ANNALS OF COMMUNICATIONS IN MATHEMATICS
Volume 5, Number 2 (2022), 80-87
ISSN: 2582-0818

SOME PERFECT SETS IN IDEAL NANO TOPOLOGICAL SPACES

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ABSTRACT. We introduce the notions of nano $\mathfrak{L}^{\star}$-perfect, nano $\mathfrak{R}^{\star}$-perfect, and nano $\mathfrak{C}^{\star}$-perfect sets in ideal nano spaces and study their properties. We obtained a characterization for compatible ideals via nano $\mathfrak{R}^{\star}$-perfect sets and and investigate further their important properties

1. INTRODUCTION

An ideal I [13] o n a space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

(1) $A \in I$ and $B \subset A$ imply $B \in I$ and
(2) $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a space (X, τ) with an ideal I on X if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^{\star} : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^{\star}(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [1]. The closure operator defined by $cl^{\star}(A) = A \cup A^{\star}(I, \tau)$ [12] is a Kuratowski closure operator which generates a topology $\tau^{\star}(I, \tau)$ called the $\star$-topology which is finer then τ. We will simply write $A^{\star}$ for $A^{\star}(I, \tau)$ and $\tau^{\star}$ for $\tau^{\star}(I, \tau)$. If I is an ideal on X, then (X, τ, I) is called an ideal topological space or an ideal space.

In this paper, we introduce nano $\mathfrak{L}^{\star}$-perfect, nano $\mathfrak{R}^{\star}$-perfect, and nano $\mathfrak{C}^{\star}$-perfect sets in ideal nano spaces and study their properties. We obtained a characterization for compatible ideals via nano $\mathfrak{R}^{\star}$-perfect sets.

2. PRELIMINARIES

Definition 2.1. [7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

2010 *Mathematics Subject Classification.* 54C60, 54E55.
Key words and phrases. nano $\mathfrak{L}^{\star}$-perfect, nano $\mathfrak{R}^{\star}$-perfect and nano $\mathfrak{C}^{\star}$-perfect.
Received: July 06, 2022. Accepted: October 10, 2022. Published: November 30, 2022.
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(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.
(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.
(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then R(X) satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$,
(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset O of U are denoted by $I_n(O)$ and $C_n(O)$, respectively.

A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [4] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \,|x \in G_n, G_n \in \mathcal{N}\}$, denotes [4] the family of nano open sets containing x.

In future an ideal nano topological spaces $(U, \mathcal{N}, I)$ is referred as a space.

Definition 2.3. [4] Let $(U, \mathcal{N}, I)$ be a space with an ideal I on U. Let $(.)^\star_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U).

For a subset $O \subseteq U$, $O^\star_n(I, \mathcal{N}) = \{x \in U : G_n \cap O \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $O^\star_n$ for $O^\star_n(I, \mathcal{N})$.

Theorem 2.1. [4] *Let $(U, \mathcal{N}, I)$ be a space and O and B be subsets of U. Then*

(1) $O \subseteq B \Rightarrow O^\star_n \subseteq B^\star_n$,
(2) $O^\star_n = C_n(O^\star_n) \subseteq C_n(O)$ *($O^\star_n$ is a n-closed subset of $C_n(O)$),*
(3) $(O^\star_n)^\star_n \subseteq O^\star_n$,
(4) $(O \cup B)^\star_n = O^\star_n \cup B^\star_n$,
(5) $V \in \mathcal{N} \Rightarrow V \cap O^\star_n = V \cap (V \cap O)^\star_n \subseteq (V \cap O)^\star_n$,
(6) $J \in I \Rightarrow (O \cup J)^\star_n = O^\star_n = (O - J)^\star_n$.

Theorem 2.2. [4] *Let $(U, \mathcal{N}, I)$ be a space with an ideal I and $O \subseteq O^\star_n$, then $O^\star_n = C_n(O^\star_n) = C_n(O)$.*

Definition 2.4. [6] A subset A of a space $(U, \mathcal{N}, I)$ is $n\star$-dense in itself (resp. $n\star$-perfect and $n\star$-closed) if $O \subseteq O^\star_n$ (resp. $O = O^\star_n$, $O^\star_n \subseteq O$).

The complement of a $n\star$-closed set is said to be $n\star$-open.

Definition 2.5. [3] A subset O of U in a nano topological space $(U, \mathcal{N})$ is called nano-codense (briefly n-codense) if $U - O$ is n-dense.

