ANNALS OF COMMUNICATIONS IN MATHEMATICS
Volume 4, Number 1 (2021), 26-34
ISSN: 2582-0818

$e^{\star}$-CONNECTEDNESS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

S. SIVASANGARI, R. BALAKUMAR AND G. SARAVANAKUMAR*

ABSTRACT. In this paper the concept of types of intuitionistic fuzzy $e^{\star}$-connected and intuitionistic fuzzy $e^{\star}$-extremally disconnected in intuitionistic fuzzy topological spaces are introduced and studied. Here we introduce the concepts of intuitionistic fuzzy $e^{\star}C_5$-connectedness, intuitionistic fuzzy $e^{\star}C_S$-connectedness, intuitionistic fuzzy $e^{\star}C_M$-connectedness, intuitionistic fuzzy $e^{\star}$-strongly connectedness, intuitionistic fuzzy $e^{\star}$-super connectedness, intuitionistic fuzzy $e^{\star}C_i$-connectedness $(i = 1, 2, 3, 4)$, and obtain several properties and some characterizations concerning connectedness in these spaces.

1. INTRODUCTION

Ever since the introduction of fuzzy sets by Zadeh [15], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by Chang [2]. Atanassov [1] introduced the notion of intuitionistic fuzzy sets, Coker [3] introduced the intuitionistic fuzzy topological spaces. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Turnali and Coker [14]. The initiation of $e^{\star}$-open sets in topological spaces is due to Ekici [5, 6, 7, 8, 9]. Sobana et.al [12] were introduced the concept of fuzzy $e^{\star}$-open sets in intuitionistic fuzzy topological spaces (briefly., IFTS's). In this paper we have introduced some types of intuitionistic fuzzy $e^{\star}$-connected and intuitionistic fuzzy $e^{\star}$-extremally disconnected spaces and studied their properties and characterizations.

2. PRELIMINARIES

We recall the following definition.

Definition 2.1. [1] Let X be a nonempty fixed set and I be the closed interval in $[0,\ 1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form $A = \{< x,\ \mu_A(x), \nu_A(x) > ; x \in X\}$ where the mappings $\mu_A(x) : X \to I$ and $\nu_A(x) : X \to I$ denote the degree of membership (namely) $\mu_A(x)$ and the degree of non membership (namely) $\nu_A(x)$ for each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

2010 *Mathematics Subject Classification.* 54A40, 54A99, 03E72, 03E99.
Key words and phrases. $IFe^{\star}$-connected,$IFe^{\star}C_5$-connected,$IFe^{\star}$-strongly connected, $IFe^{\star}\ C_M$-disconnected,$IFe^{\star}C_S$-connected,$IFe^{\star}$-extremally disconnected.
Received: January 21, 2021. Accepted: March 8, 2021. Published: March 31, 2021.
*Corresponding author.

Definition 2.2. [1] Let A and B are intuitionistic fuzzy sets of the form $A = \{< x,\ \mu_A(x), \nu_A(x) >: x \in X\}$ and $B = \{< x,\ \mu_B(x), \nu_B(x) >: x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;
(ii) $\overline{A}\ (or A^c) = \{< x,\ \nu_A(x), \mu_A(x) >: x \in X\}$;
(iii) $A \cap B = \{< x,\ \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >: x \in X\}$;
(iv) $A \cup B = \{< x,\ \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >: x \in X\}$;
(v) $[\,]A = \{< x,\ \mu_A(x),\ 1 - \mu_A(x) >: x \in X\}$;
(vi) $\langle\,\rangle A = \{< x,\ 1 - \nu_A(x),\ \nu_A(x) >: x \in X\}$;

We will use the notation $A = \{< x,\ \mu_A,\ \nu_A >: x \in X\}$ instead of $A = \{< x,\ \mu_A(x), \nu_A(x) >: x \in X\}$.

Definition 2.3. [4] $0_\sim = \{< x,\ 0,\ 1 >; x \in X\}$ and $1_\sim = \{< x,\ 1,\ 0 >; x \in X\}$. Let $\alpha,\ \beta \in [0,\ 1]$ such that $\alpha + \beta \leq 1$. An intuitionistic fuzzy point $(IFP)_{p(\alpha,\ \beta)}$ is intuitionistic fuzzy set defined by $p_{(\alpha,\ \beta)}(x)= \begin{cases} (\alpha,\ \beta) \text{ if x=p} \\ (0,\ 1) \text{ otherwise} \end{cases}$

Definition 2.4. [3] An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family T of intuitionistic fuzzy sets in X satisfying the following axioms:

(i) $0_\sim,\ 1_\sim \in T$;
(ii) $G_1 \cap G_2 \in T$, for any $G_1,\ G_2 \in T$;
(iii) $\cup G_i \in T$ for any arbitrary family $\{G_i; i \in J\} \subseteq T$.

In this paper by $(X,\ T)$ or simply by X we will denote the intuitionistic fuzzy topological space(IFTS). Each IFS which belongs to T is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.5. [10] Let $p_{(\alpha,\ \beta)}$ be an IFP in IFTS X. An IFS A in X is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(\alpha,\ \beta)}$ if there exists an IFOS B in X such that $p_{(\alpha,\ \beta)} \in B \subseteq A$.

Let X and Y are two non-empty sets and $f : (X,\ T) \rightarrow (Y,\ S)$ be a function [3]. If $B = \{< y,\ \mu_B(y),\ \nu_B(y) >; y \in Y\}$ is an IFS in Y, then the pre-image of B under f is denoted and defined by $f^{-1}(B) = \{< x,\ f^{-1}(\mu_B(x)),\ f^{-1}(\nu_B(x)) >; x \in X\}$. Since $\mu_B(x),\ \nu_B(x)$ are fuzzy sets, we explain that $f^{-1}(\mu_B(x)) = \mu_B(x)(f(x)), f^{-1}(\nu_B(x)) = \nu_B(x)(f(x))$.

