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# ON VARIOUS ALMOST IDEALS OF SEMIRINGS

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ABSTRACT. In this paper, we study various almost ideals (shortly $\mathscr{A}$-ideals), quasi $\mathscr{A}$-ideals, bi quasi $\mathscr{A}$-ideals, tri $\mathscr{A}$-ideals and tri quasi $\mathscr{A}$-ideals in semiring and give some characterizations. Some relevant counter examples are also indicated. We develop the implications ideal $\Longrightarrow$ quasi ideal $\Longrightarrow$ bi quasi ideal $\Longrightarrow$ tri quasi ideal $\Longrightarrow$ tri quasi $\mathscr{A}$-ideal $\Longrightarrow$ bi quasi $\mathscr{A}$-ideal $\Longrightarrow$ bi $\mathscr{A}$-ideal $\Longrightarrow$ quasi $\mathscr{A}$-ideal $\Longrightarrow$ $\mathscr{A}$-ideal and reverse implications do not holds with examples. We show that the union of $\mathscr{A}$-ideals (bi $\mathscr{A}$-ideals, quasi $\mathscr{A}$-ideals, bi quasi $\mathscr{A}$-ideals) is a $\mathscr{A}$-ideal (bi $\mathscr{A}$-ideal, quasi $\mathscr{A}$-ideal, bi quasi $\mathscr{A}$-ideal) in semiring.

## 1. INTRODUCTION

Vandiver introduced the idea of semirings as a generalization of rings [19]. The notion of quasi ideal was introduced by Otto Steinfeld both in semigroups and rings [18]. Shabir et al [17] characterized the semirings by the properties of quasi-ideals. Quasi-ideals of different classes of semirings have been characterized by many authors in [2, 5]. The notion of bi-ideals in semigroups introduced by Lajos [6]. The concept of a bi-ideal is a very interesting and important thing in semiring. Bi ideal is a generalization of left ideal and right ideal. Many mathematicians proved important results and characterizations of algebraic structures by using various ideals. Rao introduced bi-quasi-ideals of semigroups. The notion of tri-ideal is a generalization of quasi ideal, bi-ideal, ideal and properties of tri ideals of a semiring [11]. Grosek and Satko introduced the notion of $\mathscr{A}$-ideal of semigroup [4]. In this paper, we give some properties of various $\mathscr{A}$-ideals in semiring. Our aim in this paper is threefold.
(1) To study the relationship between quasi $\mathscr{A}$-ideal and bi quasi $\mathscr{A}$-ideal in a semiring.
(2) To characterize tri $\mathscr{A}$-ideal in a semiring.
(3) To characterize bi quasi ideal and tri quasi ideal in a semiring.

## 2. PRELIMINARIES

**Definition 2.1.** A non-empty subset $I$ of a semiring $(S, +, \cdot)$ is called a subsemiring of $S$ if $i_1 + i_2 \in I$ and $i_1 \cdot i_2 \in I$, for all $i_1, i_2 \in I$.

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**Definition 2.2.** Suppose that $I, B$ and $Q$ are non-empty subsets of a semiring $(S, +, \cdot)$. Then
(i) $I$ is called a right (left) ideal of $S$ if $I$ is a subsemiring of $S$ and $IS \subseteq I$ (respectively $SI \subseteq I$). If $I$ is a right and left ideal of $S$, then $I$ is called an ideal of $S$.
(ii) A subsemiring $B$ of $S$ is called a bi ideal if $BSB \subseteq B$.
(iii) A subsemiring $Q$ of $S$ is called a quasi ideal if $QS \cap SQ \subseteq Q$.

**Definition 2.3.** [11] Suppose that $I$ and $Q$ are non-empty subsets of a semiring $(S, +, \cdot)$. Then
(i) $I$ is called a right (left) tri ideal of $S$ if $I^2SI \subseteq I(ISI^2 \subseteq I)$.
(ii) $I$ is called a tri ideal of $S$ if $I$ is a right tri ideal and left tri ideal of $S$.
(iii) $Q$ is called a right (left) bi quasi ideal if $Q$ is a subsemigroup of $S$ and
$QS \cap QSQ \subseteq Q(SQ \cap QSQ \subseteq Q)$.
(iv) $Q$ is called a bi quasi ideal of $S$ if $Q$ is a left bi quasi ideal and right bi quasi ideal of $S$.

