ANNALS OF COMMUNICATIONS IN MATHEMATICS
Volume 4, Number 3 (2021), 254-260
ISSN: 2582-0818

NEUTROSOPHIC REGULAR SEMI CONTINUOUS FUNCTIONS

R. VIJAYALAKSHMI* AND R. R. PRAVEENA

ABSTRACT. In this paper, we introduce and study the concept of regular semi continuous, regular semi irresolute, regular semi-$T_{1/2}$ space, regular semi homeomorphisms and regular semi c-homeomorphisms in neutrosophic topological spaces. Moreover, we investigate the relationship among neutrosophic regular semi continuous, neutrosophic regular semi irresolute, neutrosophic regular semi homeomorphism and neutrosophic regular semi C-homeomorphisms mappings. Finally, we have given some counter examples to show that these types of mappings are not equivalent.

1. INTRODUCTION

The study of fuzzy set was initiated by Zadeh [18] in 1965. Thereafter the paper of Chang [3] paved the way for the subsequent tremendous growth of the numerous fuzzy topology concepts. Currently Fuzzy Topology has been observed to be very beneficial in fixing many realistic problems. Several mathematicians have tried almost all the pivotal concepts of General Topology for extension to the fuzzy settings. In 1983, Atanassov [1] introduced the concept of intuitionistic fuzzy set which was generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. Later, Coker [4] introduced the concept of intuitionistic fuzzy topological spaces, by using the notion of the intuitionitic fuzzy set. Smarandache [7, 8, 9] introduced the concept of Neutrosophic set. Neutrosophic set is classified into three independent functions namely, membership function, indeterminancy and non membership function that are independently related. In 2012, Salama and Alblowi [14, 15, 16] introduced the concept of Neutrosophic topology. Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allow more general functions to be members of fuzzy topology. In 2014, Salama et. al., [15] introduced the concept of Neutrosophic closed sets and Neutrosophic continuous functions. Ishwarya and Bageerathi [10] introduced the concept of neutrosophic semiopen sets in neutrosophic topological spaces.

In general topology, the concept of regular semiopen set was introduced by Cameron [2] in 1978. Latter Vadivel and Elavarasan introduced regular semiopen sets in fuzzy topological and soft topological spaces in [5, 6, 11, 12]. Recently Vijayalakshmi and Praveena [13]

2010 *Mathematics Subject Classification.* 03E72, 54A10, 54A40.

Key words and phrases. Neutrosophic regular semi coninuous, neutrosophic regular semi irresolute, $NRST_{1/2}$-space, NRS-homeomorphisms and NRS-C-homeomorphisms.

Received: September 26, 2021. Accepted: December 27, 2021. Published: December 31, 2021.

*Corresponding author.

introduced the concept of regular semiopen and regular semiclosed sets in neutrosophic topological spaces. In this paper, we introduce and study the concept of neutrosophic regular semi continuous and neutrosophic regular semi irresolute mappings. Moreover, we investigate the relationship among neutrosophic regular semi continuous, neutrosophic regular semi irresolute, neutrosophic regular semi homeomorphism and neutrosophic regular semi C-homeomorphisms mappings. Finally, we have given some counter examples to show that these types of mappings are not equivalent.

2. PRELIMINARIES

Definition 2.1. [14] Let X be a non-empty fixed set. A Neutrosophic set [for short, Ns] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminancy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark. [14] *A Ns* $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ *can be identified to an ordered triple* $A = \langle \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ *in* $]^{-}0, 1^{+}[$ *on* X.

Remark. [14] *For the sake of simplicity, we shall use the symbol* $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ *for the Ns* $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Example 2.2. [14] Every intuitionsistic fuzzy set A is a non-empty set in X is obviously on Ns having the form $A = \{\langle x, \mu_A(x), 1 - \mu_A(x) + \gamma_A(x)\rangle : x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic sets 0_N and 1_N in X as follows:
$0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 2.3. [14] Let $A = \langle(\mu_A, \sigma_A, \gamma_A)\rangle$ be a Ns on X, then the complement of the set A(A^c or $C(A)$ for short) may be defined as $C(A) = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$.

