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PYTHAGOREAN Q-ANTI NEUTROSOPHIC IDEALS IN GAMMA SEMIGROUP

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ABSTRACT. In this article, we define the concept of Pythagorean Q-anti neutrosophic ideal in gamma semigroup, Pythagorean Q-anti neutrosophic bi-ideal in gamma semigroup, and Pythagorean Q-anti neutrosophic interior ideal in gamma semigroup. We have illustrated the definition with an example. We have shown that Pythagorean Q- anti neutrosophic bi-ideal is a fuzzy bi-ideal and Pythagorean Q-anti neutrosophic ideal is a Pythagorean Q-anti neutrosophic interior ideal. Also, we have established some of its properties in detail.

1. INTRODUCTION

Early in the 20th century, semigroups were first formally studied. A semigroup is an algebraic structure made up of an associative binary operation and a non-empty set [10]. Semigroups play a key role in several branches of mathematics, including automata theory, combinatorics, mathematical analysis, and coding and language theory. Sen and Saha [23] established a link between a regular $Gamma$-semigroup and a $Gamma$-group and developed the idea of a $Gamma$-semigroup. Adam Proposed the Q-fuzzy soft set in 2014 [1].

Zadeh proposed the idea of fuzzy sets in 1965 [38]. Atanassov[2, 3] proposed the intuitionistic fuzzy set as an expansion of the fuzzy set. Pythagorean fuzzy set was introduced by Yager[36, 37] as an interpretation of the fuzzy set. The Pythagorean fuzzy set was first proposed by Yager and Abbasov[35] this notion may be regarded as an effective conjecture of intuitionistic fuzzy sets. The main difference between intuitionistic fuzzy sets and Pythagorean fuzzy sets is that their squares have a location in the unit stretch [0,1], and the total of their membership and non-membership grades is more than 1. Similar to how this original example, an important technique other than intuitionistic fuzzy set can be used to make sense of the linked vulnerability of membership grade and non-membership grade. chinnadurai[7] Proposed fuzzy ideals in algebraic structure. The Pythagorean neutrosophic set was first described by Jansi et al.[11] as a generalisation of the neutrosophic set. The concept of the neutrosophic set, which Smarandache [25] introduced, is a generalisation of

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the intuitionistic fuzzy set. [5, 6] Many mathematicians have spoken about the intuitionistic neutrosophic set. The Neutrosophic N-Structures were introduced by Khan et al.[16] and their use in semigroups. Sardar et al.[22] presented the idea of gamma semigroup fuzzy ideals.

Jun et al.[12, 13] investigated the properties of an intuitionistic fuzzy interior ideal of a semigroup S and the fuzzification of interior ideals in semigroups. Kuroki[15] looked at a few fuzzy sets and fuzzy bi-ideal in semigroup characteristics. The fuzzification of (1,2)-ideals in semigroups was examined by Jun et al.[14] who also looked into its characteristics. Intuitionistic fuzzy sets were first discussed in the context of the gamma semigroup by Uckum et al.[26]. Majumder[39] studied the properties of an anti fuzzy ideals in gamma semigroup.

2. Preliminaries

Definition 2.1. [36] Let X be a universe of discourse, A **Pythagorean fuzzy set** (PFS) $P = \{z, \vartheta_p(x), \omega_p(x)/z \in X\}$ where $\vartheta : X \to [0,1]$ and $\omega : X \to [0,1]$ represent the degree of membership and non-membership of the object $z \in X$ to the set P subset to the condition $0 \leq (\vartheta_p(z))^2 + (\omega_p(z))^2 \leq 1$ for all $z \in X$. For the sake of simplicity a PFS is denoted as $P = (\vartheta_p(z), \omega_p(z))$.

Definition 2.2. [24] Let X be a universe of discourse, A **Neutrosophic set** (NS) $N = \{z, \vartheta_N(z), \omega_N(z), \psi_N(z)/z \in X\}$ where $\vartheta : X \to [0,1]$, $\omega : X \to [0,1]$ and $\psi : X \to [0,1]$ represent the degree of truth membership, indeterminacy-membership and false-membership of the object $z \in X$ to the set N subset to the condition $0 \leq (\vartheta_N(z))^2 + (\omega_N(z))^2 + (\psi_N(z))^2 \leq 3$ for all $z \in X$. For the sake of simplicity a NS is denoted as $N = (\vartheta_N(z), \omega_N(z), \psi_N(z))$.