Definition 2.6. [3] Let $(X,\ T)$ be an IFTS and $A = \{< x,\ \mu_A,\ \nu_A >; x \in X\}$ be an IFS in X. Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior of A are defined by

(i) $cl(A) = \bigcap\{C : C \text{ is an IFCS in } X \text{and } C \supseteq A\}$;
(ii) $int(A) = \bigcup\{D : D \text{ is an IFOS in } X \text{and } D \subseteq A\}$;

It can be also shown that $cl(A)$ is an IFCS, $int(A)$ is an IFOS in X and A is and IFCS in X if and only if $cl(A) = A$; A is an IFOS in X if and only if $int(A) = A$

Proposition 2.1. [3] *Let $(X,\ T)$ be an IFTS and $A,\ B$ be intuitionistic fuzzy sets in X. Then the following properties hold:*

(i) $cl(\overline{A}) = \overline{(int(A))},\ int(\overline{A}) = \overline{(cl(A))}$;
(ii) $int(A) \subseteq A \subseteq cl(A)$.

Definition 2.7. [12] Let A be an IFS in an IFTS(X, T). A is called an intuitionistic fuzzy $e^{\star}$-open set (IF$e^{\star}$OS, for short) in X if $A \subseteq clintcl_{\delta}(A)$. The Complement of A is called an intuitionistic fuzzy $e^{\star}$-closed set (IF$e^{\star}$CS, for short) in X.

Definition 2.8. Let A be IFS in an IFTS (X, T). A is called an intuitionistic fuzzy regular open set [13] (briefly $IFROS$) if $A = intcl(A)$ and intuitionistic fuzzy regular closed set (briefly $IFRCS$) if $A = clint(A)$

Definition 2.9. [13] Let (X, T) be an IFTS and $A = < x, \mu_A(x), \nu_A(x) >$ be a IFS in X. Then the fuzzy δ closure of A are denoted and defined by $cl_{\delta}(A) = \cap\{K : K$ is an IFRCS in X and $A \subseteq K\}$ and $int_{\delta}(A) = \cup\{G : G$ is an IFROS in X and $G \subseteq A\}$.

Definition 2.10. [3] Let (X, T) and (Y, S) be IFTS's. A function $f : (X, T) \to (Y, S)$ is called intuitionistic fuzzy continuous if $f^{-1}(B)$ is an IFOS in X for every $B \in S$.

Lemma 2.2. [14]

(i) $A \cap B = 0_{\sim} \Rightarrow A \subseteq \overline{B}$.

(ii) $A \nsubseteq B \Rightarrow A \cap B \neq 0_{\sim}$

Definition 2.11. [4] Two intuitionistic fuzzy sets A and B are said to be q-coincident (AqB) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$. If A and B are said to be not q-coincident $(A\overline{q}B)$ if and only if $A \subseteq B$.

Definition 2.12. [11] An IFTS $(X,\ T)$ is called intuitionistic fuzzy C_5-connected between two intuitionistic fuzzy sets A and B if there is no IFOS E in $(X,\ T)$ such that $A \subseteq E$ and $E\overline{q}B$.

3. Types of intuitionistic fuzzy $e^{\star}$-connectedness in intuitionistic fuzzy topological spaces

Definition 3.1. Let (X, T) and (Y, S) be IFTS's. A function $f : (X, T) \to (Y, S)$ is called intuitionistic fuzzy $e^{\star}$-continuous if $f^{-1}(B)$ is an IF$e^{\star}$OS in X for every $B \in S$.

Definition 3.2. Let (X, T) be an IFTS and $A = < x, \mu_A, \nu_A >$ be an IFS in X. Then the intuitionistic fuzzy $e^{\star}$-interior and intuitionistic fuzzy $e^{\star}$-closure are defined and denoted by:

$$e^{\star}cl(A) = \cap\{K : K \text{ is an IF}e^{\star}\text{CS in } X \text{ and } A \subseteq K\}$$

and

$$e^{\star}int(A) = \cup\{G : G \text{ is an IF}e^{\star}\text{OS in } X \text{ and } G \subseteq A\}.$$

It is clear that A is an IF$e^{\star}$CS (IF$e^{\star}$OS) in X iff $A = cl_{e^{\star}}(A)(A = int_{e^{\star}}(A))$.

Definition 3.3. Let A be IFS in an IFTS (X, T). A is called an intuitionistic fuzzy $e^{\star}$-regular open set (briefly $IFe^{\star}ROS$) if $A = e^{\star}int(e^{\star}cl(A))$ and intuitionistic fuzzy $e^{\star}$-regular closed set (briefly $IFe^{\star}RCS$) if $A = e^{\star}cl(e^{\star}int(A))$

Definition 3.4. An IFTS $(X,\ T)$ is IF$e^{\star}$-disconnected if there exists intuitionistic fuzzy $e^{\star}$-open sets $P,\ Q$ in X, $P \neq 0_{\sim},\ Q \neq 0_{\sim}$ such that $P \cup Q = 1_{\sim}$ and $P \cap Q = 0_{\sim}$. If X is not IF$e^{\star}$-disconnected then it is said to be IF$e^{\star}$-connected.

