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NEUTROSOPHIC MICRO TOPOLOGICAL SPACES

S. GANESAN* AND S. JAFARI

ABSTRACT. In this article, we introduce the concept of neutrosophic micro topological spaces. Some interesting properties like neutrosophic micro interior, neutrosophic micro closure and neutrosophic micro continuous are studied. In this connection, interrelations are discussed. Example are provided wherever necessary.

1. INTRODUCTION

Zadeh [19] introduced the notion of fuzzy sets in the year 1965. The concept of fuzzy topological spaces have been introduced and developed by Chang [4]. In 1983, Atanassov [2] introduced the concept of intuitionistic fuzzy set which was generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. Later, Coker [6] introduced the concept of intuitionistic fuzzy topological spaces, by using the notion of the intuitionitic fuzzy set. Smarandache [17, 18] introduced the concept of Neutrosophic set. Neutrosophic set is classified into three independent functions namely, membership function, indeterminancy and non membership function that are independently related. Dhavaseelan et al [1, 3, 7, 8, 9, 10, 11, 15] studied in various concept covered in neutrosophy. Lellis Thivagar et al [14] introduced the concept of neutrosophic nano topology in the year 2018. Chandrasekar [5] introduced the concept of micro toplogy in the year 2019. In this article, we introduce the concept of neutrosophic micro topological spaces. The significance of introducing hybrid structures is that the computational techniques, based on any one of these structures alone, will not always yield the best results but a fusion of two or more of them can often give better results. The rest of this article is organized as follows. Some preliminary concepts required in our work are briefly recalled in section 2. In section 3, the concept of neutrosophic micro topology is investigated with some properties on neutrosophic micro interior, neutrosophic micro closure and neutrosophic micro continuous.

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*Corresponding author.

2. PRELIMINARIES

Definition 2.1. [17, 18] A neutrosophic set (in short ns) K on a set X $\neq \emptyset$ is defined by K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ X$\}$ where P_K : X $\rightarrow$ [0,1], Q_K : X $\rightarrow$ [0,1] and R_K : X $\rightarrow$ [0,1] denotes the membership of an object, indeterminacy and non-membership of an object, for each a X to K, respectively and 0 $\leq$ P_K(a) + Q_K(a) + R_K(a) $\leq$ 3 for each a $\in$ X.

Definition 2.2. [16] Let K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ X$\}$ be a ns. We must introduce the ns $0_\sim$ and $1_\sim$ in X as follows:
$0_\sim$ may be defined as:

(1) $0_\sim$ = $\{\prec$ x, 0, 0, 1 $\succ$: x $\in$ X$\}$
(2) $0_\sim$ = $\{\prec$ x, 0, 1, 1 $\succ$: x $\in$ X$\}$
(3) $0_\sim$ = $\{\prec$ x, 0, 1, 0 $\succ$: x $\in$ X$\}$
(4) $0_\sim$ = $\{\prec$ x, 0, 0, 0 $\succ$: x $\in$ X$\}$

$1_\sim$ may be defined as:

(a) $1_\sim$ = $\{\prec$ x, 1, 0, 0 $\succ$: x $\in$ X$\}$
(b) $1_\sim$ = $\{\prec$ x, 1, 0, 1 $\succ$: x $\in$ X$\}$
(c) $1_\sim$ = $\{\prec$ x, 1, 1, 0 $\succ$: x $\in$ X$\}$
(d) $1_\sim$ = $\{\prec$ x, 1, 1, 1 $\succ$: x $\in$ X$\}$

Proposition 2.1. [16] *For any ns S, then the following conditions are satisfied:*
(1) $0_\sim \leq S$, $0_\sim \leq 0_\sim$.
(2) $S \leq 1_\sim$, $1_\sim \leq 1_\sim$.

