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INTERVAL-VALUED PYTHAGOREAN FUZZY WEAK BI-HYPERIDEALS IN HYPERSEMIGROUPS

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ABSTRACT. In this paper we introduce the concept of interval-valued Pythagorean fuzzy subsemihypergroup and interval-valued Pythagorean fuzzy weak bi-hyperideals in hypersemigroups. We show that the $(\tilde{\alpha}, \tilde{\beta})-$level set of interval-valued Pythagorean fuzzy weak bi-hyperideal is a weak bi-hyperideal in hypersemigroup. We characterize cartesian product of interval-valued Pythagorean fuzzy set and examine that the cartesian product of interval-valued Pythagorean fuzzy weak bi-hyperideals is also an interval-valued Pythagorean weak bi-hyperideal in hypersemigroups.

1. INTRODUCTION

Fuzzy set and interval-valued fuzzy set were introduced by Zadeh[14, 15]. Atanassov[2, 3] introduced intuitionistic fuzzy set in 1986. Later he developed interval-valued intuitionistic fuzzy set. In 2013 Yager[13] established Pythagorean fuzzy set theory. Kumar et al.[7] applied the concept of Pythagorean fuzzy set in decision making problem. Peng[9] extended the concept of Pythagorean fuzzy set to interval-valued Pythagorean fuzzy set in 2019. He defined some operations on this set. Chen[4] introduced interval-valued Pythagorean fuzzy outranking method in applications. Adak et al.[1] and Hussian et al.[6] applied interval-valued fuzzy set in ideals. Marty[8] extended algebraic structures to algebraic hyperstructures. The concept of hypersemigroup is a generalization of semigroup. In a classical algebraic structure the composition of two elements yields an element while in hyperstructures composition of two elements is a non empty set. Davvaz[5] studied fuzzy hyperideals and intuitionistic fuzzy hyperideals in hypersemigroups. Pibaljommee et al.[10] characterized fuzzy bi hyperideals in ordered semihypergroups and Subha et al.[12] studied fuzzy rough bi hyperideals in semihypergroups.

In this paper we introduce the concept of interval-valued Pythagorean fuzzy subsemihypergroup and interval-valued Pythagorean fuzzy weak bi-hyperideals in hypersemigroups. We show that union of two interval-valued Pythagorean fuzzy subsemihypergroups and interval-valued Pythagorean fuzzy weak bi-hyperideals are also interval-valued Pythagorean

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fuzzy subsemihypergroups and interval-valued Pythagorean fuzzy weak bi-hyperideals respectively. In the last section we discuss the cartesian product of two interval-valued Pythagorean fuzzy weak bi-hyperideals in hypersemigroups.

2. PRELIMINARIES

In this section we recall some basic definitions such as hypersemigroup, interval-valued fuzzy set etc.

Definition 2.1. [5] Let $\mathscr{T}$ be a non-empty universe set and $\mathcal{F}(\mathscr{T})$ is the collection of all subsets of $\mathscr{T}$. The hyperoperation $\bullet$ on $\mathscr{T}$ is defined by

$\bullet : \mathscr{T} \times \mathscr{T} \to \mathcal{F}(\mathscr{T})$

The set $\mathscr{T}$ with the hyperoperation $\bullet$ is called hypergroupoid(say $\mathscr{T}^{\bullet}$). For any $t_1, t_2 \in \mathscr{T}$, the image of $(t_1, t_2) \in \mathscr{T} \times \mathscr{T}$ is denoted by $t_1 \bullet t_2$.

Let $\mathscr{T}_1$ and $\mathscr{T}_2$ be the subsets of $\mathcal{F}(\mathscr{T})$ then the hyperoperation($\star$) between $\mathscr{T}_1$ and $\mathscr{T}_2$ is defined by

$$\mathscr{T}_1 \star \mathscr{T}_2 = \bigcup_{(t_1,t_2)\in \mathscr{T}_1 \times \mathscr{T}_2} t_1 \bullet t_2 \tag{2.1}$$

where

$\star : \mathcal{F}(\mathscr{T}) \times \mathcal{F}(\mathscr{T}) \to \mathcal{F}(\mathscr{T})$.

Definition 2.2. [5] A hypergroupoid[11] ($\mathscr{T}^{\bullet}$) is called a hypersemigroup(say $\mathscr{T}^{\star}$) if $(\{c\} \star \{d\}) \star \{e\} = \{c\} \star (\{d\} \star \{e\})$ for all $c, d, e \in \mathscr{T}$.

Definition 2.3. [11] A subset S of $\mathscr{T}^{\star}$ is said to be a subsemihypergroup if $S \star S \subseteq S$.

Definition 2.4. [9] Let $\mathscr{T}^{\star}$ be a hypersemigroup and $D[0,1]$ be a collection of sub intervals of $[0,1]$. An interval-valued Pythagorean fuzzy set $\mathcal{P} =< \tilde{\delta}_M(x), \tilde{\delta}_{NM}(x) >$ is of the form $\mathcal{P} = \{< x, [\delta^-_M(x), \delta^+_M(x)], [\delta^-_{NM}(x), \delta^+_{NM}(x)] >: 0 \leq \delta^+_M(x)^2 + \delta^+_{NM}(x)^2 \leq 1\}$.
In short $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$
where

$\tilde{\delta}_M : \mathscr{T}^{\star} \longrightarrow D[0,1]$ and $\tilde{\delta}_{NM} : \mathscr{T}^{\star} \longrightarrow D[0,1]$.

$\tilde{\delta}_M$ is a membership grade and $\tilde{\delta}_{NM}$ is a non-membership grade of $x \in \mathscr{T}$.

Definition 2.5. [9] Let $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ and $\mathcal{Q} =< \tilde{\zeta}_M, \tilde{\zeta}_{NM} >$ be any two interval-valued Pythagorean fuzzy sets of $\mathscr{T}^{\star}$. Then
(i) $\mathcal{P} \cup \mathcal{Q} =< \tilde{\delta}_M \vee \tilde{\zeta}_M, \tilde{\delta}_{NM} \wedge \tilde{\zeta}_{NM} >$ is a union of $\mathcal{P}$ and $\mathcal{Q}$ and
(ii) $\mathcal{P} \cap \mathcal{Q} =< \tilde{\delta}_M \wedge \tilde{\zeta}_M, \tilde{\delta}_{NM} \vee \tilde{\zeta}_{NM} >$ is a intersection of $\mathcal{P}$ and $\mathcal{Q}$.