Example 3.5. Let $X = \{a,\ b\}$, $T = \{0_{\sim},\ 1_{\sim},\ P\}$ where $P = \{< x,\ (\frac{a}{0.3},\ \frac{b}{0.4}),\ (\frac{a}{0.7},\ \frac{b}{0.5}) >,\ x \in X\}$, $Q = \{< x,\ (\frac{a}{0.2},\ \frac{b}{0.3}),\ (\frac{a}{0.8},\ \frac{b}{0.5}) >,\ x \in X\}$, P and Q are intuitionistic fuzzy $e^{\star}$-open sets in X, $P \neq 0_{\sim},\ Q \neq 0_{\sim}$ and $P \cup Q = P \neq 1_{\sim},\ P \cap Q = Q \neq 0_{\sim}$. Hence X is IF$e^{\star}$-connected.

Example 3.6. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0},\ \frac{b}{1}),\ (\frac{a}{1},\ \frac{b}{0}) >,\ x \in X\},\ R = \{< x,\ (\frac{a}{1},\ \frac{b}{0}),\ (\frac{a}{0},\ \frac{b}{1}) >,\ x \in X\}$, Q and R are intuitionistic fuzzy $e^{\star}$-open sets in X, $Q \neq 0_{\sim},\ R \neq 0_{\sim}$ and $Q \cup R = 1_{\sim},\ Q \cap R = 0_{\sim}$. Hence X is IFe*-disconnected.

Definition 3.7. An IFTS $(X,\ T)$ is IF$e^{\star}C_5$-disconnected if there exists IFS P in X, which is both IF$e^{\star}$OS and IF$e^{\star}$CS such that $P \neq 0_{\sim}$, and $P \neq 1_{\sim}$. If X is not IF$e^{\star}C_5$-disconnected then it is said to be IF$e^{\star}C_5$-connected.

Example 3.8. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0.6},\ \frac{b}{0.5}),\ (\frac{a}{0.3},\ \frac{b}{0.3}) >,\ x \in X\}$ Q is an IF$e^{\star}$OS in X, but Q is not IF$e^{\star}$CS since $clint(cl_{\delta}(Q)) \nsubseteq Q$

Example 3.9. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0},\ \frac{b}{1}),\ (\frac{a}{1},\ \frac{b}{0}) >,\ x \in X\}$, Q is an intuitionistic fuzzy $e^{\star}$-open sets in X. Also Q is IF$e^{\star}$CS since $clint(cl_{\delta}(Q)) = 0_{\sim} \leq Q$. Hence there exists an IFS Q in X such that $1_{\sim} \neq Q \neq 0_{\sim}$ which is both IF$e^{\star}$OS and IF$e^{\star}$CS in X. Thus X is IF$e^{\star}C_5$-disconnected.

Proposition 3.1. *IF$e^{\star}C_5$-connectedness implies IF$e^{\star}$-connectedness.*

Proof. Suppose that there exists nonempty intuitionistic fuzzy $e^{\star}$-open sets P and Q such that $P \cup Q = 1_{\sim}$ and $P \cap Q = 0_{\sim}$(IF$e^{\star}$-disconnected) then $\mu_P \vee \mu_Q = 1,\ \nu_P \wedge \nu_Q = 0$ and $\mu_P \vee \mu_Q = 0,\ \nu_P \wedge \nu_Q = 1$. In other words $\overline{Q} = P$. Hence P is IF$e^{\star}$-clopen which implies X is IF$e^{\star}C_5$-disconnected. □

But the converse need not be true as shown by the following example.

Example 3.10. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0.2},\ \frac{b}{0.3}),\ (\frac{a}{0.8},\ \frac{b}{0.5}) >,\ x \in X\}$, P and Q are IF$e^{\star}$OS in X. Also $P \cup Q \neq 1_{\sim} = P,\ P \cap Q \neq 0_{\sim} = Q$. Hence X is IF$e^{\star}$-connected. Since IFS P is both IF$e^{\star}$OS and IF$e^{\star}$CS in X, X is IF$e^{\star}C_5$-disconnected.

Proposition 3.2. *Let $f : (X,\ T) \to (Y,\ S)$ be a IF$e^{\star}$-irresolute surjection, $(X,\ T)$ is an IF$e^{\star}$-connected, then $(Y,\ S)$ is IF$e^{\star}$-connected.*

Proof. Assume that $(Y,\ S)$ is not IF$e^{\star}$-connected then there exists nonempty intuitionistic fuzzy $e^{\star}$-open sets P and Q in $(Y,\ S)$ such that $P \cup Q = 1_{\sim}$ and $P \cap Q = 0_{\sim}$. Since f is IF$e^{\star}$-irresolute mapping, $R = f^{-1}(P) \neq 0_{\sim},\ U = f^{-1}(Q) \neq 0_{\sim}$ which are intuitionistic fuzzy $e^{\star}$-open sets in X. And $f^{-1}(P) \cup f^{-1}(Q) = f^{-1}(1_{\sim}) = 1_{\sim}$ which implies $R \cup U = 1_{\sim}$. $f^{-1}(P) \cap f^{-1}(Q) = f^{-1}(0_{\sim}) = 0_{\sim}$ which implies $R \cap U = 0_{\sim}$. Thus X is IF$e^{\star}$-disconnected, which is a contradiction to our hypothesis. Hence Y is IF$e^{\star}$-connected. □

Proposition 3.3. *$(X,\ T)$ is IF$e^{\star}C_5$-connected iff there exists no nonempty intuitionistic fuzzy $e^{\star}$-open sets P and Q in X such that $P = \overline{Q}$*