Definition 2.3. [16] Let K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ X$\}$ be a ns.
(1) A ns K is an empty set i.e., K = $0_\sim$ if 0 is membership of an object and 0 is an indeterminacy and 1 is a non-membership of an object respectively. i.e., $0_\sim$ = $\{$x, (0, 0, 1) : x $\in$ X$\}$
(2) A ns K is a universal set i.e., K = $1_\sim$ if 1 is membership of an object and 1 is an indeterminacy and 0 is a non-membership of an object respectively. $1_\sim$ = $\{$x, (1, 1, 0) : x $\in$ X$\}$
(3) $K_1 \cup K_2$ = $\{$a, max $\{P_{K_1}$(a), P_{K_2}(a)$\}$, max $\{Q_{K_1}$(a), Q_{K_2}(a)$\}$, min $\{R_{K_1}$(a), R_{K_2}(a)$\}$: a $\in$ X$\}$
(4) $K_1 \cap K_2$ = $\{$a, min $\{P_{K_1}$(a), P_{K_2}(a)$\}$, min $\{Q_{K_1}$(a), Q_{K_2}(a)$\}$, max $\{R_{K_1}$(a), R_{K_2}(a)$\}$: a $\in$ X$\}$
(5) K_1^C = $\{\prec$ a, R_K(a), 1 $-$ Q_K(a), P_K(a) $\succ$: a $\in$ X$\}$

Definition 2.4. [16] A neutrosophic topology (nt) in Salama's sense on a nonempty set X is a family τ of ns in X satisfying three axioms:
(1) Empty set ($0_\sim$) and universal set ($1_\sim$) are members of τ.
(2) $K_1 \cap K_2 \in \tau$ where K_1, $K_2 \in \tau$.
(3) $\cup K_\delta \in \tau$ for every $\{K_\delta : \delta \in \Delta\} \leq \tau$.
Each ns in nt are called neutrosophic open sets. Its complements are called neutrosophic closed sets.

Definition 2.5. [13] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X $\subseteq$ U.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.
$L_R(X) = \bigcup_{x \epsilon U}\{R(X): R(X) \subseteq X\}$ where R(x) denotes the equivalence class determined by X.
(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.
$U_R(X) = \bigcup_{x \epsilon U}\{R(X): R(X) \cap X \neq \phi\}$
(3) The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X)$ and $B_R(X) = U_R(X) - L_R(X)$

Proposition 2.2. [13] *If (U, R) is an approximation space and X, Y $\subseteq$ U, then*
(1) $L_R(X) \subseteq X \subseteq U_R(X)$.
(2) $L_R(\phi) = U_R(\phi) = \phi$, $L_R(U) = U_R(U) = U$.
(3) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.
(4) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.
(5) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
(6) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.
(7) $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$, *whenever* $X \subseteq Y$.
(8) $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$.
(9) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$.
(10) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$.

Definition 2.6. [13] Let U be a universe, R be an equivalence relation on U and $\tau_R(X)$ $=\{U, \phi , L_R(X) , U_R(X) , B_R(X)\}$, where X $\subseteq$ U. Then by Proposition 2.2, $\tau_R(X)$ satisfies the following axioms
(1) U , $\phi \epsilon \tau_R(X)$.
(2) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
Then $\tau_R(X)$ is called the nano topology on U with respect to X.
The space (U, $\tau_R(X)$) is the nano topological space. The elements of are called nano open sets.

Definition 2.7. [13]
If (U, $\tau_R(X)$) is the nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then
(1) The nano interior of the set A is defined as the union of all nano open subsets contained in A and it is denoted by nint(A). That is, nint(A) is the largest nano open subset of A.
(2) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and it is denoted by ncl(A). That is, ncl(A) is the smallest nano closed set containing A.

Definition 2.8. [5] Let (U, $\tau_R(X)$) be a nano topological space. Then, $\mu_R(X) = \{N \cup (\acute{N} \cap \mu) : N, \acute{N} \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological space.

[5] The Micro topology $\mu_R(X)$ satisfies the following axioms
(1) U, $\phi \in \mu_R(X)$.
(2) The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.
(3) The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological spaces and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.9. [5] For any two Micro sets A and B in a Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$),
(1) A is a Micro closed set if and only if Mic-cl(A) = A.
(2) A is a Micro open set if and only if Mic-int(A) = (A).
(3) A $\subseteq$ B implies Mic-int(A) $\subseteq$ Mic-int(B) and Mic-cl(A) $\subseteq$ Mic-cl(B).
(4) Mic-cl(Mic-cl(A)) = Mic-cl(A) and Mic-int(Mic-int(A)) = Mic-int(A).
(5) Mic-cl (A $\cup$ B) $\supseteq$ Mic-cl(A) $\cup$ Mic-cl(B).
(6) Mic-cl(A $\cap$ B) $\subseteq$ Mic-cl(A) $\cap$ Mic-cl(B).
(7) Mic-int(A $\cup$ B) $\supseteq$ Mic-int(A) $\cup$ Mic-int(B).
(8) Mic-int(A $\cap$ B) $\subseteq$ Mic-int(A) $\cap$ Mic-int(B).
(9) Mic-cl(^{C}) $= [Mic - int()]^C$.
(10) Mic-int(A^C) $= [Mic - cl()]^C$