Definition 2.3. [11] Let X be a universe of discourse, A **Pythagorean neutrosophic set** (PNS) $P_N = \{z, \mu_p(z), \zeta_p(z), \psi_p(z)/z \in X\}$ where $\mu : X \to [0,1]$, $\zeta : X \to [0,1]$ and $\psi : X \to [0,1]$ represent the degree of membership, non-membership and indeterminacy of the object $z \in X$ to the set P_N subset to the condition $0 \leq (\mu_p(z))^2 + (\zeta_p(z))^2 + (\psi_p(z))^2 \leq 2$ for all $z \in X$. For the sake of simplicity a PNS is denoted as $P_N = (\mu_p(z), \zeta_p(z), \psi_p(z))$.

Definition 2.4. [1] Let I be a unit interval and k be a positive integer. A multi Q-fuzzy set $\tilde{A}_Q$ in U and a non-empty set Q is a set of ordered sequences
$\tilde{A_Q} = (u,q), \mu_i(u,q) : u \in U, q \in Q$ where $\mu_i : UQ \to I\ k, i = 1,2,...,k$. The function $(\mu_1(u,q), \mu_2(u,q), ..., \mu_k(u,q))$ is called the membership function of multi Q-fuzzy set $\tilde{A_Q}$ and $\mu_1(u,q) + \mu_2(u,q) + ... + \mu_k(u,q) \leq 1$,k is called the dimension of $\tilde{A_Q}$. The set of all multi Q-fuzzy sets of dimension $k \in U$ and Q is denoted by $MkQF(U)$.

3. Pythagorean Q-anti neutrosophic ideals in gamma semigroup

Throughout this paper unless otherwise stated S will denote a Γ-semigroup.

Definition 3.1. A non-empty Pythagorean Q-neutrosophic set $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ of S is called a Pythagorean Q-anti neutrosophic subsemigroup of S if it satisfies:
(i) $\mu_{P_N}(x\gamma y, q) \leq \max\{\mu_{P_N}(x,q), \mu_{P_N}(y,q)\}$,
(ii) $\zeta_{P_N}(x\gamma y, q) \geq \min\{\zeta_{P_N}(x,q), \zeta_{P_N}(y,q)\}$,
$(iii)\nu_{P_N}(x\gamma y, q) \geq \min\{\nu_{P_N}(x,q), \nu_{P_N}(y,q)\}$, for all $x, y \in S, q \in Q$ and $\gamma \in \Gamma$.

Proposition 3.1. *If Pythagorean Q-neutrosophic set $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ is a Pythagorean Q-anti neutrosophic subsemigroup of S, then the set*
$P_N = \{x \in S | \mu_{P_N}(x,q) = \mu_{P_N}(0,q), \zeta_{P_N}(x,q) = \zeta_{P_N}(0,q), \nu_{P_N}(x,q) = \nu_{P_N}(0,q), q \in Q\}$ *is a subsemigroup of S.*

Proof. Let $x, y \in S, q \in Q$, and $\gamma \in \Gamma$. Then $\mu_{P_N}(x,q) = \mu_{P_N}(y,q) = \mu_{P_N}(0,q)$, $\zeta_{P_N}(x,q) = \zeta_{P_N}(y,q) = \zeta_{P_N}(0,q)$ and $\nu_{P_N}(x,q) = \nu_{P_N}(y,q) = \nu_{P_N}(0,q)$. Since $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ is Pythagorean Q-anti neutrosophic subsemigroup of S, follows that $\mu_{P_N}(x\gamma y, q) \leq \max\{\mu_{P_N}(x,q), \mu_{P_N}(y,q)\} = \mu_{P_N}(0,q)$,
$\zeta_{P_N}(x\gamma y, q) \geq \min\{\zeta_{P_N}(x,q), \zeta_{P_N}(y,q)\} = \zeta_{P_N}(0,q)$,
$\nu_{P_N}(x\gamma y, q) \geq \min\{\nu_{P_N}(x,q), \nu_{P_N}(y,q)\} = \nu_{P_N}(0,q)$, so that $\mu_{P_N}(x\gamma y, q) = \mu_{P_N}(0,q)$, $\zeta_{P_N}(x\gamma y, q) = \zeta_{P_N}(0,q)$ and $\nu_{P_N}(x\gamma y, q) = \nu_{P_N}(0,q)$. Thus $x\gamma y \in P_N$, and consequently P_N is a subsemigroup of S. Let $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ be a Pythagorean Q-anti neutrosophic subsemigroup in S and $a + b + c \in [0,1]$ be such that $a + b + c \leq 1$.
Then we define the set $P_N^{a,b,c} = \{x \in S | \mu_{P_N}(x,q) \leq a, \zeta_{P_N}(x,q) \geq b, \nu_{P_N}(x,q) \geq c\}$. □