Proof. Suppose that P and Q are intuitionistic fuzzy $e^{\star}$-open sets in X such that $P \neq 0_{\sim} \neq Q$ and $P = \overline{Q}$. Since $P = \overline{Q},\ \overline{Q}$ is an IF$e^{\star}$OS and Q is an IF$e^{\star}$CS, and $P \neq 0_{\sim}$ implies $Q \neq 1_{\sim}$. But this is a contradiction to the fact that X is IF$e^{\star}C_5$-connected. Conversely, let P be both IF$e^{\star}$OS and IF$e^{\star}$CS in X such that $0_{\sim} \neq P \neq 1_{\sim}$. Now take $Q = \overline{P}$. Q is an IF$e^{\star}$OS and $P \neq 1_{\sim}$ which implies $Q = \overline{P} \neq 0_{\sim}$ which is a contradiction. □

Definition 3.11. An IFTS $(X,\ T)$ is IF$e^{\star}$-strongly connected if there exists no nonempty IF$e^{\star}$CS's P and Q in X such that $\mu_P + \mu_Q \leq 1,\ \nu_P + \nu_Q \geq 1$

In otherwords, an IFTS $(X,\ T)$ is IF$e^{\star}$-strongly connected if there exists no nonempty IF$e^{\star}$CS's P and Q in X such that $P \cap Q = 0_{\sim}$.

Proposition 3.4. *An IFTS $(X,\ T)$ is IF$e^{\star}$-strongly connected if there exists no IF$e^{\star}$OS's P and Q in X, $P \neq 1\!\!1 \neq Q$ such that $\mu_P + \mu_Q \geq 1,\ \nu_P + \nu Q \leq 1$*

Example 3.12. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0.2},\ \frac{b}{0.3}),\ (\frac{a}{0.8},\ \frac{b}{0.5})\ >,\ x \in X\}$, P is an IF$e^{\star}$OS in X. P and Q is an IF$e^{\star}$OS in X since $Q \subseteq clint(cl_{\delta}(Q)$. Also $\mu_P + \mu_Q \leq\ 1,\ \nu_P + \nu_Q \geq\ 1$. Hence X is IF$e^{\star}$-strongly connected.

Proposition 3.5. *Let $f : (X,\ T) \to (Y,\ S)$ be a IF$e^{\star}$-irresolute surjection. If X is an IF$e^{\star}$-strongly connected, then so is Y.*

Proof. Suppose that Y is not IF$e^{\star}$-strongly connected then there exists IF$e^{\star}$CS C and D in Y such that $C \neq 0\!\!0,\ D \neq 0\!\!0,\ C \cap D = 0\!\!0$. Since f is IF$e^{\star}$-irresolute, $f^{-1}(C),\ f^{-1}(D)$ are IF$e^{\star}$CSs in X and $f^{-1}(C) \cap f^{-1}(D) = 0\!\!0,\ f^{-1}(C) \neq 0\!\!0,\ f^{-1}(D) \neq 0\!\!0$. (If $f^{-1}(C) = 0\!\!0$, then $f(f^{-1}(C)) = C$ which implies $f(0\!\!0) = C$. So $C = 0\!\!0$ a contradiction) Hence X is IF$e^{\star}$-strongly disconnected, a contradiction. Thus $(Y,\ S)$ is IF$e^{\star}$-strongly connected. □

IF$e^{\star}$-strongly connected does not imply IF$e^{\star}C_5$-connected, and IF$e^{\star}C_5$-connected does not imply IF$e^{\star}$-strongly connected. For this purpose we see the following examples:

Example 3.13. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0.2},\ \frac{b}{0.3}),\ (\frac{a}{0.8},\ \frac{b}{0.5})\ >,\ x \in X\}$, X is IF$e^{\star}C_5$-connected. Since $Q \subseteq clint(cl_{\delta}(Q))$. Also $\mu_P + \mu_Q \leq\ 1,\ \nu_P + \nu_Q \geq\ 1$. Hence X is IF$e^{\star}$-strongly connected. But X is not IF$e^{\star}C_5$-connected, since Q is both IF$e^{\star}$OS and IF$e^{\star}$CS in X.

Example 3.14. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0.6},\ \frac{b}{0.5}),\ (\frac{a}{0.3},\ \frac{b}{0.3})\ >,\ x \in X\},\ R = \{< x,\ (\frac{a}{0.6},\ \frac{b}{0.5}),\ (\frac{a}{0.4},\ \frac{b}{0.4})\ >,\ x \in X\}$, X is IF$e^{\star}C_5$-connected. But X is not IF$e^{\star}$-strongly connected since Q and R are intuitionistic fuzzy $e^{\star}$-open sets in X such that $\mu_Q + \mu_R \geq\ 1,\ \nu_Q + \nu_R \leq\ 1$.

Definition 3.15. P and Q are non-zero intuitionistic fuzzy sets in $(X,\ T)$. Then P and Q are said to be

(i) IF$e^{\star}$-weakly separated if $e^{\star}cl(P) \subseteq \overline{Q}$ and $e^{\star}cl(Q) \subseteq \overline{P}$.
(ii) IF$e^{\star}$-q-separated if $(e^{\star}cl(P)) \cap Q = 0\!\!0 = P \cap (e^{\star}cl(Q))$.

Definition 3.16. An IFTS $(X,\ T)$ is said to be IF$e^{\star}C_5$-disconnected if there exists IF$e^{\star}$-weakly separated non-zero intuitionistic fuzzy sets P and Q in $(X,\ T)$ such that $P \cup Q = 1\!\!1$.

Example 3.17. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0},\ \frac{b}{1}),\ (\frac{a}{1},\ \frac{b}{0})\ >,\ x \in X\},\ R = \{< x,\ (\frac{a}{1},\ \frac{b}{0}),\ (\frac{a}{0},\ \frac{b}{1})\ >,\ x \in X\}$, Q and R are intuitionistic fuzzy $e^{\star}$-open sets in X,$e^{\star}cl(Q) \subseteq\ R$ and $e^{\star}cl(R) \subseteq\ Q$. Hence Q and R are IF$e^{\star}$-weakly separated and $Q \cup R =\ 1\!\!1$. So X is IF$e^{\star}C_5$-disconnected.