**Definition 2.4.** [4] Suppose that $I$ is a non-empty subset of a semigroup $S$. Then
(i) $I$ is called a right (left) $\mathscr{A}$-ideal of $S$ if $IS \cap I \neq \phi(SI \cap I \neq \phi)$.
(ii) $I$ is called a $\mathscr{A}$-ideal of $S$ if $I$ is a right $\mathscr{A}$-ideal and left $\mathscr{A}$-ideal of $S$.
(iii) A subsemigroup $B$ of $S$ is called a bi $\mathscr{A}$-ideal if $BSB \cap B \neq \phi$.
(iv) A non-empty subset $Q$ of $S$ is called a quasi $\mathscr{A}$-ideal if $[QS \cap SQ] \cap Q \neq \phi$.

## 3. Various $\mathscr{A}$-ideals

Here $S$ stands for semiring unless otherwise mentioned.

**Definition 3.1.** Suppose that $I, B$ and $Q$ are non-empty subsets of a semiring $(S, +, \cdot)$. Then
(i) $I$ is called a right (left) $\mathscr{A}$-ideal of $S$ if $I$ is a subsemiring of $S$ and $IS \cap I \neq \phi$ $(SI \cap I \neq \phi)$.
(ii) $I$ is called a $\mathscr{A}$-ideal of $S$ if $I$ is a right $\mathscr{A}$-ideal and left $\mathscr{A}$-ideal of $S$.
(iii) A subsemiring $B$ of $S$ is called a bi $\mathscr{A}$-ideal if $BSB \cap B \neq \phi$.
(iv) A subsemiring $Q$ of $S$ is called a quasi $\mathscr{A}$-ideal if $[QS \cap SQ] \cap Q \neq \phi$.

**Definition 3.2.** Suppose that $Q$ is a non-empty subset of a semiring $(S, +, \cdot)$. Then
(i) $Q$ is called a right (left) bi quasi ideal of $S$ if $Q$ is a subsemiring of $S$ and
$QS \cap QSQ \subseteq Q(SQ \cap QSQ \subseteq Q)$.
(ii) $Q$ is called a bi quasi ideal of $S$ if $Q$ is a left bi quasi ideal and right bi quasi ideal of $S$.

**Definition 3.3.** Suppose that $Q$ is a non-empty subset of a semiring $(S, +, \cdot)$. Then
(i) $Q$ is called a right (left) bi quasi $\mathscr{A}$- ideal of $S$ if $Q$ is a subsemiring of $S$ and
$[QS \cap QSQ] \cap Q \neq \phi$ $([SQ \cap QSQ] \cap Q \neq \phi)$.
(ii) $Q$ is called a bi quasi $\mathscr{A}$- ideal of $S$ if $Q$ is a left bi quasi $\mathscr{A}$- ideal and right bi quasi $\mathscr{A}$- ideal of $S$.

**Theorem 3.1.** *Every ideal (bi ideal, quasi ideal) is a $\mathscr{A}$-ideal (bi $\mathscr{A}$-ideal, quasi $\mathscr{A}$-ideal).*

**Proof.** Suppose that $I$ is an ideal of $S$, then $I$ is a subsemiring of $S$ and $IS \subseteq I$ and $SI \subseteq I$. Now, $IS \cap I \subseteq I \cap I \neq \phi$ and $SI \cap I \subseteq I \cap I \neq \phi$. Hence $I$ is a $\mathscr{A}$-ideal of $S$.

Converse of the Theorem 3.1 may not be true by the following counter Example.

**Example 3.4.** (i) The semiring $S_1 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbb{Z}_2 \right\}$ and $I = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\}$. Clearly $I$ is a subsemiring and $\mathscr{A}$-ideal of $S_1$ but $I$ is not an ideal of $S_1$ by $IS_1 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \right\} \not\subseteq I$ and

$S_1 I = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \right\} \not\subseteq I.$

Let $Q = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\}$ is a quasi $\mathscr{A}$-ideal but $Q$ is not a quasi ideal of $S_1$ by $QS_1 \cap S_1 Q = S_1 \cap S_1 = S_1 \not\subseteq Q$.

(ii) The semiring $S_2 = \left\{ \begin{pmatrix} 0 & r_1 & r_2 & r_3 & r_4 & r_5 \\ 0 & 0 & r_6 & r_7 & r_8 & r_9 \\ 0 & 0 & 0 & r_{10} & r_{11} & r_{12} \\ 0 & 0 & 0 & 0 & r_{13} & r_{14} \\ 0 & 0 & 0 & 0 & 0 & r_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| r_i'^s \text{ are real numbers} \right\}$.