Theorem 3.2. *Let $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ be a Pythagorean Q-anti neutrosophic subsemigroup of S. Then $P_N^{a,b,c}$ is a subsemigroup of semigroup S for every $(a,b,c) \in Im(\mu_{P_N}) \times Im(\zeta_{P_N}) \times Im(\nu_{P_N})$ with $a + b + c \leq 1$.*

Proof. Let $x, y \in P_N^{a,b,c}, \gamma \in \Gamma$ and $q \in Q$. Then $\mu_{P_N}(x,q) \leq a, \zeta_{P_N}(x,q) \geq b, \nu_{P_N}(x,q) \geq c, \mu_{P_N}(y,q) \leq a, \zeta_{P_N}(y,q) \geq b, \nu_{P_N}(y,q) \geq c$ which implies that
$\mu_{P_N}(x\gamma y, q) \leq \max\{\mu_{P_N}(x,q), \mu_{P_N}(y,q)\} \leq a$
$\zeta_{P_N}(x\gamma y, q) \geq \min\{\zeta_{P_N}(x,q), \zeta_{P_N}(y,q)\} \geq a$
$\nu_{P_N}(x\gamma y, q) \geq \min\{\nu_{P_N}(x,q), \nu_{P_N}(y,q)\} \geq a$.
Thus $x - y, q \in P_N^{a,b,c}$. Therefore $P_N^{a,b,c}$ is a subsemigroup of semigroup S.
A semigroup S is said to be a monoid if there exists an identity element $e \in S$ such that $xe, q = ex, q = x, q$ for all $x \in S$ and $q \in Q$. □

Note that every Pythagorean Q-anti neutrosophic left(right) ideal of S is a Pythagorean Q-anti neutrosophic subsemigroup of S. But the converse is not true.

Definition 3.2. A non-empty Pythagorean Q neutrosophic set $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ of S is called a Pythagorean Q-anti neutrosophic left ideal of S if it satisfies:
(i) $\mu_{P_N}(x\gamma y, q) \leq \mu_{P_N}(y,q)$,
(ii) $\zeta_{P_N}(x\gamma y, q) \geq \zeta_{P_N}(y,q)$,
(iii) $\nu_{P_N}(x\gamma y, q) \geq \nu_{P_N}(y,q)$, for all $x, y \in S$, $q \in Q$ and $\gamma \in \Gamma$.

Definition 3.3. A non-empty Pythagorean Q neutrosophic set $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ of S is called a Pythagorean Q-anti neutrosophic right ideal of S if it satisfies:
(i) $\mu_{P_N}(x\gamma y, q) \leq \mu_{P_N}(x,q)$,
(ii) $\zeta_{P_N}(x\gamma y, q) \geq \zeta_{P_N}(x,q)$,
(iii) $\nu_{P_N}(x\gamma y, q) \geq \nu_{P_N}(x,q)$, for all $x, y \in S$, $q \in Q$ and $\gamma \in \Gamma$.

Lemma 3.3. *Let Pythagorean Q neutrosophic set $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ be a Pythagorean Q-anti neutrosophic subgroup of S such that $\mu_{P_N}(x,q) \leq \mu_{P_N}(y,q)(or(\mu_{P_N}(y,q) \leq \mu_{P_N}(x,q)))$, $\zeta_{P_N}(x,q) \geq \zeta_{P_N}(y,q)(or(\zeta_{P_N}(y,q) \geq \zeta_{P_N}(x,q)))$ and*
$\nu_{P_N}(x,q) \geq \nu_{P_N}(y,q)(or(\nu_{P_N}(y,q) \geq \nu_{P_N}(x,q)))$ for all $x, y \in S$, $q \in Q$ and $\gamma \in \Gamma$. Then $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ is a Pythagorean Q-anti neutrosophic left(right) ideal of S.