3. INTERVAL-VALUED PYTHAGOREAN FUZZY SUBSEMIHYPERGROUP OF HYPERSEMIGROUPS

In this section we introduce the notion of interval-valued Pythagorean fuzzy subsemihypergroup and study some properties of this structure.

Definition 3.1. An interval-valued Pythagorean fuzzy set $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ of $\mathscr{T}^{\star}$ is said to be an interval-valued Pythagorean fuzzy subsemihypergroup if for $f \in \mathscr{T}^{\star}$ we have
(i) $\min\{\tilde{\delta}_M(u), \tilde{\delta}_M(v)\} \leq \inf_{f\in u\star v} \tilde{\delta}_M(f)$ and
(ii) $\max\{\tilde{\delta}_{NM}(u), \tilde{\delta}_{NM}(v)\} \geq \sup_{f\in u\star v} \tilde{\delta}_{NM}(f)$ for all $u, v \in \mathscr{T}^{\star}$.

Example 3.2. Let $\mathscr{T}^\star = \{p, q, r, s, t\}$ be a hypersemigroup with the Cayley table:

$\bullet$	p	q	r	s	t
p	$\{p,q\}$	$\{p,q\}$	$\{p,q\}$	$\{p,q\}$	$\{p,q\}$
q	$\{p,q\}$	$\{p,q\}$	$\{p,q\}$	$\{p,q\}$	$\{p,q\}$
r	$\{p,q\}$	$\{p,q\}$	$\{r\}$	$\{r\}$	$\{t\}$
s	$\{p,q\}$	$\{p,q\}$	$\{r\}$	$\{s\}$	$\{t\}$
t	$\{p,q\}$	$\{p,q\}$	$\{r\}$	$\{r\}$	$\{t\}$

Define an interval-valued Pythagorean fuzzy set $\mathcal{P}$ as

$\mathcal{P}$	$\tilde{\delta}_M$	$\tilde{\delta}_{NM}$
p	$[0.7, 0.8]$	$[0.1, 0.2]$
q	$[0.4, 0.5]$	$[0.4, 0.5]$
r	$[0.3, 0.5]$	$[0.5, 0.6]$
s	$[0, 0.2]$	$[0.3, 0.5]$
t	$[0.3, 0.5]$	$[0.5, 0.6]$

Since $\min\{\tilde{\delta}_M(p), \tilde{\delta}_M(r)\} = [0.3, 0.5]$ and $\inf\limits_{p,q\in p\star r}\{\tilde{\delta}_M(p), \tilde{\delta}_M(q)\} = [0.4, 0.5]$
which implies that $\min\{\tilde{\delta}_M(p), \tilde{\delta}_M(r)\} \leq \inf\limits_{p,q\in p\star r}\{\tilde{\delta}_M(p), \tilde{\delta}_M(q)\}$
and also we have
$\max\{\tilde{\delta}_{NM}(p), \tilde{\delta}_{NM}(r)\} = [0.5, 0.6]$ and $\sup\limits_{p,q\in p\star r}\{\tilde{\delta}_{NM}(p), \tilde{\delta}_{NM}(q)\} = [0.4, 0.5]$
which implies that $\max\{\tilde{\delta}_{NM}(p), \tilde{\delta}_{NM}(r)\} \geq \sup\limits_{p,q\in p\star r}\{\tilde{\delta}_{NM}(p), \tilde{\delta}_{NM}(q)\}$.
Similarly these inequalities hold for all $p, q, r, s, t \in \mathscr{T}^\star$.
Hence $\mathcal{P}$ is an interval-valued Pythagorean fuzzy subsemihypergroup of $\mathscr{T}^\star$.

Theorem 3.1. *If $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ and $\mathcal{Q} =< \tilde{\zeta}_M, \tilde{\zeta}_{NM} >$ are any two interval-valued Pythagorean fuzzy subsemihypergroups of $\mathscr{T}^\star$ then $\mathcal{P}\cap\mathcal{Q}$ is also an interval-valued Pythagorean fuzzy subsemihypergroup of $\mathscr{T}^\star$.*

Proof. Consider for $f \in \mathscr{T}^\star$

$$\begin{aligned}
\min\{\tilde{\delta}_M \cap \tilde{\zeta}_M(u), \tilde{\delta}_M \cap \tilde{\zeta}_M(v)\} &= \min\Big\{\min\{\tilde{\delta}_M(u), \tilde{\zeta}_M(u)\}, \min\{\tilde{\delta}_M(v), \tilde{\zeta}_M(v)\}\Big\} \\
&= \min\Big\{\min\{\tilde{\delta}_M(u), \tilde{\delta}_M(v)\}, \min\{\tilde{\zeta}_M(u), \tilde{\zeta}_M(v)\}\Big\} \\
&\leq \min\Big\{\inf_{f\in u\star v} \tilde{\delta}_M(f), \inf_{f\in u\star v} \tilde{\zeta}_M(f)\Big\} \\
&\leq \inf_{f\in u\star v}\Big\{\min\{\tilde{\delta}_M(f), \tilde{\zeta}_M(f)\}\Big\} \\
&\leq \inf_{f\in u\star v} \tilde{\delta}_M \cap \tilde{\zeta}_M(f)\ \forall\, u, v \in \mathscr{T}^\star.
\end{aligned}$$

Now consider for $f \in \mathscr{T}^\star$

$$\begin{aligned}
&\max\{\tilde{\delta}_{NM} \cap \tilde{\zeta}_{NM}(u), \tilde{\delta}_{NM} \cap \tilde{\zeta}_{NM}(v)\} \\
&\quad = \max\Big\{\max\{\tilde{\delta}_{NM}(u), \tilde{\zeta}_{NM}(u)\}, \max\{\tilde{\delta}_{NM}(v), \tilde{\zeta}_{NM}(v)\}\Big\} \\
&\quad = \max\Big\{\max\{\tilde{\delta}_{NM}(u), \tilde{\delta}_{NM}(v)\}, \max\{\tilde{\zeta}_{NM}(u), \tilde{\zeta}_{NM}(v)\}\Big\}
\end{aligned}$$

$$\leq \max\left\{\sup_{f\in u\star v}\tilde{\delta}_{NM}(f),\ \sup_{f\in u\star v}\tilde{\zeta}_{NM}(f)\right\}$$
$$\geq \sup_{f\in u\star v}\left\{\max\{\tilde{\delta}_{NM}(f),\tilde{\zeta}_{NM}(f)\}\right\}$$
$$\geq \sup_{f\in u\star v}\tilde{\delta}_{NM}\cup\tilde{\zeta}_{NM}(f)\ \forall\, u,v\in\mathscr{T}^{\star}.$$