Clearly $B = \left\{ \begin{pmatrix} 0 & b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| b_i'^s \text{ are real numbers} \right\}$ is a bi $\mathscr{A}$-ideal of $S_2$, but $B$ is not a bi ideal of $S_2$. Since the arbitrary element $b = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \in B$ and $r = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \in S_2$ such that $brb \notin B$.

**Theorem 3.2.** *Every quasi ideal (bi ideal) is a bi quasi ideal.*

**Proof.** Suppose that $Q$ is a quasi ideal of $S$, then $QS \cap SQ \subseteq Q$. Now, $QS \cap QSQ \subseteq QS \cap SQ \subseteq Q$ and $SQ \cap QSQ \subseteq SQ \cap QS \subseteq Q$. Hence $Q$ is a bi quasi ideal of $S$.

Converse of the Theorem 3.2 is not true by the following Example.

**Example 3.5.** In Example 3.4, the semiring $S_2$ is not regular by $a = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \in S_2$ there is no $x \in S_2$ such that $a = axa$.

Let $Q = \left\{ \begin{pmatrix} 0 & 0 & x_1 & 0 & 0 & 0 \\ 0 & 0 & x_2 & 0 & x_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| x_i'^s \text{ are real numbers} \right\}$ is a subsemiring of $S_2$.

Now, $QS_2 = \left\{ \begin{pmatrix} 0 & 0 & 0 & y_1 & y_2 & y_3 \\ 0 & 0 & 0 & y_4 & y_5 & y_6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| y_i'^s \text{ are real numbers} \right\}$ and

$S_2 Q = \left\{ \begin{pmatrix} 0 & 0 & z_1 & 0 & z_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| z_i'^s \text{ are real numbers} \right\}$.

Thus, $S_2 Q \cap QS_2 Q \subseteq Q$ and $QS_2 \cap QS_2 Q \subseteq Q$. Hence, $Q$ is a bi quasi ideal but $Q$ is not a quasi ideal of $S_2$ by $QS_2 \cap S_2 Q = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 & u & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| u \text{ is a real numbers} \right\} \not\subseteq Q$

**Corollary 3.3.** *Every bi quasi $\mathscr{A}$-ideal is a quasi $\mathscr{A}$-ideal.*

**Proof.** Suppose that $Q$ is a bi quasi $\mathscr{A}$-ideal of $S$, then $Q^2 \subseteq Q$ and $[QS \cap QSQ] \cap Q \neq \phi$ and $[SQ \cap QSQ] \cap Q \neq \phi$. Now, $\phi \neq [QS \cap QSQ] \cap Q \subseteq QS \cap Q$ and $\phi \neq [QS \cap QSQ] \cap Q \subseteq SQ \cap Q$. Thus, $\phi \neq [QS \cap QSQ] \cap Q \subseteq [QS \cap SQ] \cap Q$. Hence $Q$ is a quasi $\mathscr{A}$-ideal of $S$.
Converse of the Corollary 3.3 is not true by the following Example.

**Example 3.6.** Let $S = \left\{ \begin{pmatrix} 0 & r_1 & r_2 & r_3 \\ 0 & 0 & r_4 & r_5 \\ 0 & 0 & 0 & r_6 \\ 0 & 0 & 0 & r_7 \end{pmatrix} \middle| r_i^{'s} \text{ are real numbers} \right\}$.
Let $Q = \left\{ \begin{pmatrix} 0 & 0 & x_1 & 0 \\ 0 & 0 & x_2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 \end{pmatrix} \middle| x_i^{'s} \text{ are real numbers} \right\}$ is a quasi $\mathscr{A}$-ideal of $S$ but $Q$ is not a bi quasi $\mathscr{A}$-ideal of $S$ by $[r^{'}q \cap qr^{''}q] \cap q = \phi$ and $[qr^{'} \cap qr^{''}q] \cap q = \phi$, where $q = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in Q$ and $r^{'} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in S$ and $r^{''} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \in S$.

**Theorem 3.4.** *Every bi $\mathscr{A}$-ideal is a quasi $\mathscr{A}$-ideal.*

**Proof.** Suppose that $B$ is a bi $\mathscr{A}$-ideal of $S$, $B$ is a subsemiring of $S$ and $BSB \cap B \neq \phi$. Now, $\phi \neq BSB \cap B \subseteq BS \cap B$ and $\phi \neq BSB \cap B \subseteq SB \cap B$. Thus, $\phi \neq BSB \cap B \subseteq [BS \cap SB] \cap B$. Hence $B$ is a quasi $\mathscr{A}$-ideal of $S$.