Definition 3.18. An IFTS $(X,\ T)$ is said to be IF$e^{\star}C_M$-disconnected if there exists IF$e^{\star}$-q-separated non-zero intuitionistic fuzzy sets P and Q in $(X,\ T)$ such that $P \cup Q = 1\!\!1$.

Example 3.19. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0},\ \frac{b}{1}),\ (\frac{a}{1},\ \frac{b}{0})\ >,\ x \in X\},\ R = \{< x,\ (\frac{a}{1},\ \frac{b}{0}),\ (\frac{a}{0},\ \frac{b}{1})\ >,\ x \in X\}$, Q and R are intuitionistic fuzzy $e^{\star}$-open sets in X,$(e^{\star}cl(Q)) \cap R =\ 0\!\!0$ and $Q \cap (e^{\star}cl(R)) =\ 0\!\!0$ which implies Q and R are IF$e^{\star}$-q-separated and $Q \cup R =\ 1\!\!1$. Hence X is IF$e^{\star}C_M$-disconnected.

Remark. *An IFTS $(X,\ T)$ is IF$e^{\star}C_S$-connected if and only if $(X,\ T)$ is IF$e^{\star}C_M$-connected.*

Definition 3.20. An IFTS $(X,\ T)$ is said to be IF$e^{\star}$-super disconnected if there exists an IF$e^{\star}$-regular open set P in X such that $0_{\sim} \neq P \neq 1_{\sim}$. X is called IF$e^{\star}$-super connected if X is not IF$e^{\star}$-super disconnected.

Example 3.21. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{1},\ \frac{b}{0}),\ (\frac{a}{0},\ \frac{b}{1}) >,\ x \in X\}$, $R = \{< x,\ (\frac{a}{0},\ \frac{b}{1}),\ (\frac{a}{1},\ \frac{b}{0}) >,\ x \in X\}$, Q and R are intuitionistic fuzzy $e^{\star}$-open sets in X and $e^{\star}int(e^{\star}cl(Q)) = Q$. This implies Q is an IF$e^{\star}$-regular open set in X. Hence X is an IF$e^{\star}$-super disconnected.

Proposition 3.6. *Let $(X,\ T)$ be an IFTS. Then the following are equivalent:*

- *(i) X is IF$e^{\star}$-super connected*
- *(ii) For each IF$e^{\star}$OS $P \neq 0_{\sim}$ in X, we have $e^{\star}clP = 1_{\sim}$*
- *(iii) For each IF$e^{\star}$CS $P \neq 1_{\sim}$ in X, we have $e^{\star}intP = 0_{\sim}$*
- *(iv) There exists no IF$e^{\star}$OS's P and Q in X such that $P \neq 0_{\sim} \neq Q$ and $P \subseteq \overline{Q}$*
- *(v) There exists no IF$e^{\star}$OS's P and Q in X such that $P \neq 0_{\sim} \neq Q$,$Q = \overline{e^{\star}clP}$ and $P = \overline{e^{\star}clQ}$*
- *(vi) There exists no IF$e^{\star}$CS's P and Q in X such that $P \neq 1_{\sim} \neq Q$,$Q = \overline{e^{\star}intP}$ and $P = \overline{e^{\star}intQ}$*

Proof. (i) $\Rightarrow$ (ii): Assume that there exists an $P \neq 0_{\sim}$ such that $e^{\star}cl(P) \neq 1_{\sim}$. Take $P = e^{\star}int(e^{\star}cl(P))$. Then P is proper $e^{\star}$-regular open set in X which contradicts that X is IF$e^{\star}$-super connectedness.

(ii) $\Rightarrow$ (iii): Let $P \neq 1_{\sim}$ be an IF$e^{\star}CS$ in X. If we take $Q = \overline{P}$ then Q is an IF$e^{\star}$OS in X and $Q \neq 0_{\sim}$. Hence by (ii) $e^{\star}cl(Q) = 1_{\sim} \Rightarrow \overline{e^{\star}cl(Q)} = 0_{\sim} \Rightarrow e^{\star}int(\overline{Q}) = 0_{\sim} \Rightarrow e^{\star}int(A) = 0_{\sim}$.

(iii) $\Rightarrow$ (iv): Let P and Q are IF$e^{\star}$OS in X such that $P \neq 0_{\sim} \neq Q$ and $P \subseteq \overline{Q}$. Since $\overline{Q}$ is an IF$e^{\star}$CS in X, $\overline{Q} \neq 1_{\sim}$ by (iii) $e^{\star}int\overline{Q} = 0_{\sim}$. But $P \subseteq \overline{Q}$ implies $0_{\sim} \neq P = e^{\star}int(P) \subseteq e^{\star}int(\overline{Q}) = 0_{\sim}$ which is a contradiction.

(iv) $\Rightarrow$ (i): Let $0_{\sim} \neq P \neq 1_{\sim}$ be an IF$e^{\star}$-regular open set in X. If we take $Q = \overline{e^{\star}cl(P)}$, we get $Q \neq 0_{\sim}$. (If not $Q = 0_{\sim}$ implies $\overline{e^{\star}cl(P)} = 0_{\sim} \Rightarrow e^{\star}cl(P) = 1_{\sim} \Rightarrow P = e^{\star}int(e^{\star}cl(P)) = e^{\star}int(1_{\sim}) = 1_{\sim} \Rightarrow P = 1_{\sim}$ a contradiction to $P \neq 1_{\sim}$). We also have $P \subseteq \overline{Q}$ which is also a contradiction. Therefore X is IF$e^{\star}$-super connected.