Thus $\mathcal{P}\cap\mathcal{Q}$ is an interval-valued Pythagorean fuzzy subsemihypergroup $\mathscr{T}^{\star}$. □

Definition 3.3. Let $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ be an interval-valued Pythagorean fuzzy set in $\mathscr{T}^{\star}$. For any $\tilde{\alpha}\in D[0,1]$ and $\tilde{\beta}\in D[0,1]$, we define $(\tilde{\alpha},\tilde{\beta})$-level set of $\mathcal{P}$ as
$\mathcal{P}^{(\tilde{\alpha},\tilde{\beta})} =< \tilde{\delta}_M^{\tilde{\alpha}}, \tilde{\delta}_{NM}^{\tilde{\beta}} >$
where $\tilde{\delta}_M^{\tilde{\alpha}} = \{x\in\mathscr{T}^{\star} : \tilde{\delta}_M(x)\geq\tilde{\alpha}\}$ and
$\tilde{\delta}_{NM}^{\tilde{\beta}} = \{x\in\mathscr{T}^{\star} : \tilde{\delta}_{NM}(x)\leq\tilde{\beta}\}$ for all $x\in\mathscr{T}^{\star}$.

Theorem 3.2. *Let $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ be an interval-valued Pythagorean fuzzy set of $\mathscr{T}^{\star}$. For $\tilde{\alpha},\tilde{\beta}\in D[0,1]$, the $(\tilde{\alpha},\tilde{\beta})-$level cut of $\mathcal{P}(\mathcal{P}^{(\tilde{\alpha},\tilde{\beta})})$ is a subsemihypergroup then $\mathcal{P}$ is an interval-valued Pythagorean fuzzy subsemihypergroup of $\mathscr{T}^{\star}$.*

Proof. Let $\tilde{\delta}_M^{\tilde{\alpha}}$ be a subsemihypergroup of $\mathscr{T}^{\star}$ then for $u,v\in\tilde{\delta}_M^{\tilde{\alpha}}$ we have

$$u\star v\in\tilde{\delta}_M^{\tilde{\alpha}} \tag{3.1}$$

Suppose if $\min\{\tilde{\delta}_M(u),\tilde{\delta}_M(v)\}\not\leq\inf_{f\in u\star v}\tilde{\delta}_M(f)$ then we have
$\min\{\tilde{\delta}_M(u),\tilde{\delta}_M(v)\} > \inf_{f\in u\star v}\tilde{\delta}_M(f)$.
Then there exists some $t_0\in D[0,1]$ such that
$\min\{\tilde{\delta}_M(u),\tilde{\delta}_M(v)\} > t_0 > \inf_{f\in u\star v}\tilde{\delta}_M(f)$. This implies that
$\min\{\tilde{\delta}_M(u),\tilde{\delta}_M(v)\} > t_0$ and $\inf_{f\in u\star v}\tilde{\delta}_M(f) < t_0$.
$\Longrightarrow \min\{\tilde{\delta}_M(u),\tilde{\delta}_M(v)\} > t_0$ and $\tilde{\delta}_M(f) < t_0$ for $f\in u\star v$
$\Longrightarrow u\star v\notin\tilde{\delta}_M^{\tilde{\alpha}}$ and either $u\in\tilde{\delta}_M^{\tilde{\alpha}}$ or $v\in\tilde{\delta}_M^{\tilde{\alpha}}$.
This is a contradiction to Equation (3.1). Hence $\min\{\tilde{\delta}_M(u),\tilde{\delta}_M(v)\}\leq\inf_{f\in u\star v}\tilde{\delta}_M(f)$.

Now, Since $\tilde{\delta}_{NM}^{\tilde{\beta}}$ is a subsemihypergroup then for $u,v\in\tilde{\delta}_{NM}^{\tilde{\beta}}$ we have

$$u\star v\in\tilde{\delta}_{NM}^{\tilde{\beta}} \tag{3.2}$$

Suppose if $\max\{\tilde{\delta}_{NM}(u),\tilde{\delta}_{NM}(v)\}\not\geq\sup_{f\in u\star v}\tilde{\delta}_{NM}(f)$ then we have
$\max\{\tilde{\delta}_{NM}(u),\tilde{\delta}_{NM}(v)\} < \sup_{f\in u\star v}\tilde{\delta}_{NM}(f)$.
Then there exists some $t_1\in D[0,1]$ such that
$\max\{\tilde{\delta}_{NM}(u),\tilde{\delta}_{NM}(v)\} < t_1 < \sup_{f\in u\star v}\tilde{\delta}_{NM}(f)$. This implies that
$\max\{\tilde{\delta}_{NM}(u),\tilde{\delta}_{NM}(v)\} < t_1$ and $\sup_{f\in u\star v}\tilde{\delta}_{NM}(f) > t_1$
$\Longrightarrow \max\{\tilde{\delta}_{NM}(u),\tilde{\delta}_{NM}(v)\} < t_1$ and $\tilde{\delta}_{NM}(f) > t_1$ for $f\in u\star v$
$\Longrightarrow u\star v\notin\tilde{\delta}_{NM}^{\tilde{\beta}}$ and either $u\in\tilde{\delta}_{NM}^{\tilde{\beta}}$ or $v\in\tilde{\delta}_{NM}^{\tilde{\beta}}$.
Which is a contradiction to Equation (3.2).
Hence $\max\{\tilde{\delta}_{NM}(u),\tilde{\delta}_{NM}(v)\}\geq\sup_{f\in u\star v}\tilde{\delta}_{NM}(f)$.

