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LACUNARY SEQUENCES WITH ALMOST AND STATISTICAL CONVERGENCE

ABDUL HAMID GANIE

ABSTRACT. In this manuscript, our concern is to introduce the new approach of studying the lacunary almost statistical convergence and strongly almost convergence of the generalized difference sequences of fuzzy numbers. Some interesting and basic properties concerning them will be studied.

1. INTRODUCTION AND BACKGROUND

The space of all sequences will be denoted by Υ. Any subspace of Υ is known as sequence space. We denote the set of whole numbers by $\mathbb{N}$; the set of real $\mathbb{R}$ and the set of complex numbers by $\mathbb{C}$, respectively. Also, the spaces of bounded and convergent sequences $\zeta = \{\zeta_n\}_{n=0}^{\infty}$ by l_∞ and c, respectively.

Definition 1.1. We call a sequence ζ to be statistically convergent to a number β if for given ϵ positive and small, we have

$$\lim_r \frac{1}{r}|\{k \leq r : |\zeta_k - \beta| \geq \epsilon|\} = 0, \tag{1.1}$$

and we write S-limit $\zeta = \beta$ or $\zeta_k \to \beta(S)$. Here the set of all statistically convergent sequences will be abbreviated by S.

It was Fast, who gave the study of statistical convergence (see, [9]) and later it was discussed by several others (see, [2, 5, 11, 21, 24, 27]).

Definition 1.2. Let T denote the shift operator on Υ, then the Banach limit B on l_∞ is analysed in [12], [18], [22] and satisfies the following:

$$B(T\zeta) = B(\zeta) \text{ and } B(e) = 1,\ e = (1, 1, 1, ...).$$

Definition 1.3. In [18], a sequence $\zeta = \{\zeta_n\}$ is known as almost convergent sequence if $B(\zeta)$ (Banach limits) are same and is known as F-limit of ζ.

The author in his paper has established the well known following result/remark:

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Remark. *A sequence $\zeta = \{\zeta_n\} \in l_\infty$ is said to almost convergent to $B(\zeta)$ iff*

$$\lim_{m\to\infty} \nu_{mn}(\zeta) = B(\zeta),$$

where, $\nu_{mn}(\zeta) = \frac{1}{m}\sum_{j=0}^{m-1} T^j\zeta_n,\ (T^0 = 0)$ uniformly in $n \geq 0$ and here we shall abbreviate such sequences by f.

It was further analysed and studied by Ganie et al (see, [12], [13]), Nanda (see, [20])), Freedman (see, [10]) and many more.

Definition 1.4. A non-decreasing sequence $\theta = \{b_r\}$ is called a lacunary if $b_0 = 0$ and $h_r = b_r - b_{r-1} \to \infty$ as $r \to \infty$. The intervals computed by θ will be abbreviated by $I_r = (b_{r-1}, b_r]$, and ratios viz., $\frac{b_r}{b_{r-1}}$ will be denoted by q_r. A interesting relationship between $|\sigma_1|$ and the space N_θ (see [7]) is given by

$$N_\theta = \left\{ \xi : \text{for some } \alpha,\ \lim_r \left(\frac{1}{h_r} \sum_{j\in I_r} |\xi_j - \alpha| \right) = 0 \right\}.$$

With this notion, the target of the paper is to analyze a study of convergence called statistical convergence (1.1) in a similar fashion as that of N_θ is related to $|\sigma_1|$.

Definition 1.5. If the lacunary sequence is θ, then the number sequence ξ is said to be S_θ-convergent to η if given $\epsilon > 0$,

$$\lim_r \frac{1}{h_r} |\{k \in I_r : |\xi_k - \eta| \geq \epsilon\}| = 0. \tag{1.2}$$

Here we abbreviate S_θ-limit $\xi = \eta$ or $\xi_k \to \eta(S_\theta)$, and define

$$S_\theta = \{\xi : for\ some\ \eta\ , S_\theta - \lim\ \xi = \eta\}\ ,$$

where limits in (1.1) and (1.2) may be pressout by employing matrix transformations of the characteristic function χ_P of the set

$$P = P(\xi, \eta, \epsilon) = \{r \in \mathbb{N} : |\xi_r - \eta| \geq \epsilon\}\ .$$

Here the limit is $\lim_j (C_1\chi_P)_j = 0$, where C_1 is the Ceşaro mean, the limit in (2) is $\lim_n (C_\theta\chi_K)_n = 0$, where C_θ is the matrix given by

$$C_\theta[n,k] = \begin{cases} \frac{1}{h_r}, & \text{if } k \in I_r, \\ 0, & \text{if } k \notin I_r. \end{cases}$$

In this pattern S_θ-convergence is in fact a part of "A-density convergence" (see, [11]).