Definition 2.6. [4] Let $(U, \mathcal{N}, I)$ be a space. The set operator $C_n^\star$ called a nano $\star$-closure is defined by $C_n^\star(O) = O \cup O_n^\star$ for $O \subseteq U$.

It can be easily observed that $C_n^\star(O) \subseteq C_n(O)$.

Theorem 2.3. [5] *In a space $(U, \mathcal{N}, I)$, if O and B are subsets of U, then the following results are true for the set operator n-$cl^\star$.*

(1) $O \subseteq C_n^\star(O)$,
(2) $C_n^\star(\phi) = \phi$ *and* $C_n^\star(U) = U$,
(3) $If O \subset B$, *then* $C_n^\star(O) \subseteq C_n^\star(B)$,
(4) $C_n^\star(O) \cup C_n^\star(B) = C_n^\star(O \cup B)$.
(5) $C_n^\star(C_n^\star(O)) = C_n^\star(O)$.

Definition 2.7. A subset O of a space $(U, \mathcal{N}, I)$ is said to be

(1) nano-I-open (resp. nI-open) [5] if $O \subseteq I_n(O_n^\star)$.
(2) nano regular-I-closed (resp. regular-nI-closed) [10] if $O = (I_n(O))_n^\star$.

3. Some perfect sets in ideal nano spaces

Definition 3.1. A subset O of an ideal nano space $(U, \mathcal{N}, I)$, is called a

(1) nano-I-dense (resp. nI-dense if $O_n^\star = U$.
(2) nano $\mathfrak{L}^\star$-perfect (resp. $n\mathfrak{L}^\star$-perfect) if $O - O^\star \in I$.
(3) nano $\mathfrak{R}^\star$-perfect (resp. nano $n\mathfrak{R}^\star$-perfect) if $O^\star - O \in I$.
(4) nano $\mathfrak{C}^\star$-perfect (resp.(resp. $n\mathfrak{L}^\star$-perfect)) if O is both $n\mathfrak{L}^\star$-perfect and $n\mathfrak{R}^\star$-perfect.

Example 3.1. *Let* $U = \{o_1, o_2, o_3, o_4\}$ *with* $U/R = \{\{o_2\}, \{o_4\}, \{o_1, o_3\}\}$ *and* $X = \{o_3, o_4\}$. *Then the nano topology* $\mathcal{N} = \{\phi, \{o_4\}, \{o_1, o_3\}, \{o_1, o_3, o_4\}, U\}$ *and* $I = \{\phi, \{o_3\}\}$. *Clearly the sets*

(1) $\{\phi, \{o_1, o_4\}, \{o_1, o_2, o_4\}, \{o_1, o_3, o_4\}, U\}$ *is* nI*-dense.*
(2) $\{\phi, \{o_3\}, \{o_2, o_3\}, \{o_3, o_4\}, \{o_2, o_3, o_4\}, U\}$ *is* $n\mathfrak{L}^\star$*-perfect.*
(3) $\{\phi, \{o_1, o_2\}, \{o_2, o_3\}, \{o_1, o_2, o_4\}, U\}$ *is* $n\mathfrak{R}^\star$*-perfect.*
(4) $\{\phi, \{o_2, o_3\}, U\}$ *is* $n\mathfrak{C}^\star$*-perfect.*

Remark. *Every* $n\star$*-perfect set is both* $n\mathfrak{R}^\star$*-perfect and* $n\mathfrak{L}^\star$*- perfect.*

Example 3.2. *In Example 3.1, the set* $\{o_2, o_3\}$ *is both* $n\mathfrak{R}^\star$*-perfect and* $n\mathfrak{L}^\star$*- perfect but not* $n\star$*-perfect set.*

Proposition 3.3. *If a subset* O *of an ideal nano space* $(U, \mathcal{N}, I)$ *is* $n\mathfrak{C}^\star$*-perfect, then* $O \Delta O_n^\star \in I$.

Proof.

Since O is both $n\mathfrak{L}^\star$-perfect and $n\mathfrak{R}^\star$-perfect, $O - O_n^\star \in I$ and $O_n^\star - O \in I$. By the finite additive property of ideals, $(O - O_n^\star) \cup (O_n^\star - O) \in I$. Hence $O \Delta O_n^\star \in I$. □

Proposition 3.4. *In an ideal nano space* $(U, \mathcal{N}, I)$*, every* $n\star$*-closed set is* $n\mathfrak{R}^\star$*-perfect.*

Proof.