□

4. Interval-valued Pythagorean fuzzy weak bi-hyperideals of hypersemigroups

In this section we introduce the concept of interval-valued Pythagorean fuzzy weak bi-hyperideals of hypersemigroups.

Definition 4.1. A subset W of $\mathscr{T}^\star$ is said to be a weak bi-hyperideal if $W \star W \star W \subseteq W$.

Definition 4.2. An interval-valued Pythagorean fuzzy subsemihypergroup $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ is said to be an interval-valued Pythagorean fuzzy weak bi-hyperideal of $\mathscr{T}^\star$ if for $f \in \mathscr{T}^\star$ we have
(i) $\min\{\tilde{\delta}_M(u), \tilde{\delta}_M(a), \tilde{\delta}_M(v)\} \leq \inf\limits_{f \in u\star a\star v} \tilde{\delta}_M(f)$ and
(ii) $\max\{\tilde{\delta}_{NM}(u), \tilde{\delta}_{NM}(a), \tilde{\delta}_{NM}(v)\} \geq \sup\limits_{f \in u\star a\star v} \tilde{\delta}_{NM}(f)$ for all $a, u, v \in \mathscr{T}^\star$.

Example 4.3. Let $\mathscr{T}^\star = \{p, q, r, s, t\}$ be a hypersemigroup with Cayley table:

$\bullet$	x	y	z
x	$\{z\}$	$\{z\}$	$\{z\}$
y	$\{x, z\}$	$\{z\}$	$\{z\}$
z	$\{z\}$	$\{z\}$	$\{z\}$

Define an interval-valued Pythagorean fuzzy set $\mathcal{P}$ as

$\mathcal{P}$	$\tilde{\delta}_M$	$\tilde{\delta}_{NM}$
x	$[0.7, 0.8]$	$[0.1, 0.2]$
y	$[0.5, 0.6]$	$[0.4, 0.6]$
z	$[0.9, 1]$	$[0.3, 0.5]$

By routine calculation we can easily check that $\mathcal{P}$ is an interval-valued Pythagorean fuzzy weak bi-hyperideal of $\mathscr{T}^\star$.

Theorem 4.1. *Let $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ and $\mathcal{Q} =< \tilde{\zeta}_M, \tilde{\zeta}_{NM} >$ be any two interval-valued Pythagorean fuzzy weak bi-hyperideals of $\mathscr{T}^\star$ then $\mathcal{P} \cap \mathcal{Q}$ is also an interval-valued Pythagorean fuzzy weak bi-hyperideal of $\mathscr{T}^\star$.*

Proof. Let $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ and $\mathcal{Q} =< \tilde{\zeta}_M, \tilde{\zeta}_{NM} >$ be two interval-valued Pythagorean fuzzy weak bi-hyperideals of $\mathscr{T}^\star$. Consider for $f \in \mathscr{T}^\star$

$$\begin{aligned}
&\min\{\tilde{\delta}_M \cap \tilde{\zeta}_M(u), \tilde{\delta}_M \cap \tilde{\zeta}_M(a), \tilde{\delta}_M \cap \tilde{\zeta}_M(v)\} \\
&\quad = \min\left\{\min\{\tilde{\delta}_M(u), \tilde{\zeta}_M(u)\}, \min\{\tilde{\delta}_M(a), \tilde{\zeta}_M(a)\}, \min\{\tilde{\delta}_M(v), \tilde{\zeta}_M(v)\}\right\} \\
&\quad = \min\left\{\min\{\tilde{\delta}_M(u), \tilde{\delta}_M(a), \tilde{\delta}_M(v)\}, \min\{\tilde{\zeta}_M(u), \tilde{\zeta}_M(a), \tilde{\zeta}_M(v)\}\right\} \\
&\quad \leq \min\left\{\inf_{f \in u\star a\star v} \tilde{\delta}_M(f), \inf_{f \in u\star a\star v} \tilde{\zeta}_M(f)\right\} \\
&\quad \leq \inf_{f \in u\star a\star v} \left\{\min\{\tilde{\delta}_M(f), \tilde{\zeta}_M(f)\}\right\} \\
&\quad \leq \inf_{f \in u\star v} \tilde{\delta}_M \cap \tilde{\zeta}_M(f) \;\forall\, u, v \in \mathscr{T}^\star.
\end{aligned}$$

Now consider for $f \in \mathscr{T}^\star$

$\max\{\tilde{\delta}_{NM} \cap \tilde{\zeta}_{NM}(u), \tilde{\delta}_{NM} \cap \tilde{\zeta}_{NM}(a), \tilde{\delta}_{NM} \cap \tilde{\zeta}_{NM}(v)\}$
$= \max\Big\{\max\{\tilde{\delta}_{NM}(u), \tilde{\zeta}_{NM}(u)\}, \max\{\tilde{\delta}_{NM}(a), \tilde{\zeta}_{NM}(a)\}, \max\{\tilde{\delta}_{NM}(v), \tilde{\zeta}_{NM}(v)\}\Big\}$
$= \max\Big\{\max\{\tilde{\delta}_{NM}(u), \tilde{\delta}_{NM}(a), \tilde{\delta}_{NM}(v)\}, \max\{\tilde{\zeta}_{NM}(u), \tilde{\zeta}_{NM}(a), \tilde{\zeta}_{NM}(v)\}\Big\}$
$\geq \max\left\{\sup_{f\in u\star a\star v} \tilde{\delta}_{NM}(f), \sup_{f\in u\star a\star v} \tilde{\zeta}_{NM}(f)\right\}$
$\geq \sup_{f\in u\star a\star v}\Big\{\max\{\tilde{\delta}_{NM}(f), \tilde{\zeta}_{NM}(f)\}\Big\}$
$\geq \sup_{f\in u\star v} \tilde{\delta}_{NM} \cap \tilde{\zeta}_{NM}(f) \; \forall\, u, v \in \mathscr{T}^\star.$
Thus $\mathcal{P} \cap \mathcal{Q}$ is an interval-valued Pythagorean fuzzy weak bi-hyperideal of $\mathscr{T}^\star$. □