Definition 2.4. [14] Let X be a non-empty set and Ns's A and B in the form $A = \{\langle x, \mu_A, \sigma_A, \gamma_A\rangle : x \in X\}$ and $B = \{\langle x, \mu_B, \sigma_B, \gamma_B\rangle : x \in X\}$. Then $(A \subseteq B)$ may defined as: $(A \subseteq B) \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x), \gamma_A(x) \ge \gamma_B(x) \forall x \in X$.

Definition 2.5. [14] Let X be a non-empty set and $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$ are Ns's. Then $A \cap B$ and $A \cup B$ may defined as:

(i) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle$
(ii) $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle$

Definition 2.6. [14] A Neutrosophic topology (for short, NT or nt) is a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:

(i) $0_N, 1_N \in \tau_N$,
(ii) $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
(iii) $\cup G_i \in \tau_N$ for every $\{G_i : i \in J\} \subseteq \tau_N$.

Throughout this paper, the pair of (X, τ_N) is called a neutrosophic topological space (for short, nts). The elements of τ_N or τ are called neutrosophic open set (for short, nos). A neutrosophic set F is neutrosophic closed set (for short, ncs)if and only if F^c is nos.

Definition 2.7. [14] Let (X, τ_N) be nts and $A = \langle x, \mu_A, \sigma_A, \gamma_A\rangle$ be a Ns in X. Then the neutrosophic closure and neutrosophic interior of A are defined by $NCl(A) = \cap\{K : K$ is a NCS in X and $A \subseteq K\}$, $NInt(A) = \{G : G$ is a NOS in X and $G \subseteq A\}$. It can be also shown that $NCl(A)$ is NCS and $NInt(A)$ is a NOS in X. A is NOS if and only if $A = NInt(A)$, A is NCS if and only if $A = NCl(A)$.

Definition 2.8. [17] Let $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ be a Ns on a nts (X, τ_N) then A is called:

(a) neutrosophic regular open (for short, nro) iff $A = NInt(NCl(A))$.
(b) neutrosophic regular closed (for short, nrc) iff $A = NCl(NInt(A))$.

Definition 2.9. [17] Let $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ be a Ns and $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$ be a Ns on a nts (X, τ_N) then A is called neutrosophic semi-open (for short, nso) iff $A \subseteq NInt(NCl(A))$.

Definition 2.10. [13] Let $(X,\ \tau)$ be a nts. Then A is called

(1) neutrosophic regular semiopen (for short, nrso) if there exists an nro set B in X such that $B \subseteq A \subseteq NCl(B)$.
(2) neutrosophic regular semiclosed (for short, nrsc) if there exists an nrc set B in X and $NInt(B) \subseteq A \subseteq B$.

We shall denote the family of all nrso sets (nrsc sets) of a nts (X, τ) by $NRSOS(X)$, $NRSCS(X)$.

Definition 2.11. [13] Let $(X,\ \tau)$ be a nts. Then

(1) the neutrosophic regular closure of A, denoted by $nrcl(A)$, and is defined by $nrcl(A) = \bigcap\{B|B \supseteq A, B \text{ is nrc }\}$.
(2) the neutrosophic regular interior of A, denoted by $nrint(A)$, and is defined by $nrint(A) = \bigcup\{B|B \subseteq A, B \text{ is nro }\}$.
(3) the neutrosophic regular semiclosure of A defined by $nrscl(A) = \bigcap\{B \mid A \subseteq B \text{ and } B \in NRSCS(X, \tau)\}$ is a neutrosophic set.
(4) the neutrosophic regular semiinterior of A defined by $nrsint(A) = \bigcup\{B \mid B \subseteq A \text{ and } B \in NRSOS(X, \tau)\}$ is a neutrosophic set.

Definition 2.12. [16] Let (X, τ) and (Y, σ) be any two nts's. A map $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic continuous (for short, NC) if the inverse image of every neutrosophic closed set in (Y, σ) is neutrosophic closed set in (X, τ).

3. Neutrosophic regular semi continuous, open and closed functions

Definition 3.1. Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be two nts's. A Neutrosophic function $f : X \to Y$ is said to be

(1) neutrosophic regular continuous (for short, NRC)if for each nos A of Y, the inverse image $f^{-1}(A)$ is a nro set of X.
(2) neutrosophic regular semi continuous (for short, $NRSC$) if for each nos A of Y, the inverse image $f^{-1}(A)$ is a nrso set of X.
(3) neutrosophic regular semi irresolute (for short, $NRSI$) if for each nrso set A of Y, the inverse image $f^{-1}(A)$ is a nrso set of X.
(4) neutrosophic regular semiopen function (for short, NRS-O) if for each nos B of X, the image $f(B)$ is a nrso set of Y.
(5) neutrosophic regular semiclosed function (for short, NRS-C) if for each ncs set B of X, the image $f(B)$ is a nrsc set of Y.