Proof. Let $\mu_{P_N}(x,q) \leq \mu_{P_N}(y,q)$, $\zeta_{P_N}(x,q) \geq \zeta_{P_N}(y,q)$ and $\nu_{P_N}(x,q) \geq \nu_{P_N}(y,q)$ for all $x, y \in S$, $q \in Q$ and $\gamma \in \Gamma$.
Then we have
$\mu_{P_N}(x\gamma y, q) \leq \max\{\mu_{P_N}(x,q), \mu_{P_N}(y,q)\} = \mu_{P_N}(y,q)$,
$\zeta_{P_N}(x\gamma y, q) \geq \min\{\zeta_{P_N}(x,q), \zeta_{P_N}(y,q)\} = \zeta_{P_N}(y,q)$,
$\nu_{P_N}(x\gamma y, q) \geq \min\{\nu_{P_N}(x,q), \nu_{P_N}(y,q)\} = \nu_{P_N}(y,q)$. Hence $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ is a Pythagorean Q-anti neutrosophic left ideal of S. Similarly if we take $\mu_{P_N}(y,q) \leq \mu_{P_N}(x,q)$, $\zeta_{P_N}(y,q) \geq \zeta_{P_N}(x,q)$ and $\nu_{P_N}(y,q) \geq \nu_{P_N}(x,q)$ for all $x, y \in S$, $q \in Q$ and $\gamma \in \Gamma$, then prove that $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ is a Pythagorean Q-anti neutrosophic right ideal of S. □

Definition 3.4. A Pythagorean neutrosophic subsemigroup $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ of S is called a Pythagorean Q-anti neutrosophic bi-ideal of S if it satisfies:
(i)$\mu_{P_N}(x\gamma a\beta y, q) \leq \max\{\mu_{P_N}(x,q), \mu_{P_N}(y,q)\}$,
(ii) $\zeta_{P_N}(x\gamma a\beta y, q) \geq \min\{\zeta_{P_N}(x,q), \zeta_{P_N}(y,q)\}$,
(iii) $\nu_{P_N}(x\gamma a\beta y, q) \geq \min\{\nu_{P_N}(x,q), \nu_{P_N}(y,q)\}$, for all $x, y \in S$, $q \in Q$ and $\gamma, \beta \in \Gamma$.

Example 3.5. Let $P_N = \{0, a, b, c\}$, $q \in Q$ and $\Gamma = \{\gamma, \beta\}$be non-empty set of binary operations defined as follows.

γ	0	a	b	c
0	0	0	0	0
a	0	b	0	a
b	0	b	0	c
c	0	0	0	b

and

β	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	0
c	a	a	a	c

Clearly S is a Γ-semigroup. A Pythagorean neutrosophic set $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ where $\mu_{P_N} : S \to [0,1]$ by $\mu_{P_N}(0,q) = 0.3, \mu_{P_N}(a,q) = 0.6, \mu_{P_N}(b,q) = 0.8 = \mu_{P_N}(c,q)$,
$\zeta_{P_N} : S \to [0,1]$ by $\zeta_{P_N}(0,q) = 0.9, \zeta_{P_N}(a,q) = 0.5, \zeta_{P_N}(b,q) = 0.4 = \zeta_{P_N}(c,q)$ and $\nu_{P_N} : S \to [0,1]$ by $\nu_{P_N}(0,q) = 0.7, \nu_{P_N}(a,q) = 0.5, \nu_{P_N}(b,q) = 0.3 = \nu_{P_N}(c,q)$.
Thus $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ is a Pythagorean Q- anti neutrosophic bi-ideal of S.

Theorem 3.4. *Let $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ be a Pythagorean neutrosophic ideal of S. If S is an intra-regular, then $P_N(a,q) = P_N(a\beta a, q)$ for all $a \in S, q \in Q, \beta \in \Gamma$.*