Theorem 4.2. *Let $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ be an interval-valued Pythagorean fuzzy set of $\mathscr{T}^\star$. For $\tilde{\alpha}, \tilde{\beta} \in D[0,1]$, the $(\tilde{\alpha}, \tilde{\beta})-$level cut of $\mathcal{P}$ is a weak bi-hyperideal of $\mathscr{T}^\star$ then $\mathcal{P}$ is an interval-valued Pythagorean fuzzy weak bi- hyperideal of $\mathscr{T}^\star$.*

Proof. Let $\tilde{\delta}_M^{\tilde{\alpha}}$ be a weak bi-hyperideal of $\mathscr{T}^\star$. By Theorem 3.2 $\mathcal{P}$ is an interval-valued Pythagorean fuzzy subsemihypergroup. Then for $a, u, v \in \tilde{\delta}_M^{\tilde{\alpha}}$ we have

$$u \star a \star v \in \tilde{\delta}_M^{\tilde{\alpha}} \tag{4.1}$$

Suppose if $\min\{\tilde{\delta}_M(u), \tilde{\delta}_M(a), \tilde{\delta}_M(v)\} \nleq \inf_{f\in u\star a\star v} \tilde{\delta}_M(f)$ then we have

$\min\{\tilde{\delta}_M(u), \tilde{\delta}_M(a), \tilde{\delta}_M(v)\} > \inf_{f\in u\star a\star v} \tilde{\delta}_M(f).$

Then there exists some $t_\phi \in D[0,1]$ such that

$\min\{\tilde{\delta}_M(u), \tilde{\delta}_M(a), \tilde{\delta}_M(v) > t_\phi > \inf_{f\in u\star a\star v} \tilde{\delta}_M(f)$. This implies that

$\min\{\tilde{\delta}_M(u), \tilde{\delta}_M(a), \tilde{\delta}_M(v) > t_\phi$ and $\inf_{f\in u\star a\star v} \tilde{\delta}_M(f) < t_\phi$.

$\Longrightarrow \min\{\tilde{\delta}_M(u), \tilde{\delta}_M(a), \tilde{\delta}_M(v)\} > t_\phi$ and $\tilde{\delta}_M(f) < t_\phi$ for $f \in u \star a \star v$
$\Longrightarrow u \star a \star v \notin \tilde{\delta}_M^{\tilde{\alpha}}$ and either $u \in \tilde{\delta}_M^{\tilde{\alpha}}$ or $a \in \tilde{\delta}_M^{\tilde{\alpha}}$ or $v \in \tilde{\delta}_M^{\tilde{\alpha}}$.
This is a contradiction to Equation (4.1).
Hence $\min\{\tilde{\delta}_M(u), \tilde{\delta}_M(a), \tilde{\delta}_M(v)\} \leq \inf_{f\in u\star a\star v} \tilde{\delta}_M(f)$.

Now, since $\tilde{\delta}_{NM}^{\tilde{\beta}}$ is a weak bi-hyperideal then for $a, u, v \in \tilde{\delta}_{NM}^{\tilde{\beta}}$ we have

$$u \star a \star v \in \tilde{\delta}_{NM}^{\tilde{\beta}} \tag{4.2}$$

Suppose if $\max\{\tilde{\delta}_{NM}(u), \tilde{\delta}_{NM}(a), \tilde{\delta}_{NM}(v)\} \ngeq \sup_{f\in u\star a\star v} \tilde{\delta}_{NM}(f)$ then we have

$\max\{\tilde{\delta}_{NM}(u), \tilde{\delta}_{NM}(a), \tilde{\delta}_{NM}(v)\} < \sup_{f\in u\star a\star v} \tilde{\delta}_{NM}(f).$

Then there exists some $t_\xi \in D[0,1]$ such that

$\max\{\tilde{\delta}_{NM}(u), \tilde{\delta}_{NM}(a), \tilde{\delta}_{NM}(v)\} < t_\xi < \sup_{f\in u\star a\star v} \tilde{\delta}_{NM}(f)$. This implies that

$\max\{\tilde{\delta}_{NM}(u), \tilde{\delta}_{NM}(a), \tilde{\delta}_{NM}(v)\} < t_\xi$ and $\sup_{f\in u\star a\star v} \tilde{\delta}_{NM}(f) > t_\xi$

$\Longrightarrow \max\{\tilde{\delta}_{NM}(u), \tilde{\delta}_{NM}(a), \tilde{\delta}_{NM}(v)\} < t_\xi$ and $\tilde{\delta}_{NM}(f) > t_\xi$ for $f \in u \star a \star v$
$\Longrightarrow u \star a \star v \notin \tilde{\delta}_{NM}^{\tilde{\beta}}$ and either $u \in \tilde{\delta}_{NM}^{\tilde{\beta}}$ or $a \in \tilde{\delta}_{NM}^{\tilde{\beta}}$ or $v \in \tilde{\delta}_{NM}^{\tilde{\beta}}$.
This is a contradiction to Equation (4.2).
Hence $\max\{\tilde{\delta}_{NM}(u), \tilde{\delta}_{NM}(a), \tilde{\delta}_{NM}(v)\} \geq \sup_{f\in u\star a\star v} \tilde{\delta}_{NM}(f)$.

□

5. Cartesian product of interval-valued Pythagorean fuzzy weak bi-hyperideals in hypersemigroups

In this section we discuss some properties of cartesian product of interval-valued fuzzy weak bi-hyperideals in hypersemigroups.

Definition 5.1. Let $\mathscr{T}^\star$ be a hypersemigroup. The cartesian product of two interval-valued Pythagorean fuzzy sets $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ and $\mathcal{Q} =< \tilde{\zeta}_M, \tilde{\zeta}_{NM} >$ is denoted by $\mathcal{P} \times \mathcal{Q}$ and is defined by
$\mathcal{P} \times \mathcal{Q} = \{< (p,q)/(\tilde{\delta}_M \times \tilde{\zeta}_M(p,q), \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(p,q)) > \ for\ all\ (p,q) \in \mathscr{T}^\star \times \mathscr{T}^\star\}$
Where $\tilde{\delta}_M \times \tilde{\zeta}_M(p,q) = \min\{\tilde{\delta}_M(p), \tilde{\zeta}_M(q)\}$,
$\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(p,q) = \max\{\tilde{\delta}_{NM}(p), \tilde{\zeta}_{NM}(q)\}$ for all $(p,q) \in \mathscr{T}^\star \times \mathscr{T}^\star$.