Definition 2.10. [14] Let U be a non-empty set and R be an equivalence relation on U. Let F be a neutrosophic set in U with the membership function μ_F , the indeterminancy function σ_F and the non-membership function ν_F. The neutrosophic nano lower, neutrosophic nano upper approximation and neutrosophic nano boundary of F in the approximation (U,R) denoted by $\underline{N}$(F), $\overline{N}$(F)and BN(F) are respectively defined as follows:
(1) $\underline{N}$(F) = $\{\prec$ x, $\mu_{\underline{R}A}$(x) $\sigma_{\underline{R}A}$(x) $\nu_{\underline{R}A}$(x) $\succ$: y $\in$ [X]$_R, x\in$ U$\}$
(2) $\overline{N}$(F) = $\{\prec$ x, $\mu_{\overline{R}A}$(x) $\sigma_{\overline{R}A}$(x) $\nu_{\overline{R}A}$(x) $\succ$: y $\in$ [X]$_R, x\in$ U$\}$
(3) BN(F)= $\overline{N}$(F) $-$ $\underline{N}$(F)
where $\mu_{\underline{R}A}$(x) = $\bigwedge_{y\in[X]_R}$ μA(y), $\sigma_{\underline{R}A}$(x) = $\bigwedge_{y\in[X]_R}$ σA(y), $\nu_{\underline{R}A}$(x) = $\bigwedge_{y\in[X]_R}$ νA(y).
$\mu_{\overline{R}A}$(x) = $\bigwedge_{y\in[X]_R}$ μA(y), $\sigma_{\overline{R}A}$(x) = $\bigwedge_{y\in[X]_R}$ σA(y), $\nu_{\overline{R}A}$(x) = $\bigwedge_{y\in[X]_R}$ νA(y).

Definition 2.11. [14] Let U be a universe, R an equivalence relation on U and F a neutrosophic set in U and if the collection τ_N(F) = $\{0_N$, 1_N, $\underline{N}$(F), $\overline{N}$(F), BN(F)$\}$ forms a topology then it is said to be a neutrosophic nano topology. We call (U, τ_N(F)) as the neutrosophic nano topological space. The elements of τ_N(F) are called neutrosophic nano open sets.

Definition 2.12. [14] Let U be a nonempty set and the neutrosophic sets A and B in the form A = $\{\prec$ x : μA(x), σA(x), νA(x)$\succ$, x $\in$ U$\}$, B = $\{\prec$ x : μB(x), σB(x), νB(x)$\succ$, x $\in$ U$\}$. Then the following statements hold
(1) 0_N = $\{$x, (0, 0, 1) : x $\in$ U$\}$.
(2) 1_N = $\{$x, (1, 1, 0) : x $\in$ U$\}$.
(3) A $\cup$ B = $\{$a, max $\{\mu$A(x), μB(x)$\}$, max $\{\sigma$A(x), σB(x)$\}$, min $\{\nu$A(x), νB(x)$\}$: x $\in$ U$\}$.
(4) A $\cap$ B = $\{$a, min $\{\mu$A(x), μB(x)$\}$, min $\{\sigma$A(x), σB(x)$\}$, max $\{\nu$A(x), νB(x)$\}$: x $\in$ U$\}$.
(5) A_1^C = $\{\prec$ a, νA(x), 1 $-\sigma$A(x), μA(x) $\succ$: x $\in$ U$\}$.
(6) A $\subseteq$ B iff μA(x) $\leq$ μB(x), σA(x) $\leq$ σB(x), νA(x) $\leq$ νB(x) for all x $\in$ U.
(7) A = B iff A $\subseteq$ B, B $\subseteq$ A

Definition 2.13. [14] If (U, τ_N(F)) is a neutrosophic nano topological space with respect to neutrosophic subset of U and if A be any neutrosophic subset of U, then the neutrosophic nano interior of A is defined as the union of all neutrosophic nano open subsets of A and it is denoted by $N_F$int(A). That is, $N_F$int(A) is the largest neutrosophic nano open subset of A. The neutrosophic nano closure of A is defined as the intersection of all neutrosophic nano closed sets containing A and it is denoted by $N_F$cl(A). That is, $N_F$cl(A) is the smallest neutrosophic nano closed set containing A.