Converse of the Theorem 3.4 may not be true by the following counter Example.

**Example 3.7.** The semiring $S$ in Example 3.6,
$B = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & x_1 & x_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 \end{pmatrix} \middle| x_i^{'s} \text{ are are real numbers} \right\}$ is a subsemiring of $S$ and
$SB = \left\{ \begin{pmatrix} 0 & 0 & y_1 & y_2 \\ 0 & 0 & 0 & y_3 \\ 0 & 0 & 0 & y_4 \\ 0 & 0 & 0 & y_5 \end{pmatrix} \middle| y_i^{'s} \text{ are are real numbers} \right\}$ and
$BS = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z_2 \end{pmatrix} \middle| z_i^{'s} \text{ are are real numbers} \right\}$.
Hence $[BS \cap SB] \cap B \neq \phi$. Thus, $B$ is a quasi $\mathscr{A}$-ideal of $S$ but $B$ is not a bi $\mathscr{A}$-ideal of $S$ by $br^{'}b \cap b = \phi$, where $b = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in B$ and $r^{'} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \in S$.

**Theorem 3.5.** *Every quasi $\mathscr{A}$-ideal is a $\mathscr{A}$-ideal.*

**Proof.** Suppose that $Q$ is a quasi $\mathscr{A}$-ideal of $S$, then $[QS \cap SQ] \cap Q \neq \phi$. Now, $\phi \neq [QS \cap SQ] \cap Q \subseteq SQ \cap Q$ and $\phi \neq [QS \cap SQ] \cap Q \subseteq QS \cap Q$. Hence $Q$ is a $\mathscr{A}$-ideal of $S$.

Converse of the Theorem 3.5 not true by the following Example.

**Example 3.8.** The semiring $S = \left\{ \begin{pmatrix} 0 & r_1 & r_2 \\ 0 & 0 & r_3 \\ 0 & 0 & 0 \end{pmatrix} \middle| r_i^{'s} \text{ are real numbers} \right\}$ and
$Q = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & q_1 \\ 0 & 0 & 0 \end{pmatrix} \middle| q_1 \text{ is a real numbers} \right\}$ is a $\mathscr{A}$-ideal but $Q$ is not a quasi $\mathscr{A}$-ideal of $S$ by $[qr^{'} \cap r^{''}q] \cap q = \phi$, where $q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \in Q$, $r^{'} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \in S$ and $r^{''} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \in S$.

**Theorem 3.6.** *Every bi quasi ideal is a bi quasi $\mathscr{A}$-ideal.*

**Proof.** Suppose that $Q$ is a bi quasi ideal of $S$, then $QS \cap QSQ \subseteq Q$ and $SQ \cap QSQ \subseteq Q$. Now, $[QS \cap QSQ] \cap Q \subseteq Q \cap Q \neq \phi$ and $[SQ \cap QSQ] \cap Q \subseteq Q \cap Q \neq \phi$. Hence $Q$ is a bi quasi $\mathscr{A}$-ideal of $S$.

Converse of the Theorem 3.6 is not true as by the Example.

**Example 3.9.** The semiring $S$ in Example 3.6,

$Q = \left\{ \begin{pmatrix} 0 & 0 & x_1 & 0 \\ 0 & 0 & x_2 & x_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_4 \end{pmatrix} \middle| x_i^{'s} \text{ are real numbers} \right\}$ is a subsemiring of $S$,

$SQ = \left\{ \begin{pmatrix} 0 & 0 & y_1 & y_2 \\ 0 & 0 & 0 & y_3 \\ 0 & 0 & 0 & y_4 \\ 0 & 0 & 0 & y_5 \end{pmatrix} \middle| y_i^{'s} \text{ are real numbers} \right\}$,

$QS = \left\{ \begin{pmatrix} 0 & 0 & 0 & z_1 \\ 0 & 0 & 0 & z_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z_3 \end{pmatrix} \middle| z_i^{'s} \text{ are real numbers} \right\}$ and

$QSQ = \left\{ \begin{pmatrix} 0 & 0 & 0 & l_1 \\ 0 & 0 & 0 & l_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_3 \end{pmatrix} \middle| l_i^{'s} \text{ are real numbers} \right\}$. Thus, $[SQ \cap QSQ] \cap Q = [QS \cap QSQ] \cap Q = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & u \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v \end{pmatrix} |u, v \text{ are real numbers} \right\} \neq \phi$.

