not always ns-continuous.

Example 3.2. (1) Let $U = \{Y_1, Y_2, Y_3\}$, $\frac{U}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X = \{Y_1\}$ then $\mathcal{N} = \{\phi, \{Y_1\}, U\}$ and let $U' = \{Y_1, Y_2, Y_3\}$, $\frac{U'}{R} = \{\{Y_2\}, \{Y_1, Y_3\}\}$, $X' = \{Y_2, Y_3\}$ then $\mathcal{N}' = \{\phi, \{Y_2\}, \{Y_1, Y_3\}, U'\}$. Clearly $f : (U,\mathcal{N}) \to (U',\mathcal{N}')$ is defined by $f(Y_1) = Y_1, f(Y_2) = Y_2 \; and \; f(Y_3) = Y_3$. Thus f is ngs-continuous but not $n\circledS$-continuous.

(2) Let $U = \{Y_1, Y_2, Y_3\}$, $\frac{U}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X = \{Y_1\}$ then $\mathcal{N} = \{\phi, \{Y_1\}, U\}$ and let $U' = \{Y_1, Y_2, Y_3\}$, $\frac{U'}{R} = \{\{Y_2\}, \{Y_1, Y_3\}\}$, $X' = \{Y_1, Y_3\}$ then $\mathcal{N}' = \{\phi, \{Y_1, Y_3\}, U'\}$. Clearly $f : (U,\mathcal{N}) \to (U',\mathcal{N}')$ is defined by $f(Y_1) = Y_2, \; f(Y_2) = Y_3 \; and \; f(Y_3) = Y_1$. Thus f is $n\circledS_s$-continuous but not $n\circledS$-continuous.

(3) Let $U = \{Y_1, Y_2, Y_3\}$, $\frac{U}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X = \{Y_1\}$ then $\mathcal{N} = \{\phi, \{Y_1\}, U\}$ and let $U' = \{Y_1, Y_2, Y_3\}$, $\frac{U'}{R} = \{\{Y_2\}, \{Y_1, Y_3\}\}$, $X' = \{Y_1, Y_3\}$ then $\mathcal{N}' = \{\phi, \{Y_1, Y_3\}, U'\}$. Clearly $f : (U,\mathcal{N}) \to (U',\mathcal{N}')$ is defined by $f(Y_1) = Y_1, \; f(Y_2) = Y_2 \; and \; f(Y_3) = Y_3$. Thus f is ns-continuous but not $n\circledS$-continuous.

Remark. In a nano topological spaces,

(1) nⓈ$_s$-continuity and ng-continuity are independent.
(2) nⓈ$_s$-continuity and ngs-continuity are independent.

Example 3.3. (1) Let $U = \{Y_1, Y_2, Y_3\}$, $\frac{U}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X = \{Y_1\}$ then $\mathcal{N} = \{\phi, \{Y_1\}, U\}$ and let $U' = \{Y_1, Y_2, Y_3\}$, $\frac{U'}{R} = \{\{Y_2\}, \{Y_1, Y_3\}\}$, $X' = \{Y_2, Y_3\}$ then $\mathcal{N}' = \{\phi, \{Y_2\}, \{Y_1, Y_3\}, U'\}$. Clearly $f : (U, \mathcal{N}) \to (U', \mathcal{N}')$ is defined by $f(Y_1) = Y_1, f(Y_2) = Y_2\ and\ f(Y_3) = Y_3$. Thus f is ng-continuous but not nⓈ$_s$-continuous.
(2) Let $U = \{Y_1, Y_2, Y_3\}$, $\frac{U}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X = \{Y_1\}$ then $\mathcal{N} = \{\phi, \{Y_1\}, U\}$ and let $U' = \{Y_1, Y_2, Y_3\}$, $\frac{U'}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X' = \{Y_2, Y_3\}$ then $\mathcal{N}' = \{\phi, \{Y_2, Y_3\}, U'\}$. Clearly $f : (U, \mathcal{N}) \to (U', \mathcal{N}')$ is defined by $f(Y_1) = Y_1, f(Y_2) = Y_2\ and\ f(Y_3) = Y_3$. Thus f is nⓈ$_s$-continuous but not ng-continuous.
(3) Let $U = \{Y_1, Y_2, Y_3\}$, $\frac{U}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X = \{Y_1\}$ then $\mathcal{N} = \{\phi, \{Y_1\}, U\}$ and let $U' = \{Y_1, Y_2, Y_3\}$, $\frac{U'}{R} = \{\{Y_2\}, \{Y_1, Y_3\}\}$, $X' = \{Y_2, Y_3\}$ then $\mathcal{N}' = \{\phi, \{Y_2\}, \{Y_1, Y_3\}, U'\}$. Clearly $f : (U, \mathcal{N}) \to (U', \mathcal{N}')$ is defined by $f(Y_1) = Y_1, f(Y_2) = Y_2\ and\ f(Y_3) = Y_3$. Thus f is ngs-continuous but not nⓈ$_s$-continuous.
(4) Let $U = \{Y_1, Y_2, Y_3\}$, $\frac{U}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X = \{Y_1\}$ then $\mathcal{N} = \{\phi, \{Y_1\}, U\}$ and let $U' = \{Y_1, Y_2, Y_3\}$, $\frac{U'}{R} = \{\{Y_1\}, \{Y_2, Y_3\}\}$, $X' = \{Y_2, Y_3\}$ then $\mathcal{N}' = \{\phi, \{Y_2, Y_3\}, U'\}$. Clearly $f : (U, \mathcal{N}) \to (U', \mathcal{N}')$ is defined by $f(Y_1) = Y_1, f(Y_2) = Y_2\ and\ f(Y_3) = Y_3$. Thus f is nⓈ$_s$-continuous but not ngs-continuous.