Definition 2.14. [5] A map f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) is called Micro-continuous if f^{-1}(V) is a Micro closed set of (U, $\tau_R(X)$, $\mu_R(X)$) for every Micro closed set V of (L, $\tau'_R(Y)$, $\mu'_R(Y)$).

3. NEUTROSOPHIC MICRO TOPOLOGICAL SPACES

Definition 3.1. Let (U, τ_N(F)) be a neutrosophic nano topological space. Then, $\lambda_N(F)$ = $\{$S $\cup$ ($\acute{S} \cap \lambda$) : S, $\acute{S} \in \tau_N$(F) and $\lambda \notin \tau_N$(F)$\}$ is called the neutrosophic micro topology on U with respect to F. The triplet (U, τ_N(F), $\lambda_N(F)$) is called neutrosophic micro topological space.

Definition 3.2. The neutrosophic micro topology $\lambda_N(F)$ satisfies the following axioms

(1) U, $\phi \in \lambda_N(F)$.
(2) The union of the elements of any sub-collection of $\lambda_N(F)$ is in $\lambda_N(F)$.
(3) The intersection of the elements of any finite sub collection of $\lambda_N(F)$ is in $\lambda_N(F)$.

Then $\lambda_N(F)$ is called the neutrosophic micro topology on U with respect to F. The triplet (U, τ_N(F), $\lambda_N(F)$) is called neutrosophic micro topological spaces and the elements of $\lambda_N(F)$ are called neutrosophic micro open sets and the complement of a neutrosophic micro open set is called a neutrosophic micro closed set.

Definition 3.3. If (U, τ_N(F), $\lambda_N(F)$) is a neutrosophic micro topological space with respect to neutrosophic subset of U and if A is any neutrosophic subset of U, then the neutrosophic nano interior of A is defined as the union of all neutrosophic micro open subsets of A and it is denoted by N_{mi}-int(A). That is, N_{mi}-int(A) is the largest neutrosophic micro open subset of A. The neutrosophic micro closure of A is defined as the intersection of all neutrosophic micro closed sets containing A and it is denoted by N_{mi}-cl(A). That is, N_{mi}-cl(A) is the smallest neutrosophic micro closed set containing A.

Theorem 3.1. *Let (U, τ_N(F), $\lambda_N(F)$) be a neutrosophic micro topological space with respect to F where F is a neutrosophic subset of U. Let A and B be neutrosophic subsets of U. Then the following statements hold :*

(1) A $\subseteq$ N_{mi}-cl(A).
(2) A is a neutrosophic micro closed set if and only if N_{mi}-cl(A) = A.
(3) N_{mi}-cl($0_\sim$) = $0_\sim$ and N_{mi}-cl($1_\sim$) = $1_\sim$.
(4) A $\subseteq$ B implies N_{mi}-cl(A) $\subseteq$ N_{mi}-cl(B).
(5) N_{mi}-cl(A $\cup$ B) $\subseteq$ N_{mi}-cl(A) $\cup$ N_{mi}-cl(B).
(6) N_{mi}-cl(A $\cap$ B) $\subseteq$ N_{mi}-cl(A) $\cap$ N_{mi}-cl(B).
(7) N_{mi}(N_{mi}-cl(A)) = N_{mi}-cl(A).
(8) N_{mi}-int(U$-$ A) = X $-$ N_{mi}-cl(A).

Proof. (1) By definition of neutrosophic micro closure, A $\subseteq$ N_{mi}-cl(A).

(2) If A is neutrosophic micro closed, then A is the smallest neutrosophic micro closed set containing itself and hence N_{mi}-cl(A) = A. Conversely, if N_{mi}-cl(A) = A, then A is the smallest neutrosophic micro closed set containing itself and hence A is neutrosophic micro closed.
(3) Since $0_\sim$ and $1_\sim$ are neutrosophic micro closed in (U, τ_N(F), $\lambda_N(F)$), N_{mi}-cl($0_\sim$) = $0_\sim$ and N_{mi}-cl($1_\sim$) = $1_\sim$.