Hence $Q$ is a bi quasi $\mathscr{A}$-ideal but $Q$ is not a bi quasi ideal of $S$ by $QS \cap QSQ = \left\{ \begin{pmatrix} 0 & 0 & 0 & x \\ 0 & 0 & 0 & y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z \end{pmatrix} \middle| x, y, z \text{ are real numbers} \right\} \nsubseteq Q$.

**Theorem 3.7.** *If $Q$ is a $\mathscr{A}$-ideal (bi $\mathscr{A}$-ideal, quasi $\mathscr{A}$-ideal, bi quasi $\mathscr{A}$-ideal) of $S$ and $Q \subseteq Q^{'} \subseteq S$, then $Q^{'}$ is a $\mathscr{A}$-ideal (bi $\mathscr{A}$-ideal, quasi $\mathscr{A}$-ideal, bi quasi $\mathscr{A}$-ideal) of $S$.*

**Proof.** Suppose that $Q$ is a bi quasi $\mathscr{A}$ ideal of $S$ with $Q \subseteq Q^{'} \subseteq S$. Then $\phi \neq [QS \cap QSQ] \cap Q \subseteq [Q^{'}S \cap Q^{'}SQ^{'}] \cap Q^{'}$ and $\phi \neq [SQ \cap QSQ] \cap Q \subseteq [SQ^{'} \cap Q^{'}SQ^{'}] \cap Q^{'}$. Therefore $Q^{'}$ is a bi quasi $\mathscr{A}$ ideal of $S$.

**Corollary 3.8.** *The union of $\mathscr{A}$-ideals (bi $\mathscr{A}$-ideals, quasi $\mathscr{A}$-ideals, bi quasi $\mathscr{A}$-ideals) of $S$ is a $\mathscr{A}$-ideal (bi $\mathscr{A}$-ideal, quasi $\mathscr{A}$-ideal, bi quasi $\mathscr{A}$-ideal) of $S$.*

**Proof.** Let $I_1$ and $I_2$ be any two $\mathscr{A}$-ideals of $S$. Then $I_1 \subseteq I_1 \cup I_2$, by Theorem 3.7, $I_1 \cup I_2$ is a $\mathscr{A}$-ideal of $S$.

## 4. Various Tri $\mathscr{A}$-ideals

**Definition 4.1.** Suppose that $I$ and $Q$ are non-empty subsets of a semiring $(S, +, \cdot)$. Then
(i) $I$ is called a right (left) tri ideal of $S$ if $I$ is a subsemiring of $R$ and $I^2SI \subseteq I(ISI^2 \subseteq I)$.
(ii) $Q$ is called a right (left) tri quasi ideal of $S$ if $Q$ is a subsemiring of $R$ and
$QS \cap Q^2SQ \subseteq Q$ ($SQ \cap QSQ^2 \subseteq Q$).
(iii) $Q$ is called a tri quasi ideal of $S$ if $Q$ is a right tri quasi ideal and left tri quasi ideal of $S$.

**Definition 4.2.** Suppose that $I$ and $Q$ are non-empty subsets of a semiring $(S, +, \cdot)$. Then
(i) $I$ is called a right (left) tri $\mathscr{A}$-ideal of $S$ if $I$ is a subsemiring of $S$ and $I^2SI \cap I \neq \phi$ ($ISI^2 \cap I \neq \phi$).
(ii) $I$ is called a tri $\mathscr{A}$-ideal of $S$ if $I$ is a right tri $\mathscr{A}$-ideal and left tri $\mathscr{A}$-ideal of $S$.
(iii) $Q$ is called a right (left) tri quasi $\mathscr{A}$-ideal of $S$ if $Q$ is a subsemiring of $S$ and
$[QS \cap Q^2SQ] \cap Q \neq \phi$ ($[SQ \cap QSQ^2] \cap Q \neq \phi$).
(iv) $Q$ is called a tri quasi $\mathscr{A}$- ideal of $S$ if $Q$ is a right tri quasi $\mathscr{A}$- ideal and left tri quasi $\mathscr{A}$-ideal of $S$.

**Theorem 4.1.** *Every ideal (bi ideal, quasi ideal, tri ideal) is a tri $\mathscr{A}$-ideal.*

Converse of the Theorem 4.1 may not true by the following Example.