Let O be a $n\star$-closed set. Therefore, $O_n^\star \subseteq O$. Hence $O_n^\star - O = \phi = I$. Therefore, O is an $n\mathfrak{R}^\star$-perfect set. □

Remark. *The converses of Proposition 3.4 is not true.*

Example 3.5. *Let $U = \{o_1, o_2, o_3\}$ with $U/R = \{\{o_1\}, \{o_2, O_3\}\}$ and $X = \{o_1\}$. Then the nano topology $\mathcal{N} = \{\phi, \{o_1\}, U\}$ and $I = \{\phi, \{o_2\}\}$.*

Clearly, the set $\{o_3\}$ is $n\mathfrak{R}^\star$-perfect but not $n\star$-closed set.

Corollary 3.6. *In an ideal nano space $(U, \mathcal{N}, I)$,*

(1) U and ϕ are $n\mathfrak{R}^\star$-perfect sets,
(2) every $n\star$-closed set is $n\mathfrak{R}^\star$-perfect,
(3) for any subset O of an ideal nano topological space, $C_n(O), O_n^\star, C_n^\star(O)$ are $n\mathfrak{R}^\star$-perfect sets,
(4) every regular-nI-closed set is $n\mathfrak{R}^\star$-perfect.

Proof.
The proof follows from Proposition 3.4. □

Remark. *The converses of Corollary 3.6(4) is not true.*

Example 3.7. *In Example 3.1, the set $\{o_1, o_2\}$ is $n\mathfrak{R}^\star$-perfect but not regular-nI-closed.*

Proposition 3.8. *If a subset O of an ideal nano topological space is such that $O \in I$, then O is $n\mathfrak{C}^\star$-perfect.*

Proof.
Since $O \in I, O_n^\star = \phi$. Then $O - O_n^\star = O \in I$ and $O_n^\star - O = \phi \in I$. Hence O is both an $n\mathfrak{L}^\star$-perfect and $n\mathfrak{R}^\star$-perfect set. □

Corollary 3.9. *Let O be a subset of an ideal nano space. Consider the following.*

(1) If $O \in I$, then every subset of O is a $n\mathfrak{C}^\star$-perfect set.
(2)) If O is $n\mathfrak{R}^\star$-perfect, then $O_n^\star - O$ is $n\mathfrak{C}^\star$-perfect.
(3) If O is $n\mathfrak{L}^\star$-perfect set, then $O - O_n^\star$ is a $n\mathfrak{C}^\star$-perfect set.
(4) If O is $n\mathfrak{C}^\star$-perfect, then $O \Delta O_n^\star$ is a $n\mathfrak{C}^\star$-perfect set.

Proof.
The proof follows from Proposition 3.8.
□

Proposition 3.10. *In an ideal nano space $(U, \mathcal{N}, I)$, every $n\star$-dense set is $n\mathfrak{L}^\star$-perfect set.*

Proof.
Let O be a $n\star$-dense set of U. Then $O \subseteq O_n^\star$. Therefore, $O - O_n^\star = \phi \in I$. Hence O is an $n\mathfrak{L}^\star$-perfect set. □

Remark. *The converses of the above Proposition 3.10 need not to be true.*

Example 3.11. *In Example 3.5, the set $\{o_2\}$ is $n\mathfrak{L}^\star$-perfect set but not $n\star$-dense.*

Corollary 3.12. *In an ideal nano space $(U, \mathcal{N}, I)$*

(1) every nI-dense set is $n\mathfrak{L}^\star$-perfect,
(2) every nI-open set is $n\mathfrak{L}^\star$-perfect,
(3) every regular-nI-closed set is $n\mathfrak{L}^\star$-perfect.

Proof.
Since all the above sets are $n\star$-dense, by Proposition 3.10, these sets are $n\mathfrak{L}^\star$-perfect.
□

Remark. *The converses of the above Corollary 3.12 need not to be true.*

Example 3.13. *In Example 3.5,*

(1) *the set $\{o_2\}$ is $n\mathfrak{L}^\star$-perfect but not nI-dense.*
(2) *the set $\{o_2\}$ is $n\mathfrak{L}^\star$-perfect but not nI-open.*
(3) *In Example 3.1,*
    (a) *the set $\{o_3\}$ is $n\mathfrak{L}^\star$-perfect but not regular-nI-closed.*

Proposition 3.14. *In an ideal nano space $(U, \mathcal{N}, I)$,*

(1) *ϕ is $n\mathfrak{L}^\star$-perfect set.*
(2) *U is a $n\mathfrak{L}^\star$-perfect set if the ideal is n-codense.*

Proof.

