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SPECIAL CHARACTERIZATIONS OF RECTANGLES IN CONNECTION WITH TRIMORPHIC NUMBERS

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ABSTRACT. This paper consists of two sections A and B. Section A exhibits rectangles, where, in each rectangle, the area added with its semi-perimeter is a Trimorphic number. Section B presents rectangles, where, in each rectangle, the area minus its semi-perimeter is a Trimorphic number.

1. INTRODUCTION

In [1-16], the diophantine problems relate geometrical representations with special numbers, namely, Armstrong numbers, Sphenic numbers, Harshad numbers, etc. The above results motivated us for obtaining rectangles with special characterizations in connection with Trimorphic numbers.

It seems that the above problems has not been considered earlier.

2. METHOD OF ANALYSIS:

Let R be a rectangle with dimensions x and y . Let A and S be represents the Area and Semi-perimeter of R.

2.1. **Section-A: A+S = Trimorphic number with digits 2, 3, 4, 5**

. The problem under consideration is mathematically equivalent to solving the binary quadratic diophantine equation represented by

$$xy + (x + y) = \alpha \tag{A-1}$$

where α is a Trimorphic number in turn.

Rewrite (A-1) as

$$x = \frac{\alpha - y}{y + 1} \tag{A-2}$$

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Given α, it is possible to find x in integers for suitable y in integers. The following Table:1 exhibits the Trimorphic number with their corresponding rectangles satisfying (A-1):

Table 1: $\mathbf{A} + \mathbf{S} = \alpha$

α	R(x,y)	Observation	
		Primitive rectangles	Non-Primitive rectangles
25	$(1,12),(12,1)$	2	–
49	$(1,24),(4,9),(9,4),(24,1)$	4	–
51	$(1,25),(3,12),(12,3),(25,1)$	2	2
75	$(1,37),(3,18),(18,3),(37,1)$	2	2
76	$(6,10),(10,6)$	–	2
99	$(1,49),(3,24),(4,19),(19,4),$ $(24,3),(49,1)$	4	2
125	$(1,62),(2,41),(5,20),$ $(6,17),(8,13),(13,8),$ $(17,6),(20,5),(41,2),$ $(62,1)$	8	2
249	$(1,124),(4,49),(9,24),$ $(24,9),(49,4),(124,1)$	4	2
999	$(1,499),(3,249),(4,199),$ $(7,124),(9,99),(19,49),$ $(24,39),(39,24),(49,19),$ $(99,9),(124,7),(199,4),$ $(249,3),(499,1)$	8	6
1249	$(1,624),(4,249),(9,124),$ $(24,49),(49,24),(124,9),$ $(249,4),(624,1)$	8	–
3751	$(1,1875),(3,937),(6,535),$ $(7,468),(13,267),(27,133),$ $(55,66),(66,55),(133,27),$ $(267,13),(468,7),(535,6),$ $(937,3),(1875,1)$	12	2
4375	$(1,2187),(3,1093),(7,546),$ $(546,7),(1093,3),(2187,1)$	4	2
4999	$(1,2499),(3,1249),(4,999),$ $(7,624),(9,499),(19,249),$ $(24,199),(39,124),(49,99),$ $(99,49),(124,39),(199,24),$ $(249,19),(499,9),(624,7),$ $(999,4),(1249,3),(2499,1)$	18	–
5001	$(1,2500),(40,121),(60,81),$ $(81,60),(121,40),(2500,1)$	4	2
9999	$(1,4999),(3,2499),(4,1999),$ $(7,1249),(9,999),(15,624),$	12	10

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Table 1 – *Continued from previous page*

α	R(x,y)	Observation	
		Primitive rectangles	Non-Primitive rectangles
	$(19,499),(24,399),(39,249),$ $(49,199),(79,124),(124,79),$ $(199,49),(249,39),(399,24),$ $(499,19),(624,15),(999,9),$ $(1249,7),(1999,4),(2499,3),$ $(4999,1)$		
18751	$(1,9375),(3,4687),(7,2343),$ $(15,1171),(31,585),(63,292),$ $(292,63),(585,31),(1171,15),$ $(2343,7),(4687,3),(9375,1)$	12	–
31249	$(1,15624),(4,6249),$ $(9,3124),(24,1249),$ $(49,624),(124,249),$ $(249,124),(624,49),$ $(1249,24),(3124,9),$ $(6249,4),(15624,1)$	12	–
49999	$(1,24999),(3,12499),$ $(4,9999),(7,6249),$ $(9,4999),(15,3124),$ $(19,2499),(24,1999),$ $(39,1249),(49,999),$ $(79,624),(99,499),$ $(124,399),(199,249),$ $(249,199),(399,124),$ $(499,99),(624,79),$ $(999,49),(1249,39),$ $(1999,24),(2499,19),$ $(3124,15),(4999,9),$ $(6249,7),(9999,4),$ $(12499,3),(24999,1)$	28	–
50001	$(1,25000),(22,2173),$ $(45,1086),(1086,45),$ $(2173,22),(25000,1)$	4	2