**Example 4.3.** Let $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbb{Z}_2 \right\}$ as a semiring.

(i) The subsemiring $I_1 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ is a tri $\mathscr{A}$-ideal of $S$ but $I_1$ is not an ideal of $S$ by $I_1 S = S \nsubseteq I_1$ and $SI_1 = S \nsubseteq I_1$.

(ii) The subsemiring $B = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\}$ is a tri $\mathscr{A}$-ideal of $S$ but $B$ is not a bi ideal of $S$ by $BSB = S \nsubseteq B$.

(iii) The subsemiring $Q = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}$ is a tri $\mathscr{A}$-ideal of $S$ but $Q$ is not a quasi ideal of $S$ by $QS \cap SQ = S \nsubseteq Q$.

(iv) $I_2 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ is a tri $\mathscr{A}$-ideal of $S$ but $I_2$ is not a tri ideal of $S$ by $I_2^2 S I_2 \nsubseteq I_2$ and $I_2 S I_2^2 \nsubseteq I_2$ of $S$.

**Theorem 4.2.** *Every tri $\mathscr{A}$-ideal is a $\mathscr{A}$-ideal (bi $\mathscr{A}$-ideal).*

**Proof.** Suppose that $I$ is a tri $\mathscr{A}$-ideal of $S$, then $I$ is a subsemiring of $S$ and $I^2 SI \cap I \neq \phi$ and $ISI^2 \cap I \neq \phi$. Now, $\phi \neq I^2 SI \cap I \subseteq ISI \cap I \subseteq IS \cap I$ and $\phi \neq ISII \cap I \subseteq SI \cap I$. Hence $I$ is a $\mathscr{A}$-ideal of $S$.

Converse of the Theorem 4.2 may not be true as in the given Example.

**Example 4.4.** (i) Let $S_1 = \left\{ \begin{pmatrix} 0 & r_1 & r_2 & r_3 \\ 0 & 0 & r_4 & r_5 \\ 0 & 0 & 0 & r_6 \\ 0 & 0 & 0 & 0 \end{pmatrix} \middle| r_i^{'s} \text{ are real numbers} \right\}$ is a semiring.

Let $I = \left\{ \begin{pmatrix} 0 & 0 & i_1 & i_2 \\ 0 & 0 & i_3 & i_4 \\ 0 & 0 & 0 & i_5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \middle| i_i^{'s} \text{ are are real numbers} \right\}$. Clearly $I$ is a $\mathscr{A}$-ideal of $S_1$ but $I$ is not a tri $\mathscr{A}$-ideal of $S_1$ by $i^2 ri \cap i = \phi$ and $iri^2 \cap i = \phi$, where $i = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \in I$ and $r = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \in S_1$.

(ii) Let $S_2 = \left\{ \begin{pmatrix} 0 & r_1 & r_2 & r_3 & r_4 & r_5 \\ 0 & 0 & r_6 & r_7 & r_8 & r_9 \\ 0 & 0 & 0 & r_{10} & r_{11} & r_{12} \\ 0 & 0 & 0 & 0 & r_{13} & r_{14} \\ 0 & 0 & 0 & 0 & 0 & r_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| r_i^{'s} \text{ are real numbers} \right\}$ is a semiring.

Clearly $B = \left\{ \begin{pmatrix} 0 & b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_2 & b_3 & b_4 \\ 0 & 0 & 0 & 0 & b_5 & b_6 \\ 0 & 0 & 0 & 0 & 0 & b_7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| b_i^{'s} \text{ are real numbers} \right\}$ is a bi $\mathscr{A}$-ideal but $B$ is not a tri $\mathscr{A}$-ideal of $S_2$ by $brb^2 \cap b = \phi$ and $b^2 rb \cap b = \phi$, where $b = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \in B$ and

$r = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \in S_2$.

**Theorem 4.3.** *Every tri $\mathscr{A}$-ideal is a quasi $\mathscr{A}$-ideal.*

**Proof.** Suppose that $Q$ is a tri $\mathscr{A}$-ideal of $S$, then $Q^2SQ \cap Q \neq \phi$ and $QSQ^2 \cap Q \neq \phi$. Since $Q$ is a subsemiring of $S$, $\phi \neq Q^2SQ \cap Q \subseteq QSQ \cap Q \subseteq QS \cap Q$ and $\phi \neq Q^2SQ \cap Q \subseteq QSQ \cap Q \subseteq SQ \cap Q$. Hence $\phi \neq Q^2SQ \cap Q \subseteq [QS \cap SQ] \cap Q$. Similarly, $\phi \neq QSQ^2 \cap Q \subseteq [QS \cap SQ] \cap Q$. Hence $Q$ is a quasi $\mathscr{A}$-ideal of $S$.