Proof. Let a be any element of S. Then since S is an intra-regular, there exists $x, y \in S$,$q \in Q$ and $\alpha, \beta, \gamma \in \Gamma$ such that $a, q = x\alpha a\beta a\gamma y, q$. Hence $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ be a Pythagorean Q-anti neutrosophic ideal, $\mu_{P_N}(a,q) = \mu_{P_N}(x\alpha a\beta a\gamma y, q) \leq \mu_{P_N}(x\alpha a\beta a, q)$ $\leq \mu_{P_N}(a\beta a, q) =\leq \mu_{P_N}(a,q)$,
$\zeta_{P_N}(a,q) = \zeta_{P_N}(x\alpha a\beta a\gamma y, q) \geq \zeta_{P_N}(x\alpha a\beta a, q) \geq \zeta_{P_N}(a\beta a, q) =\geq \zeta_{P_N}(a,q)$,
$\nu_{P_N}(a,q) = \nu_{P_N}(x\alpha a\beta a\gamma y, q) \geq \nu_{P_N}(x\alpha a\beta a, q) \geq \nu_{P_N}(a\beta a, q) =\geq \nu_{P_N}(a,q)$.
Hence we have $\mu_{P_N}(a,q) = \mu_{P_N}(a\beta a, q)$, $\zeta_{P_N}(a,q) = \zeta_{P_N}(a\beta a, q)$ and $\nu_{P_N}(a,q) = \nu_{P_N}(a\beta a, q)$.
Therefore $P_N(a,q) = P_N(a\beta a, q)$ for all $a \in S, q \in Q, \beta \in \Gamma$. □

Theorem 3.5. *Let $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ be a Pythagorean Q-anti neutrosophic ideal of S is an inter-regular, then $P_N(a\beta b, q) = P_N(b\beta a, q)$ for all $a, b \in S, q \in Q, \beta \in \Gamma$.*

Proof. Let $a, b \in S$, $q \in Q$ and $\beta \in \Gamma$. Then Theorem 3.9 we have
$\mu_{P_N}(a\beta b, q) = \mu_{P_N}(a\beta b\beta a\beta b, q)$

$$
\begin{aligned}
&= \mu_{P_N}(a\beta(b\beta a)\beta b, q)\\
&\leq \mu_{P_N}(b\beta a, q)\\
&= \mu_{P_N}(b\beta a\beta b\beta a, q)\\
&= \mu_{P_N}(b\beta(a\beta b)\beta a, q)\\
&\leq \mu_{P_N}(a\beta b, q),
\end{aligned}
$$

$$
\begin{aligned}
\zeta_{P_N}(a\beta b, q) &= \zeta_{P_N}(a\beta b\beta a\beta b, q)\\
&= \zeta_{P_N}(a\beta(b\beta a)\beta b, q)\\
&\geq \zeta_{P_N}(b\beta a, q)\\
&= \zeta_{P_N}(b\beta a\beta b\beta a, q)\\
&= \zeta_{P_N}(b\beta(a\beta b)\beta a, q)\\
&\geq \zeta_{P_N}(a\beta b, q)
\end{aligned}
$$

and

$$
\begin{aligned}
\nu_{P_N}(a\beta b, q) &= \nu_{P_N}(a\beta b\beta a\beta b, q)\\
&= \nu_{P_N}(a\beta(b\beta a)\beta b, q)\\
&\geq \nu_{P_N}(b\beta a, q)\\
&= \nu_{P_N}(b\beta a\beta b\beta a, q)\\
&= \nu_{P_N}(b\beta(a\beta b)\beta a, q)\\
&\geq \nu_{P_N}(a\beta b, q).
\end{aligned}
$$

Hence we have $\mu_{P_N}(a\beta b, q) = \mu_{P_N}(b\beta a, q)$, $\zeta_{P_N}(a\beta b, q) = \zeta_{P_N}(b\beta a, q)$ and $\nu_{P_N}(a\beta b, q) = \nu_{P_N}(b\beta a, q)$. Therefore $P_N(a\beta b, q) = P_N(b\beta a, q)$ for all $a, b \in S, q \in Q, \beta \in \Gamma$. □

Theorem 3.6. *Let $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ be a Pythagorean Q-anti neutrosophic bi-ideal of S if and only if the fuzzy set μ_{P_N}, $\overline{\zeta_{P_N}}$ and $\overline{\nu_{P_N}}$ are fuzzy bi-ideals of S.*