Example 5.2. Let us define two interval-valued Pythagorean fuzzy sets $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ and $\mathcal{Q} =< \tilde{\zeta}_M, \tilde{\zeta}_{NM} >$ as follows:

$\mathcal{P}$	$\tilde{\delta}_M$	$\tilde{\delta}_{NM}$
c	$[0.7, 0.8]$	$[0.3, 0.4]$
d	$[0.6, 0.9]$	$[0.1, 0.2]$
e	$[0, 0.2]$	$[0, 0.1]$

$\mathcal{Q}$	$\tilde{\zeta}_M$	$\tilde{\zeta}_{NM}$
c	$[0.4, 0.8]$	$[0.3, 0.4]$
d	$[0.6, 0.9]$	$[0.3, 0.4]$
e	$[0.4, 0.6]$	$[0.5, 0.7]$

Then $\mathcal{P}\times\mathcal{Q} =< c/([0.4, 0.8], [0.3, 0.4]), d/([0.6, 0.9], [0.3, 0.4]), e/([0.4, 0.6], [0, 0.7]) >$.

Theorem 5.1. *Let $\mathscr{T}^\star$ be a hypersemigroup. If interval-valued Pythagorean fuzzy sets $\mathcal{P} =< \tilde{\delta}_M, \tilde{\delta}_{NM} >$ and $\mathcal{Q} =< \tilde{\zeta}_M, \tilde{\zeta}_{NM} >$ are interval-valued Pythagorean fuzzy weak bi-hyperideals of $\mathscr{T}^\star$ then $\mathcal{P} \times \mathcal{Q}$ is also an interval-valued Pythagorean fuzzy bi-hyperideal of $\mathscr{T}^\star \times \mathscr{T}^\star$.*

Proof. Let $\mathcal{P}$ and $\mathcal{Q}$ are interval-valued Pythagorean fuzzy weak bi-hyperideals of $\mathscr{T}^\star$. Consider for $(u_1, u_2), (v_1, v_2), (f_1, f_2) \in \mathscr{T}^\star \times \mathscr{T}^\star$,

$$
\begin{aligned}
&\min\{\tilde{\delta}_M \times \tilde{\zeta}_M(u_1, u_2), \tilde{\delta}_M \times \tilde{\zeta}_M(v_1, v_2)\} \\
&\quad = \min\left\{\min\{\tilde{\delta}_M(u_1), \tilde{\zeta}_M(u_2)\}, \min\{\tilde{\delta}_M(v_1), \tilde{\zeta}_M(v_2)\}\right\} \\
&\quad = \min\left\{\min\{\tilde{\delta}_M(u_1), \tilde{\delta}_M(v_1)\}, \min\{\tilde{\zeta}_M(u_2), \tilde{\zeta}_M(v_2)\}\right\} \\
&\quad \leq \min\left\{\inf_{f_1 \in u_1 v_1} \tilde{\delta}_M(f_1), \sup_{f_2 \in u_2 v_2} \tilde{\zeta}_M(f_2)\right\} \\
&\quad \leq \inf_{\substack{f_1 \in u_1 v_1, \\ f_2 \in u_2 v_2}} \left\{\min\{\tilde{\delta}_M(f_1), \tilde{\zeta}_M(f_2)\}\right\} \\
&\quad \leq \inf_{(f_1, f_2) \in (u_1 v_1, u_2 v_2)} \tilde{\delta}_M \times \tilde{\zeta}_M(f_1, f_2) \\
&\quad \leq \inf_{(f_1, f_2) \in (u_1, u_2)(v_1, v_2)} \tilde{\delta}_M \times \tilde{\zeta}_M(f_1, f_2),
\end{aligned}
$$

$$\begin{aligned}
&\max\{\tilde{\delta}_{NM}\times\tilde{\zeta}_{NM}(u_1,u_2),\tilde{\delta}_{NM}\times\tilde{\zeta}_{NM}(v_1,v_2)\}\\
&\qquad=\max\Big\{\max\{\tilde{\delta}_{NM}(u_1),\tilde{\zeta}_{NM}(u_2)\},\max\{\tilde{\delta}_{NM}(v_1),\tilde{\zeta}_{NM}(v_2)\}\Big\}\\
&\qquad=\max\Big\{\max\{\tilde{\delta}_{NM}(u_1),\tilde{\delta}_{NM}(v_1)\},\max\{\tilde{\zeta}_{NM}(u_2),\tilde{\zeta}_{NM}(v_2)\}\Big\}\\
&\qquad\geq\max\left\{\sup_{f_1\in u_1v_1}\tilde{\delta}_{NM}(f_1),\sup_{f_2\in u_2v_2}\tilde{\zeta}_{NM}(f_1)\right\}\\
&\qquad\geq\sup_{\substack{f_1\in u_1v_1,\\ f_2\in u_2v_2}}\Big\{\max\{\tilde{\delta}_{NM}(f_1),\tilde{\zeta}_{NM}(f_2)\}\Big\}\\
&\qquad\geq\sup_{(f_1,f_2)\in(u_1v_1,u_2v_2)}\tilde{\delta}_{NM}\times\tilde{\zeta}_{NM}(f_1,f_2)\\
&\qquad\geq\sup_{(f_1,f_2)\in(u_1,u_2)(v_1,v_2)}\tilde{\delta}_{NM}\times\tilde{\zeta}_{NM}(f_1,f_2)
\end{aligned}$$