It was Zadeh, who gave the notion of fuzzy sets and fuzzy set operations (see, [28]). Later several others have worked on various concepts and applications of such sets viz., fuzzy topological behaviour, its orderings, measures of fuzzy events and its mathematical programming. Matloka (see, [19]) worked on sequences of bounded and convergent sequences of fuzzy numbers and interpret some of their topological premises and has revealed that all convergent sequences of fuzzy numbers is bounded. As a chain it was further discussed by many others (see, [1] - [9], [14], [16], [19], [21], [23], [28]).

Definition 1.6. Let us denote the set of all closed and bounded intervals $X = [c_1, c_2]$ on the real line be denoted by D. Then, for $X, Y \in D$, we define

$$d(X : Y) = \max(|c_1 - d_1|, |c_2 - d_2|),$$

where $X = [c_1, c_2]\,, Y = [d_1, d_2]$.

Definition 1.7. For $J = [0, 1]$, λ-a fuzzy real number is a fuzzy set on $\mathbb{R}$ and is a function $\lambda : \mathbb{R} \to J$ that associates with each real number t, its grade membership $\lambda(t)$.

Definition 1.8. λ is said to be *convex* if

$$\lambda(t) \geq \lambda(s) \wedge \lambda(r) = \min(\lambda(s), \lambda(s)), \text{ where } s < t < r.$$

A fuzzy real number λ is called *normal* if there exists $t_0 \in \mathbb{R}$ such that $\lambda(t_0) = 1$.

Definition 1.9. λ is called *upper semi-continuous* if for each $\varepsilon > 0$, $\lambda^{-1}([0, a+\varepsilon))\, \forall\, a \in J$ and given $\varepsilon > 0$, $\lambda^{-1}([0, a + \varepsilon))$ is open in the usual topology of $\mathbb{R}$.

Here we abbreviate the set of all *upper semi-continuous, normal, convex* fuzzy numbers by $R(J)$. By X^α with for $0 < \alpha \leq 1$, we abbreviate the α-level set of a fuzzy real number λ viz.,

$$\lambda^\alpha = \{t \in \mathbb{R} : \lambda(t) \geq \alpha\}.$$

We call $\bar{b} \in \mathbb{R}(J)$ with $b \in \mathbb{R}$ as follows:

$$\bar{b} = \begin{cases} \bar{b}, & \text{if } t = b, \\ 0, & \text{if } t \neq b. \end{cases}$$

The absolute value $|\lambda|$ of $\lambda \in \mathbb{R}(J)$ is given by (see, [16]):

$$|\lambda|(t) = \begin{cases} \max\{\lambda(t),\ \lambda(-t)\}, & \text{if } t \geq 0, \\ 0, & \text{if } t < 0. \end{cases}$$

Let $\bar{d} : \mathbb{R}(J) \times \mathbb{R}(J) \to \mathbb{R}$ be given by

$$\bar{d}(\lambda, \mu) = \sup_{0 \leq \alpha \leq 1} d(\lambda^\alpha, \mu^\alpha).$$

Then it is trivial that $\bar{d}$ induces a metric on $\mathbb{R}(J)$ (see, [16], [25]). Note that the additive identity and multiplicative identity in $\mathbb{R}(J)$ are respectively abbreviated as $\bar{0}$ and $\bar{1}$.

Definition 1.10. A Fuzzy number is a mapping viz., $\lambda : \mathbb{R}^n \to J$, that is normal, fuzzy convex, upper-semi continuous and the closure of $\{x \in \mathbb{R}^n : \lambda(x) > 0\}$ is compact. These facts show that for every $0 < \alpha \leq 1$, the α-level set $\lambda^\alpha = \{t \in \mathbb{R}^n : \lambda(t) \geq \alpha\}$, is non-void compact convex subset of $\mathbb{R}^n$ having support $\lambda^0 = \{t \in \mathbb{R}^n : \lambda(t) > 0\}$.