(1) Since $\phi - \phi^\star_n = \phi \in I$, the empty set is an $n\mathfrak{L}^\star$-perfect set.
(2) We know that $U = U^\star_n$ if and only if the ideal I is n-codense. Then $U - U^\star_n = \phi \in I$.

□

Proposition 3.15. *Let $(U, \mathcal{N}, I)$ be an ideal nano space. Let O and K be two subsets of U such that $O \subseteq K$ and $O^\star_n = K^\star_n$, then*

(1) *K is $n\mathfrak{R}^\star$-perfect if O is $n\mathfrak{R}^\star$-perfect;*
(2) *O is $n\mathfrak{L}^\star$-perfect if K is $n\mathfrak{L}^\star$-perfect.*

Proof. (1) Let O be a $n\mathfrak{R}^\star$-perfect set. Then $O^\star_n - O \in I$. Now, $K^\star_n - K = O^\star_n - K \subseteq O^\star_n - O$. By heredity property of ideals, $K^\star_n - K \in I$. Hence K is $n\mathfrak{R}^\star$-perfect.
(2)) Let K be a $n\mathfrak{L}^\star$-perfect set. Then $K - K^\star_n \in I$. Now, $O - O^\star_n = O - K^\star_n \subseteq K - K^\star_n$. By heredity property of ideals, $O - O^\star_n \in I$. Hence O is $n\mathfrak{L}^\star$-perfect.

□

Proposition 3.16. *Let $(U, \mathcal{N}, I)$ be an ideal nano space. Let O and K be two subsets of U such that $O \subseteq K \subseteq C^\star_n(O)$, then*

(1) *K is $n\mathfrak{R}^\star$-perfect if O is $n\mathfrak{R}^\star$-perfect,*
(2) *O is $n\mathfrak{L}^\star$-perfect if K is $n\mathfrak{L}^\star$-perfect.*

Proof.

Since $O \subseteq K \subseteq C^\star_n(O)$, $O^\star_n \subseteq K^\star_n \subseteq (C^\star_n(O))^\star_n = O^\star_n$. Hence $O^\star_n = K^\star_n$. Therefore, the result follows from Proposition 3.15. □

Proposition 3.17. *Let O be a subset of an ideal nano topological space $(U, \mathcal{N}, I)$ such that O is $n\mathfrak{L}^\star$-perfect set and $O \cap O^\star_n$ is $n\mathfrak{R}^\star$-perfect; then both O and $O \cap O^\star_n$ are $n\mathfrak{C}^\star$-perfect.*

Proof.

Since O is $n\mathfrak{L}^\star$-perfect, $O - O^\star_n \in I$. By Theorem 2.1(6), for every $J \in I$, $(O \cup J)^\star_n = O^\star_n = (O - J)^\star_n$. Therefore, $(O \cup (O - O^\star_n))^\star_n = O^\star_n = (O - (O - O^\star_n))^\star_n$. This implies $O^\star_n = (O \cap O^\star_n)^\star_n$. Therefore, we have $O \cap O^\star_n \subseteq O$ with $(O \cap O^\star_n)^\star_n = O^\star_n$. By Proposition 3.15, O is $n\mathfrak{R}^\star$-perfect if $O \cap O^\star_n$ is $n\mathfrak{R}^\star$-perfect and $O \cap O^\star_n$ is $n\mathfrak{L}^\star$-perfect if O is $n\mathfrak{L}^\star$-perfect set. Hence O is $n\mathfrak{R}^\star$-perfect and $O \cap O^\star_n$ is $n\mathfrak{L}^\star$-perfect. □

Proposition 3.18. *If a subset O of an ideal nano topological space $(U, \mathcal{N}, I)$ is $n\mathfrak{R}^\star$-perfect set and $O^\star_n$ is $n\mathfrak{L}^\star$-perfect, then $O \cap O^\star_n$ is $n\mathfrak{L}^\star$-perfect.*

Proof.