(i) $\Rightarrow$ (v): Let P and Q be two IF$e^{\star}$OS in $(X,\ T)$ such that $P \neq 0_{\sim} \neq Q$, $Q = \overline{e^{\star}cl(P)}$ and $P = \overline{e^{\star}cl(Q)}$. Now we have $e^{\star}int(\overline{e^{\star}cl(P)}) = e^{\star}int(\overline{Q}) = \overline{e^{\star}cl(Q)} = P$, $P \neq 0_{\sim}$ and $P \neq 1_{\sim}$, since if $P = 1_{\sim}$ then $1_{\sim} = \overline{e^{\star}cl(Q)} \Rightarrow e^{\star}cl(Q) = 0_{\sim} \Rightarrow Q = 0_{\sim}$. But $Q \neq 0_{\sim}$. Therefore $P \neq 1_{\sim} \Rightarrow P$ is proper IF$e^{\star}$-regular open set in $(X,\ T)$ which is contradiction to (i). Hence (v) is true.

(v) $\Rightarrow$ (i): Let P be IF$e^{\star}$OS in X such that $P = e^{\star}int(e^{\star}cl(P))$, $0_{\sim} \neq P \neq 1_{\sim}$. Now take $Q = \overline{e^{\star}cl(P)}$. In this case, we get $Q \neq 0_{\sim}$ and Q is an IF$e^{\star}$OS in X and $Q = \overline{e^{\star}cl(P)}$ and $\overline{e^{\star}cl(Q)} = \overline{e^{\star}cl(\overline{e^{\star}cl(P)})} = \overline{(\overline{e^{\star}int(e^{\star}cl(P))})} = e^{\star}int(e^{\star}cl(P)) = P$. But this is a contradiction to (v). Therefore $(X,\ T)$ is IF$e^{\star}$-super connected space.

(v) $\Rightarrow$ (vi): Let P and Q be IF$e^{\star}$-closed sets in $(X,\ T)$ such that $P \neq 1_{\sim} \neq Q$, $Q = \overline{e^{\star}int(P)}$ and $P = \overline{e^{\star}int(Q)}$. Taking $C = \overline{P}$ and $D = \overline{Q}$, C and D become IF$e^{\star}$-open sets in $(X,\ T)$ and $C \neq 0_{\sim} \neq D$, $\overline{e^{\star}cl(C)} = \overline{e^{\star}cl(\overline{P})} = \overline{\overline{(e^{\star}int(P))}} = e^{\star}int(P) = \overline{Q} = D$ and similarly $\overline{e^{\star}cl(D)} = C$. But this is a contradiction to (v). Hence (vi) is true.

(vi) $\Rightarrow$ (i): We can prove this by the similar way as in (v) $\Rightarrow$ (vi). □

Proposition 3.7. *Let $f : (X,\ T) \to (Y,\ S)$ be a IF$e^{\star}$-irresolute surjection. If X is an IF$e^{\star}$-super connected, then so is Y.*

Proof. Suppose that Y is IF$e^{\star}$-super disconnected. Then there exists IF$e^{\star}$OS's C and D in Y such that $C \neq 0 \neq D,\ C \subseteq \overline{D}$. Since f is IF$e^{\star}$-irresolute, $f^{-1}(C)$ and $f^{-1}(D)$ are IF$e^{\star}$OS's in X and $C \subseteq \overline{D}$ implies $f^{-1}(C) \subseteq f^{-1}(D) = \overline{f^{-1}(D)}$. Hence $f^{-1}(C) \neq 0 \neq f^{-1}(\overline{D})$ which means that X is IF$e^{\star}$-super disconnected which is a contradiction. □

Definition 3.22. An IFTS $(X,\ T)$ is called intuitionistic fuzzy $e^{\star}C_5$-connected between two intuitionistic fuzzy sets P and Q if there is no IF$e^{\star}$OS E in $(X,\ T)$ such that $P \subseteq E$ and $E\overline{q}Q$.

Example 3.23. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0.2},\ \frac{b}{0.3}),\ (\frac{a}{0.8},\ \frac{b}{0.6}) >,\ x \in X\},\ R = \{< x,\ (\frac{a}{0.9},\ \frac{b}{0.6}),\ (\frac{a}{0.1},\ \frac{b}{0.2}) >,\ x \in X\}$, P is IF$e^{\star}$OS in $(X,\ T)$. Then $(X,\ T)$ is intuitionistic fuzzy $e^{\star}$-connected between P and Q.

Theorem 3.8. *If an IFTS $(X,\ T)$ is an intuitionistic fuzzy $e^{\star}C_5$-connected between two intuitionistic fuzzy sets P and Q, then it is intuitionistic fuzzy C_5-connected between two intuitionistic fuzzy sets P and Q.*

Proof. Suppose $(X,\ T)$ is not intuitionistic fuzzy C_5-connected between two intuitionistic fuzzy sets P and Q then there exists an IFOS E in $(X,\ T)$ such that $P \subseteq E$ and $E\overline{q}Q$. Since every IFOS in IF$e^{\star}$OS, there exists an IF$e^{\star}$OS E in $(X,\ T)$ such that $P \subseteq E$ and $E\overline{q}Q$ which implies $(X,\ T)$ is not intuitionistic fuzzy $e^{\star}$-connected between P and Q, a contradiction to our hypothesis. Therefore, $(X,\ T)$ is intuitionistic fuzzy C_5-connected between P and Q. □

However, the converse of the above Theorem is need not be true, as shown by the following example.