Proof. Let $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ be a Pythagorean Q-anti neutrosophic bi-ideal of S. Then clearly μ_{P_N} is a fuzzy bi-ideal of S. Let $x, a, y \in S, q \in Q, \alpha, \beta \in \Gamma$. Then

$$
\begin{aligned}
\overline{\zeta_{P_N}}(x\alpha y, q) &= 1 - \zeta_{P_N}(x\alpha y, q)\\
&\leq 1 - min\{\zeta_{P_N}(x, q), \zeta_{P_N}(y, q)\}\\
&= \max\{1 - \zeta_{P_N}(x, q), 1 - \zeta_{P_N}(y, q)\}\\
&= \max\{\overline{\zeta_{P_N}}(x, q), \overline{\zeta_{P_N}}(y, q)\},
\end{aligned}
$$

$$
\begin{aligned}
\overline{\nu_{P_N}}(x\alpha y, q) &= 1 - \nu_{P_N}(x\alpha y, q)\\
&\leq 1 - min\{\nu_{P_N}(x, q), \nu_{P_N}(y, q)\}\\
&= \max\{1 - \nu_{P_N}(x, q), 1 - \nu_{P_N}(y, q)\}\\
&= \max\{\overline{\nu_{P_N}}(x, q), \overline{\nu_{P_N}}(y, q)\},
\end{aligned}
$$

and

$$
\begin{aligned}
\overline{\zeta_{P_N}}(x\alpha a\beta y, q) &= 1 - \zeta_{P_N}(x\alpha a\beta y, q)\\
&\leq 1 - min\{\zeta_{P_N}(x, q), \zeta_{P_N}(y, q)\}\\
&= \max\{1 - \zeta_{P_N}(x, q), 1 - \zeta_{P_N}(y, q)\}\\
&= \max\{\overline{\zeta_{P_N}}(x, q), \overline{\zeta_{P_N}}(y, q)\},
\end{aligned}
$$

$$
\begin{aligned}
\overline{\nu_{P_N}}(x\alpha a\beta y, q) &= 1 - \nu_{P_N}(x\alpha a\beta y, q)\\
&\leq 1 - \min\{\nu_{P_N}(x, q), \nu_{P_N}(y, q)\}\\
&= \max\{1 - \nu_{P_N}(x, q), 1 - \nu_{P_N}(y, q)\}\\
&= \max\{\overline{\nu_{P_N}}(x, q), \overline{\nu_{P_N}}(y, q)\}.
\end{aligned}
$$

Hence $\overline{\zeta_{P_N}}, \overline{\nu_{P_N}}$ are fuzzy bi-ideal of S. Conversely, suppose that μ_{P_N}, ζ_{P_N} and ν_{P_N} are fuzzy bi-ideal of S. Let $a, x, y \in S, q \in Q, \alpha, \beta \in \Gamma$. Then

$$
\begin{aligned}
1 - \zeta_{P_N}(x\alpha y, q) &= \overline{\zeta_{P_N}}(x\alpha y, q)\\
&\leq \max\{\overline{\zeta_{P_N}}(x, q), \overline{\zeta_{P_N}}(y, q)\}
\end{aligned}
$$

$= \max\{1-\zeta_{P_N}(x,q), 1-\zeta_{P_N}(y,q)\}$
$= \min\{\zeta_{P_N}(x,q), \zeta_{P_N}(y,q)\}$
and
$1-\zeta_{P_N}(x\alpha a\beta y,q) = \overline{\zeta_{P_N}}(x\alpha a\beta y,q)$
$\leq \max\{\overline{\zeta_{P_N}}(x,q), \overline{\zeta_{P_N}}(y,q)\}$
$= 1-\min\{\zeta_{P_N}(x,q), \zeta_{P_N}(y,q)\}$,
$1-\nu_{P_N}(x\alpha y,q) = \overline{\nu_{P_N}}(x\alpha y,q)$
$\leq \max\{\overline{\nu_{P_N}}(x,q), \overline{\nu_{P_N}}(y,q)\}$
$= \max\{1-\nu_{P_N}(x,q), 1-\nu_{P_N}(y,q)\}$
$= \min\{\nu_{P_N}(x,q), \nu_{P_N}(y,q)\}$
and
$1-\nu_{P_N}(x\alpha a\beta y,q) = \overline{\nu_{P_N}}(x\alpha a\beta y,q)$
$\leq \max\{\overline{\nu_{P_N}}(x,q), \overline{\nu_{P_N}}(y,q)\}$
$= 1-\min\{\nu_{P_N}(x,q), \nu_{P_N}(y,q)\}$,
which implies that $\zeta_{P_N}(x\alpha y,q) \geq \min\{\zeta_{P_N}(x,q), \zeta_{P_N}(y,q)\}$,
$\nu_{P_N}(x\alpha y,q) \geq \min\{\nu_{P_N}(x,q), \nu_{P_N}(y,q)\}$ and
$\zeta_{P_N}(x\alpha a\beta y,q) \geq \min\{\zeta_{P_N}(x,q), \zeta_{P_N}(y,q)\}$,
$\nu_{P_N}(x\alpha a\beta y,q) \geq \min\{\nu_{P_N}(x,q), \nu_{P_N}(y,q)\}$. □