Example 3.2. Let $X = \{a, b\}$, $\tau = \{0_N, 1_N, A, B\}$, $Y = \{p, q\}$ and $\sigma = \{0_N, 1_N, C\}$, where A and B are Ns of X and C is Ns of Y, defined as follows:

$A = \left\langle (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.6}, \frac{\gamma_b}{0.5}) \right\rangle$,
$B = \left\langle (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.4}, \frac{\gamma_b}{0.5}) \right\rangle$,
$C = \left\langle (\frac{\mu_p}{0.5}, \frac{\mu_q}{0.5}), (\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}), (\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.5}) \right\rangle$.
Clearly τ and σ are NT on X and Y. If we define the function $f : X \to Y$ as $f(a) = p$ and $f(b) = q$, then f is $NRSC$ but not NRC, the Ns C is nrso set of X, since $\exists$ a nro set B such that $B \subseteq C \subseteq NCl(B)$ but not nro.

Example 3.3. Let $X = \{a, b\}$ and $\tau = \{0_N, 1_N, X, A, B\}$, $Y = \{p, q\}$ and $\sigma = \{0_N, 1_N, C\}$, where A and B are Ns of X and C is Ns of Y, defined as follows:
$A = \left\langle (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.6}, \frac{\gamma_b}{0.5}) \right\rangle$,
$B = \left\langle (\frac{\mu_a}{0.6}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.5}, \frac{\gamma_b}{0.5}) \right\rangle$,
$C = \left\langle (\frac{\mu_p}{0.4}, \frac{\mu_q}{0.5}), (\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}), (\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.5}) \right\rangle$.
Clearly τ and σ are NT on X and Y. If we define the function $f : X \to Y$ as $f(a) = p$ and $f(b) = q$, then f is NSC but not $NRSC$, the Ns C is nso set of X, since $\exists$ a nos B such that $B \subseteq C \subseteq NCl(B)$ but not nrso.

Remark. *The above definition and Examples 3.2and 3.3, it is clear that*

(1) Every NRC function is $NRSC$ but not conversely.
(2) Every $NRSC$ function is NSC but not conversely.

Remark. *A function $f : X \to Y$ is $NRSC$ if for each ncs B of Y, the inverse image $f^{-1}(B)$ is a nrsc set of X.*

Theorem 3.1. *A function $f : X \to Y$ is $NRSC$ iff $f(rsscl(A)) \subseteq NCl(f(A))$ for every Ns A of X.*

Proof. Let $f : X \to Y$ is $NRSC$. Now $NCl(f(A))$ is a ncs of Y. By NRS-continuity of f, $f^{-1}(NCl(f(A)))$ is nrsc set and $A \subseteq f^{-1}(NCl(f(A)))$. But $nrscl(A)$ is the smallest nrsc set containing A. Then $nrscl(A) \subseteq f^{-1}(NCl(f(A)))$. Thus $f(nrscl(A)) \subseteq NCl(f(A))$.

Conversely, let A be any ncs of Y. Then
$f^{-1}(A) \in X$
$\Rightarrow f(nrscl(f^{-1}(A))) \subseteq NCl(f(f^{-1}(A)))$
$\Rightarrow f(nrscl(f^{-1}(A))) \subseteq NCl(A) = A$
$\Rightarrow nrscl(f^{-1}(A)) = f^{-1}(A)$.
Thus $f^{-1}(A)$ is nrsc set. □

Theorem 3.2. *A function $f : X \to Y$ is $NRSC$ iff $f^{-1}(NInt(A)) \subseteq nrsint(f^{-1}(A))$ for every Ns A of Y.*

Proof. Let $f : X \to Y$ is $NRSC$. Now $NInt(f(A))$ is a nos of Y. By NRS-continuity of f, $f^{-1}(NInt(f(A)))$ is nrso set and $f^{-1}(NInt(f(A))) \subseteq A$. As $nrsint(A)$ is the largest nrso set containing A, $f^{-1}(NInt(f(A))) \subseteq nrsint(A)$.