(4) If A $\subseteq$ B, since B $\subseteq$ N$_{mi}$-cl(B), then A $\subseteq$ N$_{mi}$-cl(B). That is, N$_{mi}$-cl(B) is a Neutrosophic micro closed set containing A. But N$_{mi}$-cl(A) is the smallest Neutrosophic micro closed set containing A. Therefore, N$_{mi}$-cl(A) $\subseteq$ N$_{mi}$-cl(B).
(5) Since A $\subseteq$ A $\cup$ B and B $\subseteq$ A $\cup$ B, N$_{mi}$-cl(A)$\subseteq$ N$_{mi}$-cl(A $\cup$ B) and N$_{mi}$-cl(B)$\subseteq$ N$_{mi}$-cl(A $\cup$ B). Therefore, N$_{mi}$-cl(A) $\cup$ N$_{mi}$-cl(B) $\subseteq$ N$_{mi}$-cl(A $\cup$ B). By the fact that A $\cup$ B $\subseteq$ N$_{mi}$-cl(A) $\cup$ N$_{mi}$-cl(B), and since N$_{mi}$-cl(A $\cup$ B) is the smallest micro closed set containing A $\cup$ B, so N$_{mi}$-cl(A $\cup$ B) $\subseteq$ N$_{mi}$-cl(A) $\cup$ N$_{mi}$-cl(B). Thus, N$_{mi}$-cl(A $\cup$ B) = N$_{mi}$-cl(A) $\cup$ N$_{mi}$-cl(B).
(6) Since A $\cap$ B $\subseteq$ A and A $\cap$ B $\subseteq$ B, N$_{mi}$-cl(A $\cap$ B) $\subseteq$ N$_{mi}$-cl(A) $\cap$ N$_{mi}$-cl(B).
(7) Since N$_{mi}$-cl(A) is micro closed, N$_{mi}$-cl(N$_{mi}$-cl(A)) = N$_{mi}$-cl(A).
(8) For A $\leq$ U, U $-$ N$_{mi}$-cl(A) = U $-$ min $\{$L : L $\geq$ A, L is neutrosophic micro closed$\}$ = max $\{$ U $-$ L : L $\geq$ A, L is neutrosophic micro closed$\}$ = max $\{$U L : U $-$ A $\geq$ U $-$ L, U is neutrosophic micro closed$\}$ = N$_{mi}$-int(U $-$ A).

□

Theorem 3.2. *Let (U, τ_N(F), $\lambda_N(F)$) be a neutrosophic micro topological space with respect to F where F is a neutrosophic subset of U. Let A and B be neutrosophic subsets of U. Then the following statements hold :*
(1) A $\subseteq$ N$_{mi}$-int(A).
(2) A is a neutrosophic micro open set if and only if N$_{mi}$-int(A) = A.
(3) N$_{mi}$-int($0_\sim$) = $0_\sim$ and N$_{mi}$-int($1_\sim$) = $1_\sim$.
(4) A $\subseteq$ B implies N$_{mi}$-int(A) $\subseteq$ N$_{mi}$-int(B).
(5) N$_{mi}$-int(A $\cup$ B) $\subseteq$ N$_{mi}$-int(A) $\cup$ N$_{mi}$-int(B).
(6) N$_{mi}$-int(A $\cap$ B) $\subseteq$ N$_{mi}$-int(A) $\cap$ N$_{mi}$-int(B).
(7) N$_{mi}$(N$_{mi}$-int(A)) = N$_{mi}$-int(A).
(8) N$_{mi}$-cl(U $-$ A) = X$-$N$_{mi}$-int(A).