By $L(\mathbb{R}^n)$ we shall abbreviate the set of all Fuzzy number. The linear structure of $L(\mathbb{R}^n)$ makes the $\lambda + \mu$ and $b\lambda$, $b \in \mathbb{R}$, with respect of α level sets

$$|\lambda + \mu|^\alpha = |\lambda|^\alpha + |\mu|^\alpha \;\&\; |b\lambda|^\alpha = \lambda|\lambda|^\alpha$$

with $0 \leq \alpha \leq 1$. Now, for $1 \leq q < \infty$, we define

$$d_q(\lambda, \mu) = \left[\int_0^1 \delta_\infty(\lambda^\alpha, \mu^\alpha)^q d\alpha\right]^{\frac{1}{q}},$$

and

$$d_\infty(\lambda, \mu) = \sup_{0 \leq \alpha \leq 1} \delta(\lambda^\alpha, \mu^\alpha),$$

with δ_∞ is as Hausdorff metric. It is obvious that $d_\infty(\lambda, \mu) = \lim_{q \to \infty} d_q(\lambda, \mu)$ and for $q \leq r$, we have $d_q \leq d_r$. Throughout the text, we will denote d_q by d where $1 \leq q < \infty$.

Definition 1.11. In [17], the author introduced $W(\triangle)$ and has defined as follows:

$$W(\triangle) = \{\xi = (\xi_k) \in \Upsilon : (\triangle \xi_k) \in W\},$$

where $W \in \{l_\infty, c, c_0\}$ and $\triangle\xi_k = \xi_k - \xi_{k+1}$.

It was further studied by Esi (see, [8]), Ganie et al (see, [14]- [15])as follows: Let the integer be m with $m \geq 0$, then

$$H(\Delta_m \xi) = \{ \xi = (\xi_k) : \Delta_m \xi \in H \},$$

for $H = l_\infty , c$ and c_0, where $\Delta_m \xi_k = \xi_k - \xi_{k+m}$. As in [26], we have

$$\Delta_m^n \xi_k = \{ \xi \in \Upsilon : (\Delta_n^m \xi_k) \in W \},$$

where

$$\Delta_m^n \xi_k = \sum_{\mu=0}^{n} (-1)^\mu \binom{n}{r} \xi_{k+m\mu},$$

and

$$\Delta_0^n \xi_k = \xi_k \forall\, k \in \mathbb{N}.$$

2. Main results

In this protion, we shall develop the study of Fuzzy numbers by employing generalized difference operator Δ_m^n and the lacunary sequence $a = (a_r)$ and interpret their topological behaviour.

Definition 2.1. A sequence $X = (X_k)$ of Fuzzy numbers is known as lacunary almost $\Delta_m^n(\theta)$-convergent to the Fuzzy number X_0 provided that for every $\epsilon > 0$,

$$\lim_r \frac{1}{h_r} |\{k \in J_r : d\left(\Delta_m^n \lambda_{k+i}, \lambda_0\right) \geq \epsilon\}| = 0, \tag{2.1}$$

uniformly in i, where $\theta = (a_r)$ be a lacunary sequence. Here we write $\lambda_k \to \lambda_0 \left(\hat{S}\left(\Delta_m^n(\theta)\right)\right)$ or $\hat{S}\left(\Delta_m^n(\theta)\right) - \lim \lambda_k = \lambda_0$. The set of all lacunary almost $\Delta_m^n(\theta)$- statistically convergent sequences of Fuzzy numbers is abbreviated by $\hat{S}\left(\Delta_m^n(\theta)\right)$.