Conversely, take a nos $A = f^{-1}(NInt(A)) \subseteq nrsint(f^{-1}(A))$. Then $f^{-1}(A) \subseteq nrsint(f^{-1}(A))$. Thus $f^{-1}(A)$ is nrso set. □

Theorem 3.3. *A function $f : X \to Y$ is NRS-O iff $f(NInt(A)) \subseteq nrsint(f(A))$ for every Ns A of X.*

Proof. If $f : X \to Y$ is NRS-O, then $f(NInt\ (A)) = nrsint(f(NInt(A))) \subseteq nrsint(f(A))$. On the other hand, take a nos A of X. Then by hypothesis, $f(A) = f(NInt(A)) \subseteq nrsint(f(A))$. Thus $f(A)$ is nrso set in Y. □

Theorem 3.4. *Let $f : X \to Y$ be NRS-O. If B is a Ns in Y and A is nrc set containing $f^{-1}(B)$ then $\exists$ a nrsc set C such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.*

Proof. Take $C = (f(A^c))^c$. Then $f^{-1}(B) \subseteq A$. Thus $f(A^c) \subseteq B^c$. So A^c is nro set. Hence $f(A^c)$ is nrso. Therefore C is nrsc and $B \subseteq C$ and $f^{-1}(C) \subseteq A$. $\square$

Theorem 3.5. *A function $f : X \to Y$ is NRS-C iff $nrscl(f(A)) \subseteq f(NCl(A))$ for every Ns A of X.*

Definition 3.4. A nts (X, τ) is called $NRST_{1/2}$ if for each nrsc set A in X is nrc.

Theorem 3.6. *A nts (X, τ) is called $NRST_{1/2}$ iff $nrscl(A) = nrcl(A)$ for each A in X.*

Proof. Let (X, τ) be $NRST_{1/2}$. By definition of $nrscl$ and $nrcl$, we have $nrscl(A) = nrcl(A)$ for each A in X.

Conversely, suppose (X, τ) is not $NRST_{1/2}$. There exist nrsc set B in X such that B is not nrc. Hence $nrscl(B) = B$ but $nrcl(B) \neq B$. Thus, $nrscl(B) \neq nrcl(B)$. $\square$

Theorem 3.7. *Let (X, τ) and (Y, σ) be nts's. Let $f : (X, \tau) \to (Y, \sigma)$ be a function.*

(1) If (X, τ) is $NRST_{1/2}$, then f is $NRSC$ iff f is NRC.
(2) If (Y, σ) is $NRST_{1/2}$, then f is $NRSC$ iff f is $NRSI$.
(3) If (X, τ) and (Y, σ) is $NRST_{1/2}$, then f is NRC iff f is $NRSC$ iff f is $NRSI$.

Proof. (1) Let B be a ncs in Y. Since f is $NRSC$, $f^{-1}(B)$ is nrsc set of X. By Definition of $NRST_{1/2}$, $f^{-1}(B)$ is nrc set in X. Thus f is NRC.

Proof of (2) and (3) are similar. $\square$

Theorem 3.8. *Let $f : (X, \tau) \to (Y, \eta)$ and $g : (Y, \eta) \to (Z, \sigma)$ be $NRSC$ and (Y, η) is $NRST_{1/2}$. Then $g \circ f : (X, \tau) \to (Z, \sigma)$ is $NRSC$.*

Proof. Let A be a ncs in Z. Since f and g are $NRSC$, $g^{-1}(A)$ is nrsc set in Y. As Y is $NRST_{1/2}$, $g^{-1}(A)$ is nrc set in Y. Since every nrc set is nc set. Which implies $f^{-1}(g^{-1}(A))$ is nrsc set in X. Thus $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is nrsc set in X. Hence $g \circ f$ is $NRSC$. $\square$

Theorem 3.9. *Let $f : (X, \tau) \to (Y, \eta)$ be $NRSI$ and $g : (Y, \eta) \to (Z, \sigma)$ be $NRSC$. Then $g \circ f : (X, \tau) \to (Z, \sigma)$ is $NRSC$.*

Proof. Let A be a ncs in Z. Since g is $NRSC$, $g^{-1}(A)$ is nrsc set in Y. As f is $NRSI$, $f^{-1}(g^{-1}(A))$ is nrsc set in X. Thus $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is nrsc set in X. Hence $g \circ f$ is $NRSC$. $\square$

4. Neutrosophic regular semi homeomorphisms

Definition 4.1. A bijection $f : X \to Y$ is called neutrosophic regular semi homeomorphism (for short, NRS-h) if f is both $NRSC$ and NRS-O functions.