Therefore $\mathcal{P}\times\mathcal{Q}$ is an interval-valued Pythagorean fuzzy subsemihypergroup of $\mathscr{T}^\star\times\mathscr{T}^\star$.
Now,

$$\begin{aligned}
&\min\{\tilde{\delta}_{M}\times\tilde{\zeta}_{M}(u_1,u_2),\tilde{\delta}_{M}\times\tilde{\zeta}_{M}(a_1,a_2),\tilde{\delta}_{M}\times\tilde{\zeta}_{M}(v_1,v_2)\}\\
&=\min\Big\{\min\{\tilde{\delta}_{M}(u_1),\tilde{\zeta}_{M}(u_2)\},\min\{\tilde{\delta}_{M}(a_1),\tilde{\zeta}_{M}(a_2)\},\min\{\tilde{\delta}_{M}(v_1),\tilde{\zeta}_{M}(v_2)\}\Big\}\\
&=\min\Big\{\min\{\tilde{\delta}_{M}(u_1),\tilde{\delta}_{M}(a_1),\tilde{\delta}_{M}(v_1)\},\min\{\tilde{\zeta}_{M}(u_2),\tilde{\zeta}_{M}(a_2),\tilde{\zeta}_{M}(v_2)\}\Big\}\\
&\leq\min\left\{\inf_{f_1\in u_1a_1v_1}\tilde{\delta}_{M}(f_1),\inf_{f_2\in u_2a_2v_2}\tilde{\zeta}_{M}(f_2)\right\}\\
&\leq\inf_{\substack{f_1\in u_1a_1v_1,\\ f_2\in u_2a_2v_2}}\Big\{\min\{\tilde{\delta}_{M}(f_1),\tilde{\zeta}_{M}(f_2)\}\Big\}\\
&\leq\inf_{(f_1,f_2)\in(u_1a_1v_1,u_2a_2v_2)}\tilde{\delta}_{M}\times\tilde{\zeta}_{M}(f_1,f_2)\\
&\leq\inf_{(f_1,f_2)\in(u_1,u_2)(a_1,a_2)(v_1,v_2)}\tilde{\delta}_{M}\times\tilde{\zeta}_{M}(f_1,f_2)\text{ and}
\end{aligned}$$

$$\begin{aligned}
&\max\{\tilde{\delta}_{NM}\times\tilde{\zeta}_{NM}(u_1,u_2),\tilde{\delta}_{NM}\times\tilde{\zeta}_{NM}(a_1,a_2),\tilde{\delta}_{NM}\times\tilde{\zeta}_{NM}(v_1,v_2)\}\\
&=\max\Big\{\max\{\tilde{\delta}_{NM}(u_1),\tilde{\zeta}_{NM}(u_2)\},\max\{\tilde{\delta}_{NM}(a_1),\tilde{\zeta}_{NM}(a_2)\},\max\{\tilde{\delta}_{NM}(v_1),\tilde{\zeta}_{NM}(v_2)\}\Big\}\\
&=\max\Big\{\max\{\tilde{\delta}_{NM}(u_1),\tilde{\delta}_{NM}(a_1),\tilde{\delta}_{NM}(v_1)\},\max\{\tilde{\zeta}_{NM}(u_2),\tilde{\zeta}_{NM}(a_2),\tilde{\zeta}_{NM}(v_2)\}\Big\}\\
&\geq\max\left\{\sup_{f_1\in u_1a_1v_1}\tilde{\delta}_{NM}(f_1),\sup_{f_2\in u_2a_2v_2}\tilde{\zeta}_{NM}(f_2)\right\}\\
&\geq\sup_{\substack{f_1\in u_1a_1v_1,\\ f_2\in u_2a_2v_2}}\Big\{\max\{\tilde{\delta}_{NM}(f_1),\tilde{\zeta}_{NM}(f_2)\}\Big\}\\
&\geq\sup_{(f_1,f_2)\in(u_1a_1v_1,u_2a_2v_2)}\tilde{\delta}_{NM}\times\tilde{\zeta}_{NM}(f_1,f_2)\\
&\geq\sup_{(f_1,f_2)\in(u_1,u_2)(a_1,a_2)(v_1,v_2)}\tilde{\delta}_{NM}\times\tilde{\zeta}_{NM}(f_1,f_2)
\end{aligned}$$

Therefore $\mathcal{P}\times\mathcal{Q}$ is an interval-valued Pythagorean fuzzy weak bi-hyperideal of $\mathscr{T}^\star\times\mathscr{T}^\star$. □

Theorem 5.2. *Let $\mathscr{T}^\star$ be a hypersemigroup. If the interval-valued Pythagorean fuzzy sets $\mathcal{P}=<\tilde{\delta}_M,\tilde{\delta}_{NM}>$ and $\mathcal{Q}=<\tilde{\zeta}_M,\tilde{\zeta}_{NM}>$ are interval-valued Pythagorean fuzzy weak bi-hyperideals of $\mathscr{T}^\star$ then the $(\tilde{\alpha},\tilde{\beta})-$level set of $\mathcal{P}\times\mathcal{Q}$ is also an interval-valued Pythagorean fuzzy weak bi-hyperideal of $\mathscr{T}^\star\times\mathscr{T}^\star$.*

Proof. Let $(u_1,u_2),(v_1,v_2),(f_1,f_2)\in\mathscr{T}^\star\times\mathscr{T}^\star$ such that
$(u_1,u_2),(v_1,v_2)\in(\tilde{\delta}_M\times\tilde{\zeta}_M)^{\tilde{\alpha}}$. Which implies that $\tilde{\delta}_M\times\tilde{\zeta}_M(u_1,u_2)\geq\tilde{\alpha}$ and

$\tilde{\delta}_M \times \tilde{\zeta}_M(v_1, v_2) \geq \tilde{\alpha}$.
Since $\min\{\tilde{\delta}_M \times \tilde{\zeta}_M(u_1, u_2), \tilde{\delta}_M \times \tilde{\zeta}_M(v_1, v_2)\} \leq \inf_{(f_1,f_2)\in(u_1,u_2)(v_1,v_2)} \tilde{\delta}_M \times \tilde{\zeta}_M(f_1, f_2)$
$\Longrightarrow \inf_{(f_1,f_2)\in(u_1,u_2)(v_1,v_2)} \tilde{\delta}_M \times \tilde{\zeta}_M(f_1, f_2) \geq \min\{\tilde{\alpha}, \tilde{\alpha}\} \geq \tilde{\alpha}$
$\Longrightarrow \tilde{\delta}_M \times \tilde{\zeta}_M(f_1, f_2) \geq \tilde{\alpha}$ for $(f_1, f_2) \in (u_1, u_2)(v_1, v_2)$
$\Longrightarrow (f_1, f_2) \in (\tilde{\delta}_M \times \tilde{\zeta}_M)^{\tilde{\alpha}}$ for $(f_1, f_2) \in (u_1, u_2)(v_1, v_2)$
$\Longrightarrow (u_1, u_2)(v_1, v_2) \in (\tilde{\delta}_M \times \tilde{\zeta}_M)^{\tilde{\alpha}}$. Hence $(\tilde{\delta}_M \times \tilde{\zeta}_M)^{\tilde{\alpha}}$ is a subsemihypergroup of $\mathscr{T}^\star$.