Definition 3.6. A Pythagorean Q-anti neutrosophic subsemigroup $P_N = (\mu_{P_N}, \zeta_{P_N}, \nu_{P_N})$ of S is called a Pythagorean Q-anti neutrosophic interior ideal of S if it satisfies:
(i) $\mu_{P_N}(x\gamma a\beta y,q) \leq \mu_{P_N}(a,q)$,
(ii) $\zeta_{P_N}(x\gamma a\beta y,q) \geq \zeta_{P_N}(a,q)$,
(iii) $\nu_{P_N}(x\gamma a\beta y,q) \geq \nu_{P_N}(a,q)$, for all $x,y \in S$, $q \in Q$ and $\gamma,\beta \in \Gamma$.

Proposition 3.7. *Let P_N be a Pythagorean Q-anti neutrosophic ideal of S. Then P_N is a Pythagorean Q-anti neutrosophic interior ideal of S.*

Proof. Since P_N is a Pythagorean Q-anti neutrosophic ideal of S, for any $x,y \in S$, $q \in Q$ and $\gamma \in \Gamma$,
$\mu_{P_N}(x\gamma y,q) \leq \mu_{P_N}(x,q)$, $\zeta_{P_N}(x\gamma y,q) \geq \zeta_{P_N}(x,q)$, $\nu_{P_N}(x\gamma y,q) \geq \nu_{P_N}(x,q)$ are Pythagorean Q-anti neutrosophic left ideals of S and $\mu_{P_N}(x\gamma y,q) \leq \mu_{P_N}(y,q)$,
$\zeta_{P_N}(x\gamma y,q) \geq \zeta_{P_N}(y,q)$, $\nu_{P_N}(x\gamma y,q) \geq \nu_{P_N}(y,q)$ are Pythagorean Q-anti neutrosophic right ideal of S, which implies that $\mu_{P_N}(x\gamma y,q) \leq \max\{\mu_{P_N}(x,q), \mu_{P_N}(y,q)\}$,
$\zeta_{P_N}(x\gamma y,q) \geq \min\{\zeta_{P_N}(x,q), \zeta_{P_N}(y,q)\}$, $\nu_{P_N}(x\gamma y,q) \geq \min\{\nu_{P_N}(x,q), \nu_{P_N}(y,q)\}$.
Hence P_N is a Pythagorean Q-anti neutrosophic sub-semigroup of S. Now let $x,a,y \in S$, $q \in Q$ and $\alpha,\beta \in \Gamma$,
$\mu_{P_N}(x\gamma a\beta y,q) = \mu_{P_N}(x\gamma(a\beta y),q) \leq \mu_{P_N}(a\beta y,q) \leq \mu_{P_N}(a,q)$. $\zeta_{P_N}(x\gamma a\beta y,q) = \zeta_{P_N}(x\gamma(a\beta y),q) \geq \zeta_{P_N}(a\beta y,q) \geq \zeta_{P_N}(a,q)$. $\nu_{P_N}(x\gamma a\beta y,q) = \nu_{P_N}(x\gamma(a\beta y),q) \geq \nu_{P_N}(a\beta y,q) \geq \nu_{P_N}(a,q)$. Consequently, P_N is a Pythagorean Q-anti neutrosophic interior ideal of S. □

Proposition 3.8. *If $\{P_{N_i}\}_{i\in I}$ is a family of Pythagorean Q-anti neutrosophic interior ideals of S, then so is $\bigcap_{i\in I}\mu_{P_{N_i}}(x,q) = \sup\{\mu_{P_{N_i}}(x,q) : i \in I, q \in Q, x \in S\}$,
$\bigcap_{i\in I}\zeta_{P_{N_i}}(x,q) = \inf\{\zeta_{P_{N_i}}(x,q) : i \in I, q \in Q, x \in S\}$
, $\bigcap_{i\in I}\nu_{P_{N_i}}(x,q) = \inf\{\nu_{P_{N_i}}(x,q) : i \in I, q \in Q, x \in S\}$, provided it is non-empty.*