Proof. It is similar to the proof of Theorem 3.1. □

Example 3.4. Let U = $\{$P$\}$ be the universe of discourse. Let U / R = $\{$P$\}$ be an equivalence relation on U and A = $\prec$ (0.6, 0.5, 0.5)$\succ$ be a neutrosophic set on U then $\underline{N}$(A) = $\{\prec$ (0.4, 0.5, 0.5)$\succ\}$, $\overline{N}$(A) = $\{\prec$ (0.6, 0.5, 0.5)$\succ\}$, B(A) = $\{\prec$ (0.5, 0.5, 0.5)$\succ\}$. Then the collection τ_N(A) = $\{0_\sim$, $\prec$ (0.4, 0.5, 0.5)$\succ$, $\prec$ (0.6, 0.5, 0.5)$\succ$, $\prec$ (0.5, 0.5, 0.5)$\succ$, $1_\sim\}$. Then λ = $\prec$ (0.2, 0.3, 0.4)$\succ$. Neutrosophic micro open = $\lambda_N(A)$ = $\{0_\sim$, $\prec$ (0.2, 1, 0.4)$\succ$, $\prec$ (0.2, 1, 0.5)$\succ$, $\prec$ (1, 0.3, 0)$\succ$, $\prec$ (0.4, 0.5, 0.5)$\succ$, $\prec$ (0.4, 0.5, 0.4)$\succ$, $\prec$ (0.6, 0.5, 0.5)$\succ$, $\prec$ (0.6, 0.5, 0.4)$\succ$, $\prec$ (0.5, 0.5, 0.5)$\succ$, $\prec$ (0.5, 0.5, 0.4)$\succ$, $1_\sim\}$ is a neutrosophic micro topology on U and neutrosophic micro closed = $(\lambda_N(A))^C$ = $\{1_\sim^C$, $\prec$ (0.4, 0, 0.2)$\succ$, $\prec$ (0.5, 0, 0.2)$\succ$, $\prec$ (0, 0.7, 1)$\succ$, $\prec$ (0.5, 0.5, 0.4)$\succ$, $\prec$ (0.4, 0.5, 0.4)$\succ$, $\prec$ (0.5, 0.5, 0.6)$\succ$, $\prec$ (0.4, 0.5, 0.6)$\succ$, $\prec$ (0.5, 0.5, 0.5)$\succ$, $\prec$ (0.4, 0.5, 0.5)$\succ$, $0_\sim^C\}$ is also a neutrosophic micro topology on U.
We know that $0_\sim$ = $\{\prec$ x, 0, 0, 1 $\succ$: x $\in$ U$\}$, $1_\sim$ = $\{\prec$ x, 1, 1, 0 $\succ$: x $\in$ U$\}$ and $0_\sim^C$ = $\{\prec$ x, 1, 1, 0 $\succ$: x $\in$ U$\}$, $1_\sim^C$ = $\{\prec$ x, 0, 0, 1 $\succ$: x $\in$ U$\}$.
Here A = $\prec$ (0.2, 1, 0.4)$\succ$. N$_{mi}$-int(A) = $\prec$ (0, 1, 1)$\succ$ $\vee$ $\prec$ (0.2, 0, 0.4)$\succ$ $\vee$ $\prec$ (0.2, 1, 0.4)$\succ$ $\vee$ $\prec$ (0.2, 1, 0.5)$\succ$ $\vee$ $\prec$ (0.2, 0.3, 0.4)$\succ$ $\vee$ $\prec$ (0.2, 0.5, 0.5)$\succ$ $\vee$ $\prec$ (0.2, 0.5, 0.4)$\succ$ $\vee$ $\prec$ (0.2, 0.5, 0.5)$\succ$ $\vee$ $\prec$ (0.2, 0.5, 0.4)$\succ$ $\vee$ $\prec$ (0.2, 0.5, 0.5)$\succ$ $\vee$ $\prec$ (0.2, 0.5, 0.4)$\succ$ = $\prec$ (0.2, 1, 0.4)$\succ$ $\vee$ $\prec$ (0.2, 0.5, 0.4)$\succ$ $\vee$ $\prec$ (0.2, 0.5, 0.4)$\succ$ = $\prec$ (0.2, 1, 0.4)$\succ$ $\vee$ $\prec$ (0.2, 0.5, 0.4)$\succ$ = $\prec$ (0.2, 1, 0.4)$\succ$.
Therefore N$_{mi}$-int(A) = $\prec$ (0.2, 1, 0.4)$\succ$.
Hence A is a neutrosophic micro open set if and only if N$_{mi}$-int(A) = A.
Similarly, A is a neutrosophic micro closed set if and only if N$_{mi}$-cl(A) = A.

Theorem 3.3. *(U, τ_N(F), $\lambda_N(F)$) be a neutrosophic micro topological space with respect to F where F is a neutrosophic subset of U. Let A be a neutrosophic subset of U. Then*
(1) $1_\sim - N_{mi}$-int(A) = N_{mi}-cl($1_\sim - A$).
(2) $1_\sim - N_{mi}$-cl(A) = N_{mi}-int($1_\sim - A$).