Definition 2.2. Let $\theta = (a_r)$ be a lacunary sequence, $p = (p_k)$ be a sequence of strictly positive real numbers and $\lambda = (\lambda_k)$ be a sequence of Fuzzy numbers. Then, the sequence $\lambda = (\lambda_k)$ is said to be a lacunary strongly $\Delta_m^n(\theta)$-convergent if there is a Fuzzy number λ_0 such that

$$\lim_r \frac{1}{h_r} \sum_{k \in J_r} \left[d\left(\Delta_m^n \lambda_{k+i}, \lambda_0\right)\right]^{p_k} = 0, \tag{2.2}$$

uniformly in i. In this case, we write $\lambda_k \to \lambda_0\left([M_\theta^p, \Delta_m^n(\theta)]\right)$. By $[M_\theta^p, \Delta_m^n(\theta)]$, we shall denote the set of all lacunary strongly almost $\Delta_m^n(\theta)$ convergent sequence of Fuzzy numbers.

Definition 2.3. Let $\theta = (a_r)$ be a lacunary sequence. Then, the sequence $\lambda = (\lambda_k)$ of Fuzzy numbers is said to be Δ_m^n bounded if the set $\{(\Delta_m^n \lambda_k) : k \in \mathbb{N}\}$ of Fuzzy numbers is bounded. We shall denote by $l_\infty(\Delta_m^n)$, the set of all Δ_m^n-bounded sequences of Fuzzy numbers.

Theorem 2.1. *If $\lambda = (\lambda_k),\ \lambda = (\lambda_k) \in \hat{S}\left(\Delta_m^n(\theta)\right)$ and $\alpha \in \mathbb{R}$, then,* *(i)* $\hat{S}\left(\Delta_m^n(\theta)\right) - \lim(\alpha\lambda_k) = \alpha\hat{S}\left(\Delta_m^n(\theta)\right) - \lim\lambda_k$,

(ii) $\hat{S}\left(\Delta_m^n(\theta)\right) - \lim(\lambda_k + \mu_k) = \hat{S}\left(\Delta_m^n(\theta)\right) - \lim\lambda_k + \hat{S}\left(\Delta_m^n(\theta)\right) - \lim\mu_k$.

Proof. Omitted. □

Theorem 2.2. *Let $\theta = (a_r)$ be a lacunary sequence, $p = (p_k)$ be a sequence of strictly positive real numbers with $0 < h = \inf p_k \le p_k \le \sup p_k = H < \infty$, then*

(i) $\lambda_k \to \lambda_0\left([M_\theta^p, \Delta_m^n(\theta)]\right) \Rightarrow \lambda_k \to \lambda_0\left(\hat{S}\left(\Delta_m^n(\theta)\right)\right)$,

(ii) $\lambda_k \in l_\infty(\Delta_m^n)$ & $\lambda_k \in \lambda_0\left(\hat{S}\left(\Delta_m^n(\theta)\right)\right) \Rightarrow \lambda_k \to \lambda_0\left([M_\theta^p, \Delta_m^n(\theta)]\right)$.

Proof. Let $\lambda_k \to \lambda_0\left([M_\theta^p, \Delta_m^n(\theta)]\right)$. Then, for $\epsilon > 0$, we have

$$\begin{aligned}
\frac{1}{h_r}\sum_{k\in J_r}\left[d\left(\Delta_m^n\lambda_{k+i}, \lambda_0\right)\right]^{p_k} &\ge \frac{1}{h_r}\sum_{\substack{k\in J_r\\ d(\Delta_m^n\lambda_{k+i},\lambda_0)\ge\epsilon}}\left[d\left(\Delta_m^n\lambda_{k+i}, \lambda_0\right)\right]^{p_k}\\
&\ge \frac{1}{h_r}\sum_{\substack{k\in J_r\\ d(\Delta_m^n\lambda_{k+i},\lambda_0)\ge\epsilon}}\epsilon^{p_k}\\
&\ge \frac{1}{h_r}\sum_{\substack{k\in J_r\\ d(\Delta_m^n\lambda_{k+i},\lambda_0)\ge\epsilon}}\min\left(\epsilon^h, \epsilon^H\right)\\
&\ge \frac{1}{h_r}\left|\left\{k \in J_r : d\left(\Delta_m^n\lambda_{k+i}, \lambda_0\right) \ge \epsilon\right\}\right|\min\left(\epsilon^h, \epsilon^H\right)
\end{aligned}$$

uniformly in i, there by proving part (i).