Proof. Let $x,a,y \in S$, $q \in Q$ and $\alpha,\beta \in \Gamma$. Then,
$\bigcap_{i\in I}\mu_{P_{N_i}}(x\gamma y,q) = \sup\{\mu_{P_{N_i}}(x\gamma y,q) : i \in I, q \in Q\}$
$\leq \sup\{\max\{\mu_{P_{N_i}}(x,q), \mu_{P_{N_i}}(y,q)\} : i \in I, q \in Q\}$
$= \max[\sup\{\mu_{P_{N_i}}(x,q) : i \in I, q \in Q\}, \sup\{\mu_{P_{N_i}}(y,q) : i \in I, q \in Q\}]$

$$= \max\{\bigcap \mu_{P_{N_i}}(x,q), \bigcap \mu_{P_{N_i}}(y,q)\}.$$
$$\bigcap_{i\in I} \zeta_{P_{N_i}}(x\gamma y,q) = \inf\{\zeta_{P_{N_i}}(x\gamma y,q) : i\in I, q\in Q\}$$
$$\geq \inf\{min\{\zeta_{P_{N_i}}(x,q), \zeta_{P_{N_i}}(y,q)\} : i\in I, q\in Q\}$$
$$= \min[\inf\{\zeta_{P_{N_i}}(x,q) : i\in I, q\in Q\}, \inf\{\zeta_{P_{N_i}}(y,q) : i\in I, q\in Q\}]$$
$$= \min\{\bigcap \zeta_{P_{N_i}}(x,q), \bigcap \zeta_{P_{N_i}}(y,q)\}.$$
$$\bigcap_{i\in I} \nu_{P_{N_i}}(x\gamma y,q) = \inf\{\nu_{P_{N_i}}(x\gamma y,q) : i\in I, q\in Q\}$$
$$\geq \inf\{\min\{\nu_{P_{N_i}}(x,q), \nu_{P_{N_i}}(y,q)\} : i\in I, q\in Q\}$$
$$= \min[\inf\{\nu_{P_{N_i}}(x,q) : i\in I, q\in Q\}, \inf\{\nu_{P_{N_i}}(y,q) : i\in I, q\in Q\}]$$
$$= \min\{\bigcap \nu_{P_{N_i}}(x,q), \bigcap \nu_{P_{N_i}}(y,q)\}.$$

Hence $\bigcap P_{N_i}$ is a Pythagorean Q-anti neutrosophic subsemigroup of S.
Now $\bigcap_{i\in I} \mu_{P_{N_i}}(x\alpha a\beta y,q) = \sup\{\mu_{P_{N_i}}(x\alpha a\beta y,q) : i\in I, q\in Q\} \sup\{\mu_{P_{N_i}}(a,q) : i\in I, q\in Q\} = \bigcap \mu_{P_{N_i}}(a,q)$
$\bigcap_{i\in I} \zeta_{P_{N_i}}(x\alpha a\beta y,q) = \inf\{\zeta_{P_{N_i}}(x\alpha a\beta y,q) : i\in I, q\in Q\} \inf\{\zeta_{P_{N_i}}(a,q) : i\in I, q\in Q\} = \bigcap \zeta_{P_{N_i}}(a,q)$
$\bigcap_{i\in I} \nu_{P_{N_i}}(x\alpha a\beta y,q) = \inf\{\nu_{P_{N_i}}(x\alpha a\beta y,q) : i\in I, q\in Q\} \inf\{\nu_{P_{N_i}}(a,q) : i\in I, q\in Q\} = \bigcap \nu_{P_{N_i}}(a,q)$.
Consequently, $\bigcap P_{N_i}$ is a Pythagorean Q-anti neutrosophic interior ideal of S. □

4. Conclusions

We have define the notion of Pythagorean Q-anti neutrosophic ideal in gamma semigroup, Pythagorean Q-anti neutrosophic bi-ideal in gamma semigroup, and Pythagorean Q-anti neutrosophic interior ideal in gamma semigroup with suitable example. Also, the properties of Pythagorean Q- anti neutrosophic bi-ideal is a fuzzy bi-ideal and Pythagorean Q-anti neutrosophic ideal is a Pythagorean Q-anti neutrosophic interior ideal are established. Further, we have provided the definition of the Pythagorean Q-anti neutrosophic interior ideal. In this article, we have highlighted the concept of Pythagorean Q-anti neutrosophic ideal in gamma semigroup and studied some of its properties

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