Example 3.24. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0.1},\ \frac{b}{0.2}),\ (\frac{a}{0.9},\ \frac{b}{0.6}) >,\ x \in X\},\ R = \{< x,\ (\frac{a}{0.7},\ \frac{b}{0.4}),\ (\frac{a}{0.3},\ \frac{b}{0.4}) >,\ x \in X\}$ P is IFOS in $(X,\ T)$. Then $(X,\ T)$ intuitionistic fuzzy C_5-connected between Q and R. Consider IFS $D = \{< x,\ (\frac{a}{0.2},\ \frac{b}{0.3}),\ (\frac{a}{0.8},\ \frac{b}{0.5}) >,\ x \in X\}$, D is an IF$e^{\star}$OS such that $Q \subseteq D$ and $D \subseteq \overline{R}$ which implies $(X,\ T)$ is intuitionistic fuzzy $e^{\star}$-disconnected between Q and R.

Theorem 3.9. *Let $(X,\ T)$ be an IFTS and P and Q be intuitionistic fuzzy sets in $(X,\ T)$. If PqQ then $(X,\ T)$ is intuitionistic fuzzy $e^{\star}C_5$-connected between P and Q.*

Proof. Suppose $(X,\ T)$ is not intuitionistic fuzzy $e^{\star}C_5$-connected between P and Q. Then there exists an IF$e^{\star}$OS E in $(X,\ T)$ such that $P \subseteq E$ and $E \subseteq \overline{Q}$. This implies that $P \subseteq \overline{Q}$. That is $P\overline{q}Q$ which is a contradiction to our hypothesis. Therefore $(X,\ T)$ is intuitionistic fuzzy $e^{\star}C_5$-connected between P and Q. □

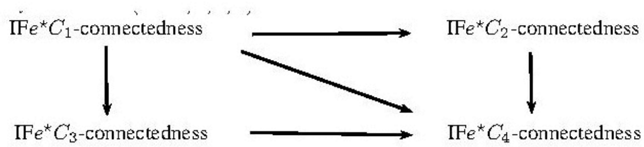
However, the converse of the above Theorem is need not be true, as shown by the following example.

Example 3.25. In Example 3.5, Consider the intuitionistic fuzzy sets $Q = \{< x,\ (\frac{a}{0.2},\ \frac{b}{0.3}),\ (\frac{a}{0.8},\ \frac{b}{0.6}) >,\ x \in X\},\ R = \{< x,\ (\frac{a}{0.9},\ \frac{b}{0.6}),\ (\frac{a}{0.1},\ \frac{b}{0.2}) >,\ x \in X\}$, P is IFeOS in $(X,\ T)$. Then $(X,\ T)$ is intuitionistic fuzzy $e^{\star}$-connected between P and Q. But P is not q-coincident with Q, since $\mu_P(x) < \nu_Q(x)$.

Definition 3.26. Let N be an IFS in IFTS $(X,\ T)$
(a) If there exists intuitionistic fuzzy $e^{\star}$-open sets M and W in X satisfying the following properties, then N is called IF$e^{\star}C_i$-disconnected (i=1,2,3,4):
$e^{\star}C_1 : N \subseteq M \cup W,\ M \cap W \subseteq \overline{N},\ N \cap M \neq 0,\ N \cap W \neq 0,$
$e^{\star}C_2 : N \subseteq M \cup W,\ N \cap M \cap W = 0,\ N \cap M \neq 0,\ N \cap W \neq 0,$

$e^\star C_3 : N \subseteq M \cup W,\ M \cap W \subseteq \overline{N},\ M \not\subseteq \overline{N},\ W \not\subseteq \overline{N}$,
$e^\star C_4 : N \subseteq M \cup W,\ N \cap M \cap W = 0_\sim, M \not\subseteq \overline{N},\ W \not\subseteq \overline{N}$,
(b) N is said to be IF$e^\star C_i$-connected $(i = 1, 2, 3, 4)$ if N is not IF$e^\star C_i$-disconnected $(i = 1, 2, 3, 4)$.

Obviously, we can obtain the following implications between several types of IF$e^\star C_i$-connected $(i = 1, 2, 3, 4)$:



Example 3.27. In Example 3.5, $M = \{< x,\ (\frac{a}{0.6},\ \frac{b}{0.5}),\ (\frac{a}{0.3},\ \frac{b}{0.3}) >,\ x \in X\}$, $W = \{< x,\ (\frac{a}{0.6},\ \frac{b}{0.5}),\ (\frac{a}{0.4},\ \frac{b}{0.4}) >,\ x \in X\}$, be IF$e^\star$OS. Consider the IFS $N = \{< x,\ (\frac{a}{0.2},\ \frac{b}{0.3}),\ (\frac{a}{0.6},\ \frac{b}{0.5}) >,\ x \in X\}$, N is IF$e^\star C_2$-connected, IF$e^\star C_3$-connected, IF$e^\star C_4$-connected but IF$e^\star C_1$-disconnected.

Example 3.28. In Example 3.5, $M = \{< x,\ (\frac{a}{0.3},\ \frac{b}{0}),\ (\frac{a}{0.4},\ \frac{b}{1}) >,\ x \in X\}$, $W = \{< x,\ (\frac{a}{0},\ \frac{b}{1}),\ (\frac{a}{1},\ \frac{b}{0}) >,\ x \in X\}$, be IF$e^\star$OS. Consider the IFS $N = \{< x,\ (\frac{a}{0.2},\ \frac{b}{0.3}),\ (\frac{a}{0.6},\ \frac{b}{0.5}) >,\ x \in X\}$, N is IF$e^\star C_2$-disconnected but IF$e^\star C_4$-connected.