(ii) Now to prove part (ii), we suppose that $\lambda_k \in l_\infty(\Delta_m^n)$ and $\lambda_k \in \lambda_0\left(\hat{S}\left(\Delta_m^n(\theta)\right)\right)$. Then, there is a constant $Q > 0$ such that

$$d\left(\Delta_m^n\lambda_{k+i}, \lambda_0\right) \le Q.$$

Then, for $\epsilon > 0$, we have

$$\begin{aligned}
\frac{1}{h_r}\sum_{k\in J_r}\left[d\left(\Delta_m^n\lambda_{k+i}, \lambda_0\right)\right]^{p_k} &\ge \frac{1}{h_r}\sum_{\substack{k\in J_r\\ d(\Delta_m^n\lambda_{k+i},\lambda_0)\ge\epsilon}}\left[d\left(\Delta_m^n\lambda_{k+i}, \lambda_0\right)\right]^{p_k}\\
&\quad+\frac{1}{h_r}\sum_{\substack{k\in J_r\\ d(\Delta_m^n\lambda_{k+i},\lambda_0)<\epsilon}}\left[d\left(\Delta_m^n\lambda_{k+i}, \lambda_0\right)\right]^{p_k}\\
&\le \max\left(Q^h, Q^H\right)\frac{1}{h_r}\left|\left\{k \in J_r : d\left(\Delta_m^n\lambda_{k+i}, \lambda_0\right) \ge \epsilon\right\}\right| + \max\left(\epsilon^h, \epsilon^H\right).
\end{aligned}$$

Therefore, we conclude that $X_0 \in \left([M_\theta^p, \Delta_m^n(\theta)]\right)$. □

From this result, we have the following result:

Corollary 2.3. $\hat{S}\left(\Delta_m^n(\theta)\right) \cap l_\infty(\Delta_m^n) = [M_\theta^p, \Delta_m^n(\theta)] \cap l_\infty(\Delta_m^n)$.

Theorem 2.4. *Let $\theta = (a_r)$ be a lacunary sequence. Then $\lambda = (\lambda_k)$ is almost $\Delta_m^n(\theta)$-statistically convergent to λ_0 if it is almost Δ_m^n-statistically convergent to the Fuzzy number λ_0 along with $\liminf_{(r)} \left(\frac{a_r}{r}\right) > 0$.*

Proof. For $\epsilon > 0$, we see that

$$|\{k \leq r : d\left(\Delta_m^n \lambda_{k+i}, \lambda_0\right) \geq \epsilon\}| \supset |\{k \in J_r : d\left(\Delta_m^n \lambda_{k+i}, \lambda_0\right) \geq \epsilon\}|$$

This gives,

$$\begin{aligned}
&\frac{1}{r}\left|\{k \leq r : d\left(\Delta_m^n \lambda_{k+i}, \lambda_0\right) \geq \epsilon\}\right| \\
&\geq \frac{1}{r}\left|\{k \in J_r : d\left(\Delta_m^n \lambda_{k+i}, \lambda_0\right) \geq \epsilon\}\right| \\
&= \frac{h_r}{r}\frac{1}{h_r}\left|\{k \in \lambda_r : d\left(\Delta_m^n \lambda_{k+i}, \lambda_0\right) \geq \epsilon\}\right|.
\end{aligned}$$

Proceeding the limits as $r \to \infty$ and employ the reality that $\liminf_{(r)} \left(\frac{a_r}{r}\right) > 0$, we conclude that λ is $\Delta_m^n(\theta)$-statistically convergent to λ_0. □

Theorem 2.5. *For $0 < p_k \leq q_k$ and $\left(\frac{q_k}{p_k}\right)$ as bounded, we have $[M_\theta^q, \Delta_m^n(\theta)] \subset [M_\theta^p, \Delta_m^n(\theta)]$.*