Since O is $n\mathfrak{R}^\star$-perfect,$O^\star_n - O \in I$. By Theorem 2.1(6), for every $J \in I$, $(O \cup J)^\star_n = O^\star_n = (O - J)^\star_n$. Therefore, $(O^\star_n \cup (O^\star_n - O))^\star_n = (O^\star_n)^\star_n = (O^\star_n - (O^\star_n - O))^\star_n$. This implies $(O^\star_n)^\star_n = (O \cap O^\star_n)^\star_n$. Therefore, we have $O \cap O^\star_n \subseteq O^\star_n$ with $(O \cap O^\star_n)^\star_n = (O^\star_n)^\star_n$.

By Proposition 3.15, $O \cap O_n^\star$ is $n\mathfrak{L}^\star$-perfect if $O_n^\star$ is $n\mathfrak{L}^\star$-perfect set. Hence $O \cap O_n^\star$ is $n\mathfrak{L}^\star$-perfect.

□

Proposition 3.19. *If O and K are $n\mathfrak{R}^\star$-perfect sets, then $O \cup K$ is an $n\mathfrak{R}^\star$-perfect set.*

Proof.

Let O and K be $n\mathfrak{R}^\star$-perfect sets. Then $O_n^\star - O \in I$ and $K_n^\star - K \in I$. By finite additive property of ideals, $(O_n^\star - O) \cup (K_n^\star - K) \in I$. Since $(A_n^\star \cup K_n^\star) - (O \cup K) \subseteq (O_n^\star - O) \cup (K_n^\star - K)$, by heredity property $(O_n^\star \cup K_n^\star) - (O \cup K) \in I$. Hence $(O \cup K)_n^\star - (O \cup K) \in I$. □

Corollary 3.20. *Finite union of $n\mathfrak{R}^\star$-perfect sets is $n\mathfrak{R}^\star$-perfect set.*

Proof.

The proof follows from Proposition 3.19. □

Example 3.21. *In Example 3.1, the two sets $O = \{o_2, o_3\}$ and $K = \{o_1, o_2, o_4\}$ in $n\mathfrak{R}^\star$-perfect, then union of $O \cup K = U$ in $n\mathfrak{R}^\star$-perfect.*

Proposition 3.22. *If O and K are $n\mathfrak{L}^\star$-perfect sets, then $O \cup K$ is $n\mathfrak{L}^\star$-perfect set.*

Proof.

Since O and K are $n\mathfrak{L}^\star$-perfect sets, $O - O_n^\star \in I$ and $K - K_n^\star \in I$. Hence by finite additive property of ideals, $(O - O_n^\star) \cup (K - K_n^\star) \in I$. Since $(O \cup K) - (O \cup K)_n^\star = (O \cup K) - (O_n^\star \cup K_n^\star) \subseteq (O - O_n^\star) \cup (K - K_n^\star)$, by heredity property $(O \cup K) - (O \cup K)_n^\star \in I$. This proves that $O \cup K$ is an $n\mathfrak{L}^\star$-perfect set. □

Corollary 3.23. *Finite union of $n\mathfrak{L}^\star$-perfect sets is $n\mathfrak{L}^\star$-perfect sets.*

Proof.

The proof follows from Proposition 3.22. □

Example 3.24. *In Example 3.1, the two sets $O = \{o_2, o_3\}$ and $K = \{o_3, o_4\}$ in $n\mathfrak{L}^\star$-perfect, then union of $O \cup K = \{o_2, o_3, o_4\}$ in $n\mathfrak{L}^\star$-perfect.*

Proposition 3.25. *If O and K are $n\mathfrak{R}^\star$-perfect sets, then $O \cap K$ is $n\mathfrak{R}^\star$-perfect set.*

Proof.

Suppose that O and K are $n\mathfrak{R}^\star$-perfect sets. Then $O_n^\star - O \in I$ and $K_n^\star - K \in I$. By finite additive property of ideals, $(O_n^\star - O) \cup (K_n^\star - K) \in I$. Since $(O_n^\star \cap K_n^\star) - (O \cap K) \subseteq (O_n^\star - O) \cup (K_n^\star - K)$, by heredity property $(O_n^\star \cap K_n^\star) - (O \cap K) \in I$. Also $(O \cap K)_n^\star - (O \cap K) \subseteq (O_n^\star \cap K_n^\star) - (O \cap K) \in I$. □

Corollary 3.26. *Finite intersection of $n\mathfrak{R}^\star$-perfect sets is a $n\mathfrak{R}^\star$-perfect set.*

Proof.

The proof follows from Proposition 3.25. □

Proposition 3.27. *Finite union of $n\mathfrak{C}^\star$-perfect sets is a $n\mathfrak{C}^\star$-perfect set.*

Proof.