2.2. **Section-B: A-S = Trimorphic number with digits 2, 3, 4, 5**

. The problem under consideration is mathematically equivalent to solving the binary quadratic diophantine equation represented by

$$xy-(x+y)=\alpha \tag{B-1}$$

where α is a Trimorphic number in turn.
Rewrite (B-1) as

$$x = \frac{\alpha + y}{y - 1} \tag{B-2}$$

Given α, it is possible to find x in integers for suitable y in integers. The following Table:2 exhibits the Trimorphic number with their corresponding rectangles satisfying (B-1):

Table 2: $\mathbf{A} - \mathbf{S} = \alpha$

α	R(x,y)	Observation	
		Primitive rectangles	Non-Primitive rectangles
24	$(2,26),(26,2)$	$-$	2
25	$(2,27),(3,14),(14,3),$ $(27,2)$	4	$-$
49	$(2,51),(3,26),(6,11),$ $(11,6),(26,3),(51,2)$	6	$-$
51	$(2,53),(3,27),(5,14),$ $(14,5),(27,3),(53,2)$	4	2
75	$(2,77),(3,39),(5,20),$ $(20,5),(39,3),(77,2)$	2	4
76	$(2,78),(8,12),(12,8),$ $(78,2)$	$-$	4
99	$(2,101),(3,51),(5,26),$ $(6,21),(21,6),(26,5),$ $(51,3),(101,2)$	4	4
125	$(2,127),(3,64),(4,43),$ $(8,19),(10,15),(15,10),$ $(19,8),(43,4),(64,3),$ $(127,2)$	8	2
249	$(2,251),(3,126),(6,51),$ $(11,26),(26,11),(51,6),$ $(126,3),(251,2)$	4	4
999	$(2,1001),(3,501),(5,251),$ $(6,201),(9,126),(11,101),$ $(21,51),(26,41),(41,26),$ $(51,21),(101,11),(126,9),$ $(201,6),(251,5),(501,3),$ $(1001,2)$	8	8
1249	$(2,1251),(3,626),(6,251),$ $(11,126),(26,51),(51,26),$ $(126,11),(251,6),(626,3),$ $(1251,2)$	10	$-$
3751	$(2,3753),(3,1877),(5,939),$ $(8,537),(9,470),(15,269),$ $(29,135),(57,68),(68,57),$ $(135,29),(269,15),(470,9),$ $(537,8),(939,5),(1877,3),$	16	$-$

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Table 2 – *Continued from previous page*

α	R(x,y)	Observation	
		Primitive rectangles	Non-Primitive rectangles
	$(3753, 2)$		
4375	$(2, 4377), (3, 2189), (5, 1095), (9, 548), (548, 9), (1095, 5), (2189, 3), (4377, 2)$	6	2
4999	$(2, 5001), (3, 2501), (5, 1251), (6, 1001), (9, 626), (11, 501), (21, 251), (26, 201), (41, 126), (51, 101), (101, 51), (126, 41), (201, 26), (251, 21), (501, 11), (626, 9), (1001, 6), (1251, 5), (2501, 3), (5001, 2)$	20	–
5001	$(2, 5003), (3, 2502), (42, 123), (62, 83), (83, 62), (123, 42), (2502, 3), (5003, 2)$	4	4
9999	$(2, 10001), (3, 5001), (5, 2501), (6, 2001), (9, 1251), (11, 1001), (17, 626), (21, 501), (26, 401), (41, 251), (51, 201), (81, 126), (126, 81), (201, 51), (251, 41), (401, 26), (501, 21), (626, 17), (1001, 11), (1251, 9), (2001, 6), (2501, 5), (5001, 3), (10001, 2)$	10	14
18751	$(2, 18753), (3, 9377), (5, 4689), (9, 2345), (17, 1173), (33, 587), (65, 294), (294, 65), (587, 33), (1173, 17), (2345, 9), (4689, 5), (9377, 3), (18753, 2)$	12	2
31249	$(2, 31251), (3, 15626), (6, 6251), (11, 3126), (26, 1251), (51, 626), (126, 251), (251, 126), (626, 51), (1251, 26), (3126, 11), (6251, 6), (15626, 3), (31251, 2)$	14	–
49999	$(2, 50001), (3, 25001), (5, 12501), (6, 10001), (9, 6251), (11, 5001), (17, 3126), (21, 2501), (26, 2001), (41, 1251), (51, 1001), (81, 626), (101, 501), (126, 401),$	30	–

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Table 2 – *Continued from previous page*

α	R(x,y)	Observation	
		Primitive rectangles	Non-Primitive rectangles
	$(201, 251), (251, 201),$ $(401, 126), (501, 101),$ $(626, 81), (1001, 51),$ $(1251, 41), (2001, 26),$ $(2501, 21), (3126, 17),$ $(5001, 11), (6251, 9),$ $(10001, 6), (12501, 5),$ $(25001, 3), (50001, 2)$		
50001	$(2, 50003), (3, 25002),$ $(24, 2175), (47, 1088),$ $(1088, 47), (2175, 24),$ $(25002, 3), (50003, 2)$	4	4

3. CONCLUSION

In this paper, we have presented rectangles such that, in each rectangle, the area added with its semi-perimeter as well as the area minus the semi-perimeter is represented by a Trimorphic number.

To conclude, one may search for rectangles with other characterization in connection with higher order Trimorphic numbers.

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