Converse of the Theorem 4.3 may not be true in the given Example.

**Example 4.5.** The semiring $S_2$ in Example 4.4,
$Q = \left\{ \begin{pmatrix} 0&0&0&q_1&q_2&0 \\ 0&0&0&0&q_3&q_4 \\ 0&0&0&0&0&q_5 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix} \middle| q_i^{'s} \text{ are real numbers} \right\}$ is a quasi $\mathscr{A}$-ideal but $Q$ is not a tri $\mathscr{A}$-ideal of $S_2$ by $q^2rq \cap q = \phi$ and $qrq^2 \cap q = \phi$, where $q = \begin{pmatrix} 0&0&0&1&1&0 \\ 0&0&0&0&1&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix} \in Q$ and
$r = \begin{pmatrix} 0&0&1&1&0&0 \\ 0&0&0&1&1&1 \\ 0&0&0&1&1&1 \\ 0&0&0&0&1&1 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix} \in S_2$.

**Corollary 4.4.** *Every tri $\mathscr{A}$-ideal is a bi quasi $\mathscr{A}$-ideal.*

**Proof.** Suppose that $Q$ is a right tri $\mathscr{A}$-ideal of $S$, then $Q^2SQ \cap Q \neq \phi$. Since $Q$ is a subsemiring of $S$, $\phi \neq Q^2SQ \cap Q \subseteq QS \cap Q$ and $\phi \neq Q^2SQ \cap Q \subseteq QSQ \cap Q$. This implies that $\phi \neq Q^2SQ \cap Q \subseteq [QS \cap QSQ] \cap Q$. Thus, $Q$ is a right bi quasi $\mathscr{A}$-ideal of $S$. Suppose that $Q$ is a left tri $\mathscr{A}$-ideal of $S$, then $Q$ is a left bi quasi $\mathscr{A}$-ideal of $S$. Hence, $Q$ is a bi quasi $\mathscr{A}$-ideal of $S$.

Converse of the Corollary 4.4 may not be true in the given Example.

**Example 4.6.** Let $S = \left\{ \begin{pmatrix} 0&0&0&0 \\ r_1&0&0&0 \\ r_2&r_3&0&0 \\ r_4&r_5&r_6&r_7 \end{pmatrix} \middle| r_i^{'s} \text{ are real numbers} \right\}$ and
$Q = \left\{ \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ q_1&q_2&0&0 \\ 0&q_3&0&q_4 \end{pmatrix} \middle| q_i^{'s} \text{ are real numbers} \right\}$ be subsemirings.
Now, $SQ = \left\{ \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ c_1&c_2&0&c_3 \end{pmatrix} \middle| c_i^{'s} \text{ are real numbers} \right\}$ and
$QS = \left\{ \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ d_1&0&0&0 \\ d_2&d_3&d_4&d_5 \end{pmatrix} \middle| d_i^{'s} \text{ are real numbers} \right\}$,
$QSQ = \left\{ \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ e_1&e_2&0&e_3 \end{pmatrix} \middle| e_i^{'s} \text{ are real numbers} \right\}$.
Thus, $[QS \cap QSQ] \cap Q \neq \phi$ and $[SQ \cap QSQ] \cap Q \neq \phi$. Hence $Q$ is a bi quasi $\mathscr{A}$-ideal of $S$. But $Q$ is not a tri $\mathscr{A}$-ideal of $S$ by $q^2r^{'}q \cap q = \phi$ and $qr^{'}q^2 \cap q = \phi$, where $q = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 1&1&0&0 \\ 0&1&0&1 \end{pmatrix} \in Q$ and $r^{'} = \begin{pmatrix} 0&0&0&0 \\ 1&0&0&0 \\ 1&1&0&0 \\ 0&1&1&0 \end{pmatrix} \in S$.

**Theorem 4.5.** *Every bi quasi ideal is a tri quasi ideal.*

**Proof.** Suppose that $Q$ is a bi quasi ideal of $S$ then $SQ \cap QSQ \subseteq Q$ and $QS \cap QSQ \subseteq Q$. Now, $SQ \cap QSQ^2 \subseteq SQ \cap QSQ \subseteq Q$ and $QS \cap Q^2SQ \subseteq QS \cap QSQ \subseteq Q$, since $Q$ is a subsemiring of $S$. Hence $Q$ is a tri quasi ideal of $S$.

Converse of Theorem 4.5 not true by the following Example.