Proof. Let $\lambda \in [M_\theta^q, \Delta_m^n(\theta)]$. Let us denote $w_{k,i} = [d\left(\Delta_m^n \lambda_{k+i}, \lambda_0\right)]^{q_k}$ and $\mu_k = \frac{p_k}{q_k}$, in order that $0 < \mu < \mu_k \leq 1 \, \forall \, k$. We abbreviate the sequences $(u_{k,i})$ and $(v_{k,i})$ as follows: Let $u_{k,i} = w_{k,i}$ and $v_{k,i} = 0$ for $w_{k,i} \geq 1$ and $u_{k,i} = 0$ and $v_{k,i} = w_{k,i}$ for $w_{k,i} < 1$. Then, for all $k \in \mathbb{N}$, it is obvious that $w_{k,i} = u_{k,i} + v_{k,i}$ and $w_{k,i}^{\mu_k} = u_{k,i}^{\mu_k} + v_{k,i}^{\mu_k}$. Hence, we conclude that $u_{k,i}^{\mu_k} = u_{k,i} \leq w_{k,i}$ and $v_{k,i}^{\mu_k} \leq v_{k,i}^{\mu_k}$. Consequently, we have

$$\begin{aligned}
\frac{1}{h_r}\sum_{k \in I_r} w_{k,i}^{\mu_k} &= \frac{1}{h_r}\sum_{k \in I_r}\left(u_{k,i}^{\mu_k} + v_{k,i}^{\mu_k}\right) \\
&\leq \frac{1}{h_r}\sum_{k \in I_r} w_{k,i} = \frac{1}{h_r}\sum_{k \in I_r} v_{k,i}^{\mu_k}.
\end{aligned}$$

But $\mu < 1 \, \forall \, m$, therefore, we see

$$\begin{aligned}
\frac{1}{h_r}\sum_{k \in I_r} v_{k,i}^{\mu_k} &= \left(\frac{1}{h_r} v_{k,i}\right)^{\mu}\left(\frac{1}{h_r}\right)^{1-\mu} \\
&\leq \left(\sum_{k \in I_r}\left[\left(\frac{1}{h_r} v_{k,i}\right)^{\mu}\right]^{\frac{1}{\mu}}\right)^{\mu}\left(\sum_{k \in I_r}\left[\left(\frac{1}{h_r}\right)^{1-\mu}\right]^{\frac{1}{1-\mu}}\right)^{1-\mu} \\
&= \left(\frac{1}{h_r}\sum_{k \in I_r} v_{k,i}\right)^{\mu}.
\end{aligned}$$

Hence by Holder's inequality we note that

$$\frac{1}{h_r}\sum_{k\in I_r} w_{k,i}^{\mu_k} \leq \frac{1}{h_r}\sum_{k\in I_r} w_{k,i} + \left(\frac{1}{h_r}\sum_{k\in I_r} v_{k,i}\right)^{\mu}.$$

This shows that $\lambda \in [M_\theta^p, \Delta_m^n(\theta)]$. □

Theorem 2.6. $[M_\theta^p, \Delta_m^n(\theta)]_\infty \subset l_\infty(\Delta_m^n)$,
where

$$[M_\theta, \Delta_m^n(\theta)]_\infty = \left\{\lambda = (\lambda_k) : \frac{1}{h_r}\sum_{k\in I_r}\left[d\left(\Delta_m^n \lambda_{k+i}, 0\right)\right] < \infty\right\}.$$

Proof. Let us suppose that $\lambda \in [M_\theta^p, \Delta_m^n(\theta)]_\infty$. Therefore, we choose a constant $\lambda > 0$ in such a way that

$$\frac{1}{h_1} d\left[\Delta_m^n \lambda_{1+i}, \bar{0}\right] \leq \frac{1}{h_r}\sum_{k\in J_r} d\left[\Delta_m^n \lambda_{k+i}, \bar{0}\right] \leq \lambda,$$

for all i and hence, we have $X \in l_\infty(\Delta_m^n)$.

Conversely, we let $X \in l_\infty(\Delta_m^n)$. Hence, there exists a constant β such that for every j, we have

$$d\left(\Delta_m^n \lambda_j, 0\right) \leq \beta,$$

and hence

$$\frac{1}{h_r}\sum_{k\in I_r} d\left[\Delta_m^n X_{k+i}, \bar{0}\right] \leq \frac{k_2}{h_r} 1 \sum_{k\in I_r} 1 \leq \beta,$$

for every k and i. We thus have, $X \in [M_\theta^p, \Delta_m^n(\theta)]_\infty$. □

REFERENCES

[1] Y. Altin, M. Et and M. Başarir, On some generalized difference sequences of fuzzy numbers, Kuwait J. Sci. Engrg., 34(1A)(2007), 1-14.