Definition 4.2. A bijection $f : X \to Y$ is called neutrosophic regular semi C-homeomorphism (for short, NRS-C-h) if f and f^{-1} are both $NRSI$.

Proposition 4.1. *For any bijection $f : X \to Y$, the following statements are equivalent:*

(1) $f^{-1} : Y \to X$ is $NRSC$.
(2) f is NRS-O.
(3) f is NRS-C.

Proof. (1)⇒(2): Let A be a nos in X. Then A^c is ncs in X. Since f^{-1} is $NRSC$, $(f^{-1})^{-1}(A^c) = f(A^c) = (f(A))^c$ is nrsc set in Y. Then $f(A)$ is nrso in Y. Thus f is a NRS-O function.

(2)⇒(3): Let f be a NRS-O function. Let A be a ncs in X. Then A^c nos in X. Since f is NRS-O function, $f(A^c) = (f(A))^c$ is nrso set in Y. Thus $f(A)$ is nrsc set in Y. So f is NRS-C.

(3)⇒(1): Let A be ncs in X. Then $f(A)$ is nrsc set in Y. Thus is $(f^{-1})^{-1}(A)$ is nrsc set in Y. So f^{-1} is $NRSC$. □

Proposition 4.2. *Let $f : X \to Y$ be a bijective and $NRSC$. Then the following statements are equivalent:*

(1) f is a NRS-O.
(2) f is a NRS-h.
(3) f is a NRS-C.

Proof. (1)⇒(2): Follows from the definition.

(2)⇒(3): Let A be a ncs in X. Then A^c is nos in X. Since f is a NRS-h, $f(A^c) = (f(A))^c$ is nrso set in Y. Then $f(A)$ is nrsc set in Y. Hence f is a NRS-C.

(3)⇒(1): Let A be a nos in X. Then A^c is ncs in X. Since f is a NRS-C, $f(A^c) = (f(A))^c$ is nrsc set in Y. Thus $f(A)$ is nrso set in Y. So f is a NRS-O function. □

Proposition 4.3. *If $f : X \to Y$ and $g : Y \to Z$ are NRS-C-h, then $g \circ f : X \to Z$ is also a NRS-C-h.*

Proof. Let A be a nrso set in Z. Then $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A)) = f^{-1}(A)$, where $A = g^{-1}(A)$. By hypothesis, A is nrso set in Y and again by hypothesis, $f^{-1}(A)$ is nrso set in X. Thus, $(g \circ f)$ is $NRSI$. Also for a nrso set B in X, we have $(g \circ f)(B) = g(f(B)) = g(C)$, where $C = f(B)$. By hypothesis, $f(B)$ is nrso set in Y and again by hypothesis, $g(C)$ is nrso set in Z. So, $(g \circ f)^{-1}$ is $NRSI$. Hence $g \circ f$ is NRS-C-h. □

Proposition 4.4. *For a nts (X, τ), the collection $nrsCh(X, \tau)$ forms a group under the composition of functions.*

Proof. Define $\Psi : nrsCh(X, \tau) \times nrsCh(X, \tau) \to nrsCh(X, \tau)$ by $\Psi(f, g) = (g \circ f)$ for every $f,\ g \in nrsCh(X, \tau)$. Then by Proposition 4.3, $(g \circ f) \in nrsCh(X, \tau)$. Thus $nrsCh(X, \tau)$ is nrsc. We know that the composition of maps is associative. The identity map $i : (X, \tau) \to (X, \tau)$ is a NRS-C-h and $i \in nrsCh(X, \tau)$. Also $i \circ f = f \circ i = f$ for every $f \in nrsCh(X, \tau)$. For any $f \in nrsCh(X, \tau)$, $f \circ f^{-1} = f^{-1} \circ f = i$. So inverse exists for each element of $nrsCh(X, \tau)$. Hence $nrsCh(X, \tau)$ is a group under composition of functions. □

Proposition 4.5. *Every NRS-h from a nrs-space into another nrs-space is a neutrosophic homeomorphism.*