Remark. *Taking complements on either side of(1) and (2) of Theorem 3.3, we get (N_{mi}-int(A)) = $1_\sim - N_{mi}$-cl($1_\sim - A$)) and (N_{mi}-cl(A)) = $1_\sim$ − (N_{mi}-int($1_\sim - A$)).*

Definition 3.5. A map f : (U, τ_N(F), $\lambda_N(F)$) → (L, τ_N'(F), λ_N'(F)) is called neutrosophic micro-continuous if f^{-1}(V) is a neutrosophic micro closed set of (U, τ_N(F), $\lambda_N(F)$) for every neutrosophic micro closed set V of (L, τ_N'(F), λ_N'(F)).

Theorem 3.4. *A map f : (U, τ_N(F), $\lambda_N(F)$) and (L, τ_N'(F), λ_N'(F)) are neutrosophic micro topological spaces. Then*
(1) Identity map from (U, τ_N(F), $\lambda_N(F)$) to (L, τ_N'(F), λ_N'(F)) is a neutrosophic micro continuous function.
(2) Any constant function which maps from (U, τ_N(F), $\lambda_N(F)$) to (L, τ_N'(F), λ_N'(F)) is a neutrosophic micro continuous function.

Proof. The proof is obvious. □

Theorem 3.5. *Let f : U → L be a map on two nms (U, τ_N(F), $\lambda_N(F)$) and (L, τ_N'(F), λ_N'(F)). Then the following statements are equivalent:*
(1) f is neutrosophic micro continuous function.
(2) f^{-1}(V) is an neutrosophic micro closed set for each neutrosophic micro closed set V in L.
(3) N_{mi}-cl(f^{-1}(V)) ≤ f^{-1}(N_{mi}-cl(V)) for V ∈ L.
(4) N_{mi}-cl(f(A)) ≥ f(N_{mi}-cl(A)) for A ∈ U.
(5) N_{mi}-int(f^{-1}(A)) ≥ f^{-1}(N_{mi}-int(A)) for A ∈ U.

Proof. (1) ⇒ (2) Let V be a neutrosophic micro closed in L. Then $f^{-1}(V)^C = f^{-1}(V^C)$ neutrosophic micro closed in U.
(2) ⇒ (3) N_{mi}-cl(f^{-1}(V)) = min {G : f^{-1}(V) ≤ G, D^C is neutrosophic micro closed}) ≤ min {f^{-1}(G) : V ≤ G , D^C is neutrosophic micro closed in L} = f^{-1}({G : V ≤ G^C is neutrosophic micro closed in L}) = f^{-1}(N_{mi}-cl(V)).
(3) ⇒ (4) Since A ≤ f^{-1}(f(A)), than N_{mi}-cl(f(A)) ≤ N_{mi}-cl(f^{-1}(f(A))) ≤ f^{-1}(N_{mi}-cl(f(A))). Therefore f(N_{mi}-cl(A)) ≤ N_{mi}-cl(f(A)).
(4) ⇒ (5) f(N_{mi}-int(f^{-1}(A)))^C = f(N_{mi}-cl(f^{-1}(A))^C) = f(N_{mi}-cl(f(A)^C)) ≤ N_{mi}-cl(f(f^{-1}(A^C))) ≤ N_{mi}-cl(A^C) = (N_{mi}-int(A))^C. This implies that N_{mi}-int(f^{-1}(B))^C ≤ f^{-1}(N_{mi}-int(A))^C= (f^{-1}(N_{mi}-int(A)))^C. Taking complement on both sides, f^{-1}(N_{mi}-int(A)) ≤ N_{mi}-int(f^{-1}(A)). □

4. CONCLUSIONS

Neutrosophic set is a general formal framework, which generalizes the concept of classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, interval intuitionistic fuzzy set, nano set, interval valued nano set, intuitionistic nano set, and interval intuitionistic nano set. In this article, we introduce the concept of neutrosophic micro topological spaces. Some interesting properties of neutrosophic micro interior, neutrosophic micro closure and neutrosophic micro continuous are studied. In future, we have extend this work in various neutrosophic micro ideal topology fields.

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S. Ganesan

PG & Research Department of Mathematics,, Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India. (Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India)

Email address: sgsgsgsgsg77@gmail.com

S. Jafari

College of Vestsjaelland South & Mathematical and Physical Science Foundation, 4200 Slagelse, Denmark

Email address: saeidjafari@topositus.com