Now let $(u_1, u_2), (v_1, v_2) \in (\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM})^{\tilde{\beta}}$. Which implies that $\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(u_1, u_2) \leq \tilde{\beta}$ and $\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(v_1, v_2) \leq \tilde{\beta}$. Since
$\max\{\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(u_1, u_2), \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(v_1, v_2)\} \geq \sup_{(f_1,f_2)\in(u_1,u_2)(v_1,v_2)} \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(f_1, f_2)$
$\Longrightarrow \sup_{(f_1,f_2)\in(u_1,u_2)(v_1,v_2)} \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(f_1, f_2) \leq \max\{\tilde{\beta}, \tilde{\beta}\} \leq \tilde{\beta}$
$\Longrightarrow \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(f_1, f_2) \leq \tilde{\beta}$ for $(f_1, f_2) \in (u_1, u_2)(v_1, v_2)$
$\Longrightarrow (f_1, f_2) \in (\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM})^{\tilde{\beta}}$ for $(f_1, f_2) \in (u_1, u_2)(v_1, v_2)$
$\Longrightarrow (u_1, u_2)(v_1, v_2) \in (\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM})^{\tilde{\beta}}$. Hence $(\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM})^{\tilde{\beta}}$ is a subsemihypergroup.

Let $(u_1, u_2), (v_1, v_2), (a_1, a_2), (f_1, f_2) \in \mathscr{T}^\star \times \mathscr{T}^\star$ such that $(u_1, u_2), (v_1, v_2), (a_1, a_2) \in (\tilde{\delta}_M \times \tilde{\zeta}_M)^{\tilde{\alpha}}$. Which implies that $\tilde{\delta}_M \times \tilde{\zeta}_M(u_1, u_2) \geq \tilde{\alpha}$, $\tilde{\delta}_M \times \tilde{\zeta}_M(a_1, a_2) \geq \tilde{\alpha}$ and $\tilde{\delta}_M \times \tilde{\zeta}_M(v_1, v_2) \geq \tilde{\alpha}$. Since
$\min\{\tilde{\delta}_M \times \tilde{\zeta}_M(u_1, u_2), \tilde{\delta}_M \times \tilde{\zeta}_M(a_1, a_2), \tilde{\delta}_M \times \tilde{\zeta}_M(v_1, v_2)\}$
$\qquad \leq \inf_{(f_1,f_2)\in(u_1,u_2)(a_1,a_2)(v_1,v_2)} \tilde{\delta}_M \times \tilde{\zeta}_M(f_1, f_2)$
$\Longrightarrow \inf_{(f_1,f_2)\in(u_1,u_2)(a_1,a_2)(v_1,v_2)} \tilde{\delta}_M \times \tilde{\zeta}_M(f_1, f_2) \geq \min\{\tilde{\alpha}, \tilde{\alpha}\} \geq \tilde{\alpha}$
$\Longrightarrow \tilde{\delta}_M \times \tilde{\zeta}_M(f_1, f_2) \geq \tilde{\alpha}$ for $(f_1, f_2) \in (u_1, u_2)(a_1, a_2)(v_1, v_2)$
$\Longrightarrow (f_1, f_2) \in (\tilde{\delta}_M \times \tilde{\zeta}_M)^{\tilde{\alpha}}$ for $(f_1, f_2) \in (u_1, u_2)(a_1, a_2)(v_1, v_2)$
$\Longrightarrow (u_1, u_2)(a_1, a_2)(v_1, v_2) \in (\tilde{\delta}_M \times \tilde{\zeta}_M)^{\tilde{\alpha}}$. Hence $(\tilde{\delta}_M \times \tilde{\zeta}_M)^{\tilde{\alpha}}$ is a weak bi-hyperideal.

Let $(u_1, u_2), (v_1, v_2), (a_1, a_2), (f_1, f_2) \in \mathscr{T}^\star \times \mathscr{T}^\star$ such that $(u_1, u_2), (v_1, v_2), (a_1, a_2) \in \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}^{\tilde{\beta}}$. Which implies that $\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(u_1, u_2) \leq \tilde{\beta}$, $\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(a_1, a_2) \leq \tilde{\beta}$ and $\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(v_1, v_2) \leq \tilde{\beta}$. Since
$\max\{\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(u_1, u_2), \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(a_1, a_2), \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(v_1, v_2)\}$
$\qquad \geq \sup_{(f_1,f_2)\in(u_1,u_2)(a_1,a_2)(v_1,v_2)} \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(f_1, f_2)$
$\Longrightarrow \sup_{(f_1,f_2)\in(u_1,u_2)(a_1,a_2)(v_1,v_2)} \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(f_1, f_2) \leq \max\{\tilde{\beta}, \tilde{\beta}\} \leq \tilde{\beta}$
$\Longrightarrow \tilde{\delta}_{NM} \times \tilde{\zeta}_{NM}(f_1, f_2) \leq \tilde{\beta}$ for $(f_1, f_2) \in (u_1, u_2)(a_1, a_2)(v_1, v_2)$
$\Longrightarrow (f_1, f_2) \in (\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM})^{\tilde{\beta}}$ for $(f_1, f_2) \in (u_1, u_2)(a_1, a_2)(v_1, v_2)$
$\Longrightarrow (u_1, u_2)(a_1, a_2)(v_1, v_2) \in (\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM})^{\tilde{\beta}}$. Hence $(\tilde{\delta}_{NM} \times \tilde{\zeta}_{NM})^{\tilde{\beta}}$ is a weak bi-hyperideal.

□

6. CONCLUSION

In this paper we presented the notion of interval-valued Pythagorean fuzzy subsemihypergroup and interval-valued Pythagorean fuzzy weak bi-hyperideals in hypersemigroups.

We characterized cartesian product of interval-valued Pythagorean fuzzy weak bi-hyperideals. We investigated some properties with suitable examples. In continuity of this paper, we study interval-valued weak bi $\Gamma-$hyperideals in $\Gamma-$hypersemigroups.

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