Proof. Let $f : X \to Y$ be a NRS-h. Then f is bijective, $NRSC$ and NRS-O. Let A be an nos in X. Since f is NRS-O and since Y is nrs-space, $f(A)$ is nos in Y. This implies f is neutrosophic open function. Let A be ncs in Y. Since f is $NRSC$ and since X is nrs-space, $f^{-1}(A)$ is ncs in X. Thus f is neutrosophic continuous. So f is a neutrosophic homeomorphism. □

Proposition 4.6. *Every NRS-h from a nrs-space into another nrs-space is a NRS-C-h.*

Proof. Let $f : X \to Y$ be a NRS-h. Then f is bijective, $NRSC$ and NRS-O. Let A be an nrsc set in Y. Then A is ncs in Y. Since f is $NRSC$, $f^{-1}(A)$ is nrsc set in X. Thus f is a $NRSI$ map. Let B be nrso set in X. Then B is nos in X. Since f is NRS-O, $f(B)$ is nrso set in Y. That is $(f^{-1})^{-1}(B)$ is nrso set in Y. Thus f^{-1} is $NRSI$. So f is NRS-C-h. □

REFERENCES

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986), 87–96.
[2] D. E. Cameron, Properties of S-closed spaces, Proc. Amer. Math. Soc., 72 (1978) 581–586.
[3] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24 (1968), 182–190.
[4] Dogan Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88 (1997), 81–89.
[5] E. Elavarasan and A. Vadivel, Regular semiopen soft sets and their applications, Annals of Communications in Mathematics, 4(1), (2021), 45–62.
[6] E. Elavarasan, r-fuzzy Rs-compactness and r-fuzzy Rs-connectedness in the sense of Sostak's, Annals of Communications in Mathematics, 3(4), (2020), 273–284.
[7] Floretin Smarandache, A Unifying Field in Logic: Neutrosophic Logic. Neutrosophy, Neutrosophic set, Neutrosophic Probability, Ameican Research Press, Rehoboth, NM, 1999.
[8] Floretin Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA, 2002.
[9] Floretin Smarandache, Neutrosophic Set: A Generalization of Intuitionistiic Fuzzy set, Journal of Defense Resourses Management, 1 (2010), 107–116.
[10] P. Ishwarya and K. Bageerathi, On Neutrosophic semi-open sets in Neutrosophic topological spaces, International Jour. of Math. Trends and Tech. 2016, 214-223.
[11] A. Vadivel and E. Elavarasan, Regular semiopen soft sets and maps in soft topological spaces, Annals Fuzzy Mathematics and Informatics, 12(6), (2016), 877–891.
[12] A. Vadivel and E. Elavarasan, r-fuzzy regular semiopen sets in smooth topological spaces, Sahand communications in Mathematical Analysis, 6(1), (2017), 1–17.
[13] Vijayalakshmi and Praveena, Regular semiopen sets in neutrosophic topological spaces, (submitted).
[14] A. A. Salama and S. A. Alblowi, Neutrosophic set and Neutrosophic topological space, ISOR J. Mathematics, 3(4), (2012), 31–35.
[15] A. A. Salama and S. A. Alblowi, Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, Journal computer Sci. Engineering, 2(7), (2012), 12–23.
[16] A. A. Salama, Florentin Smarandache and Valeri Kroumov, Neutrosophic Closed Set and Neutrosophic Continuous Function, Neutrosophic Sets and Systems, 4 (2014), 4–8.
[17] Wadel Faris Al-omeri and Florentin Smarandache, New Neutrosophic Sets via Neutrosophic Topological Spaces, New Trends in Neutrosophic Theory and Applications, 2 June 2016.
[18] Zadeh.L.A, Fuzzy set, Inform and Control, 8 (1965), 338–353.

R. VIJAYALAKSHMI
DEPARTMENT OF MATHEMATICS, ARIGNAR ANNA GOVERNMENT ARTS COLLEGE, NAMMAKKAL, TAMIL NADU-637 002, INDIA
Email address: viji$_lakshmi80@rediffmail.com$

R. R. PRAVEENA
RESEARCH SCHOLAR, DEPARTMENT OF MATHEMATICS, ANNAMALAI UNIVERSITY, ANNAMALAINAGAR, TAMIL NADU-608 002, INDIA
Email address: praveenaphd24@gmail.com