[2] S. Aytar, Statistical limit points of sequences of Fuzzy numbers, Inform Sci., 165(2004), 129-138.

[3] M. Başarir and M. Mursaleen, Some difference sequence spaces of fuzzy numbers, J. Fuzzy Math., 11(3)(2003), 1-7.

[4] A. K. Chaudhury and P. Das, Some results on fuzzy topology on fuzzy sets, Fuzzy Sets and Systems, 56(1993), 331-336.

[5] J. Cannor, A topological and functional analytic approach to statistical convergence, Appl. and Numerical Har. Anal., (1999), 403-413.

[6] P. Diamond and P. Kloeden, Metric spaces of fuzzy sets, Fuzzy Sets Syst., 35(1990), 241-249.

[7] A. Esi., On some new classes of sequences of Fuzzy numbers, Int. J. Math. Anal., 2(17)(2008), 837-844.

[8] A. Esi and B. Hazarika, Some new generalized classes of sequences of fuzzy numbers defined by an Orlicz function, Annals Fuzzy Math. Infor. 4(2)(2012), 401-406.

[9] H. Fast, Sur la convergence, Colloq. Math., (1951), 241-244.

[10] A. R. Freedman, J. J. Sember and M. Raphael, some Ceşaro type summability spaces, Proc. London Math. Soc., 37(1978), 508-520.

[11] J. Friday, On statistical convergence, Analysis, 5(1985), 301-313.

[12] A. H. Ganie and A. Antesar, Certain sequence spaces using delta operator (to appear in Adv. Stud. Contm. Math., (2020))

[13] A. H. Ganie and N. A. Sheikh, A note on almost convergent sequences and some matrix transformations, Int. J. Mod. Math. Sci., 4(2012),126-132.

[14] A. H. Ganie and N. A. Sheikh, Generalized difference sequences of fuzzy numbers, New York J. Math., 19 (2013), 431-438.

[15] A. H. Ganie and N. A. Sheikh and M. Sen, The difference sequence space defined by Orlicz functions, Int. J. Mod. Math. Sci., 6(3)(2013), 151-159.

[16] O. Kelava and S. Seikkala, On fuzzy metric spaces, Fuzzy Sets Syst., 12(1984), 215-229.

[17] H. Kizmaz, On certain sequence spaces, Canad. Math. Bull., 24(2)(1981), 169-175.

[18] G. G. Lorentz, A contribution to the theory of divergent series, Acta Math., (80)(1948) 167-190.

[19] M. Matloka, Sequences of fuzzy numbers, BUSEFAL, 28(1986), 28-37.

[20] S. Nanda, Infinite matrices and almost convergence, J. Indian Math. Soc., 40(1976), 173-184.

[21] F. Nuray, Lacunary Statistical convergence of sequences of Fuzzy numbers, Fuzzy Sets Syst., 99(1998), 353-356.

[22] S. A. Gupkari, Some new sequence spaces and almost convergence, Filomat, 22(2) (2008), 59-64.

[23] E. Savaş, A note on sequence of fuzzy numbers, Inf. Sci., 124(2000), 297-300.

[24] T. $\check{S}$alat, On statistically convergent sequences of real numbers, Math. Slovacca, 30(1980), 139-150.

[25] B. Sarma, Some sequences of Fuzzy numbers associated with a modulus function, Int. J. Fut. Rev. Com. Sci. & Comm. Engg., 3(12)(2018), 83-86.

[26] B. C. Tripathy, A. Esi, On a new type of generalized difference Cesro sequence spaces, Soochow J. Math., 31(2005), 333-340.

[27] B. C. Tripathy and M. Sen, On generalised statistically convergent sequences, Indian J. Pure and App. Maths., 32(11)(2001), 1689-1694.

[28] L. A. Zadeh, Fuzzy sets, Inform. Control, 8(1965), 338-353.

ABDUL HAMID GANIE
COLLEGE OF SCIENCES AND THEORETICAL STUDIES,, SAUDI ELECTRONIC UNIVERSITY-SAUDI ARABIA
Email address: a.ganie@seu.edu.sa