**Example 4.7.** The semiring $S$ in Example 4.6,
$Q = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & a_3 \end{pmatrix} \middle| a_i^{'s} \text{ are real numbers} \right\}$ is a subsemiring of $S$.
Now, $SQ = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ c_1 & c_2 & 0 & c_3 \end{pmatrix} \middle| c_i^{'s} \text{ are real numbers} \right\}$,
$QS = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ d_1 & d_2 & d_3 & d_4 \end{pmatrix} \middle| d_i^{'s} \text{ are real numbers} \right\}$,
$QSQ = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ e_1 & e_2 & 0 & e_3 \end{pmatrix} \middle| e_i^{'s} \text{ are real numbers} \right\}$ and
$QSQ^2 = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & f_1 & 0 & f_2 \end{pmatrix} \middle| f_i^{'s} \text{ are real numbers} \right\}$. Thus, $SQ \cap QSQ^2 \subseteq Q$. Hence $Q$ is a tri quasi ideal of $S$ but $Q$ is not a left bi quasi ideal of $S$ by $SQ \cap QSQ = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ h_1 & h_2 & 0 & h_3 \end{pmatrix} \middle| h_i^{'s} \text{ are real numbers} \right\} \nsubseteq Q$.

**Corollary 4.6.** *Every tri quasi $\mathscr{A}$-ideal is a bi quasi $\mathscr{A}$-ideal.*

**Proof.** Suppose that $Q$ is a tri quasi $\mathscr{A}$- ideal of $S$ then $[QS \cap Q^2SQ] \cap Q \neq \phi$ and $[SQ \cap QSQ^2] \cap Q \neq \phi$. Since $Q$ is a subsemiring of $S$, $\phi \neq [QS \cap Q^2SQ] \cap Q \subseteq [QS \cap QSQ] \cap Q$ and $\phi \neq [SQ \cap QSQ^2] \cap Q \subseteq [SQ \cap QSQ] \cap Q$. Hence $Q$ is a bi quasi $\mathscr{A}$-ideal of $S$.

Converse of the Corollary 4.6 is not true in the following Example.

**Example 4.8.** The semiring $S$ in Example 4.6,
$Q = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & a_3 \end{pmatrix} \middle| a_i^{'s} \text{ are real numbers} \right\}$ is a bi quasi $\mathscr{A}$-ideal of $S$ but $Q$ is not a tri quasi $\mathscr{A}$-ideal of $S$ by $[r^{'}q \cap qr^{''}q^2] \cap q = \phi$ and $[qr^{'} \cap q^2r^{''}q] \cap q = \phi$, where $q = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \in Q$ and $r^{'} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \in S$ and $r^{''} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \in S$.

**Theorem 4.7.** *If $Q$ is a tri $\mathscr{A}$-ideal (tri quasi $\mathscr{A}$-ideal) of $S$ and $Q \subseteq Q^{'} \subseteq S$, then $Q^{'}$ is a tri $\mathscr{A}$-ideal (tri quasi $\mathscr{A}$-ideal) of $S$.*

**Proof.** Suppose that $Q$ is a tri quasi $\mathscr{A}$-ideal of $S$ with $Q \subseteq Q^{'} \subseteq S$. Then $\phi \neq [QS \cap QSQ^2] \cap Q \subseteq [Q^{'}S \cap Q^{'}SQ^{'}Q^{'}] \cap Q^{'}$ and $\phi \neq [SQ \cap Q^2SQ] \cap Q \subseteq [SQ^{'} \cap Q^{'}Q^{'}SQ^{'}] \cap Q^{'}$. Therefore, $Q^{'}$ is a tri quasi $\mathscr{A}$-ideal of $S$.

**Corollary 4.8.** *The union of tri $\mathscr{A}$-ideals(tri quasi $\mathscr{A}$-ideals) of $S$ is a tri $\mathscr{A}$-ideal(tri quasi $\mathscr{A}$-ideal) of $S$.*

**Proof.** Let $Q_1$ and $Q_2$ be any two tri $\mathscr{A}$-ideals of $S$. Then $Q_1 \subseteq Q_1 \cup Q_2$, by Theorem 4.7, $Q_1 \cup Q_2$ is a tri $\mathscr{A}$-ideal of $S$.

## 5. Conclusions

The main goal of this work is to present a various ideals and $\mathscr{A}$-ideals in semirings. So in future, we should consider the various generalized ideals and generalized $\mathscr{A}$-ideals in semirings.

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