Example 3.29. In Example 3.5, $M = \{< x,\ (\frac{a}{0.8},\ \frac{b}{0.7}),\ (\frac{a}{0.2},\ \frac{b}{0.1}) >,\ x \in X\}$, $W = \{< x,\ (\frac{a}{0.6},\ \frac{b}{0.8}),\ (\frac{a}{0.4},\ \frac{b}{0.4}) >,\ x \in X\}$, be IF$e^\star$OS. Consider the IFS $N = \{< x,\ (\frac{a}{0.2},\ \frac{b}{0.3}),\ (\frac{a}{0.6},\ \frac{b}{0.5}) >,\ x \in X\}$, N is IF$e^\star C_3$-disconnected but IF$e^\star C_4$-connected.

4. Intuitionistic Fuzzy $e^\star$-Extremally Disconnectedness in Intuitionistic Fuzzy Topological Spaces

Definition 4.1. Let $(X,\ T)$ be any IFTS. X is called IF$e^\star$-extremally disconnected if the $e^\star$-closure of every IF$e^\star$OS in X is IF$e^\star$OS.

Theorem 4.1. *For an IFTS $(X,\ T)$ the following are equivalent:*

(i) $(X,\ T)$ is an IF$e^\star$-extremally disconnected space.
(ii) For each IF$e^\star$CS P, $e^\star int(P)$ is an IF$e^\star$CS.
(iii) For each IF$e^\star$OS P, $e^\star cl(P) = \overline{e^\star cl(\overline{e^\star cl(P)})}$ is an IF$e^\star$CS.
(iv) For each intuitionistic fuzzy $e^\star$-open sets P and Q with $e^\star cl(P) = \overline{Q}$, $e^\star cl(P) = \overline{e^\star cl B}$.

Proof. (i) ⇒ (ii): Let P be any IF$e^\star$CS. Then $\overline{P}$ is an IF$e^\star$OS. So $e^\star cl(\overline{P}) = \overline{e^\star int(P)}$ is an IF$e^\star$OS. Thus $e^\star int(P)$ is an IF$e^\star$CS in $(X,\ T)$.

(ii) ⇒ (iii): Let P be an IF$e^\star$OS. Then $e^\star cl(\overline{e^\star cl(P)}) = e^\star cl(e^\star int(\overline{P}))$. $\overline{e^\star cl(\overline{e^\star cl(P)})} = \overline{e^\star cl(e^\star int(\overline{P}))}$. Since P is an IF$e^\star$OS, $\overline{P}$ is an IF$e^\star$CS. So by (ii) $e^\star int(\overline{P})$ is an IF$e^\star$CS. That is $e^\star cl(e^\star int(\overline{P})) = e^\star int(\overline{P})$. Hence $\overline{e^\star cl(e^\star int(\overline{P}))} = \overline{e^\star int(\overline{P})} = e^\star cl(P)$.

(iii) ⇒ (iv): Let P and Q be any two intuitionistic fuzzy $e^\star$-open sets in $(X,\ T)$ such that $e^\star cl(P) = \overline{Q}$. (iii) implies $e^\star cl(P) = \overline{e^\star cl(\overline{e^\star cl(P)})} = \overline{e^\star cl(\overline{\overline{Q}})} = \overline{e^\star cl(Q)}$.

(iv) ⇒ (i): Let P be any IF$e^\star$OS in $(X,\ T)$. Put $Q = \overline{e^\star cl(P)}$. Then $e^\star cl(P) = \overline{Q}$. Hence by (iv) $e^\star cl(P) = \overline{e^\star cl(Q)}$. Therefore $e^\star cl(P)$ is IF$e^\star$OS in $(X,\ T)$. That is $(X,\ T)$ is an IF$e^\star$-extremally disconnected space. □

5. CONCLUSION

In this paper, we have introduced and studied the concept of types of intuitionistic fuzzy $e^{\star}$-connected and intuitionistic fuzzy $e^{\star}$-extremally disconnected in intuitionistic fuzzy topological spaces are introduced and studied. Here we introduce the concepts of intuitionistic fuzzy $e^{\star}C_5$-connectedness, intuitionistic fuzzy $e^{\star}C_S$-connectedness, intuitionistic fuzzy $e^{\star}C_M$-connectedness, intuitionistic fuzzy $e^{\star}$-strongly connectedness, intuitionistic fuzzy $e^{\star}$-super connectedness, intuitionistic fuzzy $e^{\star}C_i$-connectedness $(i = 1, 2, 3, 4)$, and obtained several properties and some characterizations concerning connectedness in these spaces.

6. ACKNOWLEDGEMENTS

The authors would like to thank from the anonymous reviewers for carefully reading of the manuscript and giving useful comments, which will help us to improve the paper.

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S. SIVASANGARI,
DEPARTMENT OF MATHEMATICS, PONNAIYAH RAMAJAYAM INSTITUTE OF SCIENCE & TECHNOLOGY (PRIST)(INSTITUTION DEEMED TO BE UNIVERSITY), THANJAVUR-613403, TAMILNADU
Email address: sivasangarimaths2020@gmail.com

R. BALAKUMAR
DEPARTMENT OF MATHEMATICS, PONNAIYAH RAMAJAYAM INSTITUTE OF SCIENCE & TECHNOLOGY (PRIST)(INSTITUTION DEEMED TO BE UNIVERSITY), THANJAVUR-613403, TAMILNADU
Email address: balaphdmaths@gmail.com

G. SARAVANAKUMAR
DEPARTMENT OF MATHEMATICS, M.KUMARASAMY COLLEGE OF ENGINEERING (AUTONOMOUS) KARUR, TAMILNADU -639 113, INDIA.
Email address: saravananguru2612@gmail.com