4. APPLICATIONS

Definition 4.1. A nano topological space $(U, \mathcal{N})$ is said to be
(1) nⓈ-$T_{\frac{1}{2}}$ if every nⓈ$_s$-closed set is ns-closed.
(2) nⓈ-T_s if every nⓈ$_s$-closed set is n-closed.

Proposition 4.1. *Let $(U, \mathcal{N})$ be a nano topological space.*
(1) For each $x \in U$, $\{x\}$ is ns-closed or its complement $U - \{x\}$ is nⓈ-closed.
(2) For each $x \in U$, $\{x\}$ is n-open and ns-closed or its complement $U - \{x\}$ is nⓈ$_s$-closed.

Proof. (1) Suppose that $\{x\}$ is not ns-closed. Then $U - \{x\}$ is not ns-open and the only ns-open set containing $U - \{x\}$ is U. Therefore, $sC_n(U - \{x\}) \subseteq I_n(U) = U$. So, $U - \{x\}$ is nⓈ-closed.
(2) Suppose that $\{x\}$ is not ns-closed. Then by (1), $U - \{x\}$ is nⓈ-closed and then nⓈ$_s$-closed. Suppose that $\{x\}$ is not n-open and let G be a ns-open set such that $U - \{x\} \subseteq G$. If $G = U$, then $sC_n(U - \{x\}) \subseteq I_n(C_n(G)) = G$. If $G = U - \{x\}$, then we have that $I_n(C_n(G)) = I_n(C_n(U - \{x\})) = I_n(U) = U$. Hence, $sC_n(U - \{x\}) \subseteq I_n(C_n(G))$. Therefore, $U - \{x\}$ is nⓈ$_s$-closed.
□

Theorem 4.2. *For a nano topological space, the next conditions are equivalent:*
(1) Every nⓈ-closed set is ns-closed.
(2) For each $x \in U$, $\{x\}$ is ns-open or ns-closed.

Proof. (1) (1) $\Longrightarrow$ (2). Suppose that for a point $x \in U$, $\{x\}$ is not ns-closed. By Proposition 4.1(1), $U - \{x\}$ is nⓈ-closed. By assumption, $U - \{x\}$ is ns-closed and hence $\{x\}$ is ns-open. Therefore, each singleton is ns-open or ns-closed.

(2) $(2) \implies (1)$. Let H be a nⓈ-closed set. We want to prove that $sC_n(H) = H$. Suppose $x \in sC_n(H)$.

Case 1: $\{x\}$ is ns-open. Then $\{x\} \cap H \neq \phi$ which implies $x \in H$.
Case 2: $\{x\}$ is ns-closed and $x \notin H$. Then $sC_n(H) - H$ contains a ns-closed set $\{x\}$ and this contradicts Proposition 2.2. Hence $x \in H$ and H is ns-closed. Therefore, Every nⓈ-closed set is nsemi-closed.

□

Theorem 4.3. *For a nano topological space, the following properties hold:*

(1) If $(U, \mathcal{N})$ is nⓈ- T_s, then for each $x \in U$ the singleton $\{x\}$ is n-open or ns-closed.
(2) $(U, \mathcal{N})$ is nⓈ- $T_{\frac{1}{2}}$ $\iff$ for each $x \in U$, $\{x\}$ is ns-open or ns-closed and n-open.
(3) If $(U, \mathcal{N})$ is nⓈ-T_s, then it is nⓈ-$T_{\frac{1}{2}}$.

Proof.

(1) Suppose that for some $x \in U$, $\{x\}$ is not ns-closed. By Proposition 4.1, $U - \{x\}$ is nⓈ-closed. Hence, $U - \{x\}$ is nⓈ$_s$-closed. Since $(U, \mathcal{N})$ is nⓈ-T_s, then $U - \{x\}$ is n-closed. Thus $\{x\}$ is n-open.
(2) Necessity. Suppose that a singleton $\{x\}$ is not ns-closed or n-open. By Proposition 4.1, $U - \{x\}$ is nⓈ$_s$-closed. Using the assumption we have that $\{x\}$ is ns-open.
Sufficiency. It follows from the assumption that every subset is ns-open and ns-closed. Then $(U, \mathcal{N})$ is nⓈ-$T_{\frac{1}{2}}$.
(3) It is straightforward from the definitions of nⓈ-T_s spaces and nⓈ-$T_{\frac{1}{2}}$ spaces.

□

5. Conclusion

In this paper we have discussed the concepts of nⓈ-closed and nⓈ$_s$-closed. These concepts can be used to derive a new real world applications in future.

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