Above the Corollaries 3.23 and 3.26, finite union of $n\mathfrak{C}^\star$-perfect sets is a $n\mathfrak{C}^\star$-perfect set. □

Proposition 3.28. *Let $(U, \mathcal{N}, I)$ be an ideal nano space and $O \subseteq U$. The set O is $n\mathfrak{R}^\star$-perfect if and only if$K \subseteq O_n^\star - O \in U \Longrightarrow K \in I$*

Proof.
Assume that O is a $n\mathfrak{R}^\star$-perfect set. Then $O_n^\star - O \in I$. By heredity property of ideals, every set $K \subseteq O_n^\star - O \in U$ is also in I. Conversely assume that $K \subseteq O_n^\star - O \in U \Longrightarrow K \in I$. Since $O_n^\star - O$ is a subset of itself, by assumption $O_n^\star - O \in I$. Hence O is $n\mathfrak{R}^\star$-perfect. □

Proposition 3.29. *Let $(U, \mathcal{N}, I)$ be an ideal nano space and $O \subseteq U$. The set O is $n\mathfrak{L}^\star$-perfect if and only if$K \subseteq O - O_n^\star \in U \Longrightarrow K \in I$*

Proof.
Assume that O is a $n\mathfrak{L}^\star$-perfect set. Then $O - O_n^\star \in I$. By heredity property of ideals, every set $K \subseteq O - O_n^\star \in U$ is also in I. Conversely assume that $K \subseteq O - O_n^\star \in U \Longrightarrow K \in I$. Since $O - O_n^\star$ is a subset of itself, by assumption $O - O_n^\star \in I$. Hence O is $n\mathfrak{L}^\star$-perfect. □

4. Conclusion

The notions of the sets, functions and spaces in ideal topological spaces are highly developed and used extensively in many practical and engineering problems, computational topology for geometric design, computer-aided geometric design, engineering design research and mathematical sciences. Also, topology plays a significant role in space time geometry and high-energy physics. Thus generalized continuity is one of the most important subjects on topological spaces. Hence we studied new types of generalizations of non-continuous functions, obtained some of their properties in ideal nano topological spaces. Moreover, the ideal nano topological version of the concepts and the results introduced in this paper may be applied by using the concepts of fuzzy sets and fuzzy functions.

5. Acknowledgement

The authors thank the referees for their valuable comments and suggestions for improvement of this paper.

References

[1] K. Kuratowski, *Topology*, Vol I. Academic Press (New York) 1966.
[2] M. Lellis Thivagar and Carmel Richard, *On nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention,1(1)(2013), 31-37.
[3] O. Nethaji, R. Asokan and I. Rajasekaran, *New generalized classes of an ideal nano topological spaces*, Bull. Int. Math. Virtual Inst., 9(3)(2019), 543-552.
[4] M. Parimala, T. Noiri and S. Jafari, *New types of nano topological spaces via nano ideals* (to appear).
[5] M. Parimala and S. Jafari, *On some new notions in nano ideal topological spaces*, International Balkan Journal of Mathematics(IBJM), 1(3)(2018), 85-92.
[6] M. Parimala, S. Jafari and S. Murali, *Nano ideal generalized closed sets in nano ideal topological spaces*, Annales Univ. Sci. Budapest., 60(2017), 3-11.
[7] Z. Pawlak, *Rough sets*, International journal of computer and Information Sciences, 11(5)(1982), 341-356.
[8] I. Rajasekaran, N. Sekar and A. Pandi, *On Ⓢ-closed sets and semi Ⓢ-closed in nano topological spaces*, Annals of Communications in Mathematics, 5(1)(2022), 55-62.
[9] I. Rajasekaran, *On $^\star b$-open sets and $^\star b$-sets in nano topological spaces*, Asia Mathematika, 5(3)(2022), 84-88.
[10] N. Sekar, R. Asokan and I. Rajasekaran, *On regular-closed sets in ideal nano topological spaces*, (communicated).
[11] N. Sekar, R. Asokan and I. Rajasekaran, *On $\mathbb{S}_{p^\star}$-open sets in ideal nano topological spaces*, Asia Mathematika, 6(2)(2022), 39-47.
[12] R. Vaidyanathaswamy, *The localization theory in set topology*, Proc. Indian Acad. Sci., 20(1945), 51-61.
[13] R. Vaidyanathaswamy, *Set topology*, Chelsea Publishing Company, New York, 1946.

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