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PRIME UP-FILTER OF THE THIRD KIND IN MEET-COMMUTATIVE UP-ALGEBRAS

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ABSTRACT. UP-algebra was introduced by Iampan 2017 as a generalization of KU-algebra. The concept of meet-commutative UP–algebras was introduced by Sawika et al. In such algebras, the notion of prime UP-filter of the first kind and the notion of prime UP-filter of the second kind are introduced. In this article, as a continuation of the previous two, the concept of prime UP–filters of the third kind was introduced and the relationship between these three types of prime UP-filters was discussed.

1. INTRODUCTION

The concept of KU-algebras was introduced in 2009 by Prabpayak and Leerawat in the article [6]. Iampan introduced the concept of UP-algebras as a generalization of KU-algebras ([1]). In [12], Somjanta et al. introduced the notion of filters in this class of algebra. Proper UP-filter in a UP-algebra was introduced by Romano 2018 ([7, 8]). Jun and Iampan then introduced and analyzed several classes of filters in UP algebras such as implicative, comparative and shift UP-filters (see, for example, [2, 3, 4]). The concept of weak implicative UP-filters in a UP-algebra was introduced and analyzed by Romano and Jun ([9]).

The concept of meet-commutative UP-algebras was introduced in article [11]. In article [5], a number of important properties of meet-commutative UP-algebras are given. In addition, in such UP-algebras, the concept of prime UP-filters was introduced and analyzed. Then, in [10] the author introduces the notion of prime UP-filters of the second kind in meet-commutative UP-algebras and connects it with the prime UP-filters of the first type.

In this article we introduce the concept of prime UP-filter of third kind in a meet-commutative UP-algebra and we analyze the interrelationships of these three types of prime UP-filters. It is shown that every prime UP-filter of the second kind in a meet-commutative UP-algebra is a prime UP-filter of the third kind (Theorem 3.1) but that the reverse does not have to be (Example 3.7). Also, it has been shown that a prime UP-filter

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of third kind in a meet-commutative UP-algebra does not have to be a prime UP-filter of the first kind and vice versa. However, in a pre-linear meet-commutative UP-algebra, all three of these observed types of prime UP-filters are coincide.

2. Preliminaries

In this section, taking from the literature, we will repeat some concepts and statements of interest for this research.

An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra* (see [1]) if it satisfies the following axioms:

(UP-1) $(\forall x, y, \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,
(UP-2) $(\forall x \in A)(0 \cdot x = x)$,
(UP-3) $(\forall x \in A)(x \cdot 0 = 0)$,
(UP-4) $(\forall x, y \in A)((x \cdot y = 0 \wedge y \cdot x = 0) \implies x = y)$.

A UP-algebra A is said to be meet-commutative (see [3], Definition 3) if it satisfies the condition

$$(\forall x, y \in A)((x \cdot y) \cdot y = (y \cdot x) \cdot x).$$

This term also appears in the paper [11] (Definition 1.15).

In a UP-algebra, the order relation '$\leqslant$' is defined as follows

$$(\forall x, y \in A)(x \leqslant y \iff x \cdot y = 0).$$

A subset F of a UP-algebra A is called a *UP-filter* in A (see [12]) if it satisfies the following conditions:

(F-1) $0 \in F$,
(F-2) $(\forall x, y \in A)((x \in F \wedge x \cdot y \in F) \implies y \in F)$.

It is clear that every UP-filter F in a UP-algebra A satisfies:

(1) $(\forall x, y \in A)((x \in F \wedge x \leqslant y) \implies y \in F)$.

We first characterize the meet-commutative UP-algebras.

Theorem 2.1 ([5], Theorem 3.1)**.** *Let A be a meet-commutative UP-algebra. Then the following holds*

(2) $(\forall x, y \in A)(x \leqslant y \implies y = (y \cdot x) \cdot x)$.

Theorem 2.2 ([5], Theorem 3.2)**.** *Let A be a meet-commutative UP-algebra. For any $x, y \in A$, the element $x \sqcup y := (x \cdot y) \cdot y = (y \cdot x) \cdot x$ is the least upper bound of x and y.*

Proposition 2.3 ([5], Proposition 3.1; [10], Proposition 2.2)**.** *Let A be a meet-commutative UP-algebra. Then*

(3) $(\forall x, y \in A)(0 \sqcup x = x,\ x \sqcup 0 = 0,\ x \sqcup x = x,$ *and* $x \sqcup y = y \sqcup x)$.
(4) $(\forall x, y, z \in A)((x \sqcup y) \sqcup z = (x \sqcup z) \sqcup (y \sqcup z))$.
(5) $(\forall x, y, z \in A)((z \cdot x) \sqcup (z \cdot y) \leqslant z \cdot (x \sqcup y))$.
(6) $(\forall x, y, z \in A)((x \sqcup y) \cdot z \leqslant (x \cdot z) \sqcup (y \cdot z))$.
(7) $(\forall x, y \in A)(x \sqcup y \leqslant (y \cdot x) \sqcup (x \cdot y))$.

3. Prime UP-filters of the third kind

The notion of prime UP-filters in a meet-commutative UP-algebra was introduced in article [5]. For the purposes of this paper, we will recognize such a UP filter as a 'prime UP filter of the first kind'.

Definition 3.1. Let F be a UP-filter in a meet-commutative UP-algebra A. Then F is said to be a *prime UP-filter of the first kind* in A if the following holds
(PF1) $(\forall x, y \in A)(x \sqcup y \in F \implies (x \in F \vee y \in F))$.

Example 3.2. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	c	0
b	0	c	0	c
c	0	b	b	0

Then $A = (A, \cdot, 0)$ is a meet-commutative UP-algebra. Subsets $\{0\}$, $\{0, b\}$ and $\{0, c\}$ are UP-filters in A. It is not difficult to verify that UP-filters $\{0, b\}$ and $\{0, c\}$ are prime of the first kind. It is clear that $\{0\}$ is not a prime UP-filter of the first kind in A because $b \sqcup c = 0 \in \{0\}$ but $b \notin \{0\}$ and $c \notin \{0\}$.

The following definition gives another type of prime UP-filter in meet-commutative UP-algebras.

Definition 3.3. ([10], Definition 3.2) Let F be a UP-filter in a meet-commutative UP-algebra A. Then F is said to be a *prime UP-filter of the second kind* in A if the following holds
(PF2) $(\forall x, y \in A)(x \cdot y \in F \vee y \cdot x \in F)$.

Example 3.4. Let A be as in Example 3.2. Then $A = (A, \cdot, 0)$ is a meet-commutative UP-algebra. Subsets $\{0, c\}$ is a prime UP-filter of the second kind in A. The subset $F := \{0, b\}$ is a prime UP-filter of the first kind but it is not a prime UP-filter of the second kind because, for example, holds $a \cdot b = c \notin F$ and $b \cdot a = c \notin F$.

In the previous example it was shown that a UP filter can be a prime UP-filter of the first kind but it does not have to be a prime UP-filter of the second kind. However, it has been shown ([10], Theorem 3.1) that if F satisfies the condition (PF2), then it satisfies the condition (PF1) also, i.e. any prime UP-filter of the second kind in a meet-commutative UP-algebra A is a prime UP-filter of the first kind in A.

The following definition introduces the term 'prime UP–filter of the third kind' in meet-commutative UP–algebras.

Definition 3.5. Let F be a UP–filter in a meet-commutative UP-algebra A. Then F is said to be a *prime UP-filter of the third kind* in A if the following holds
(PF3) $(\forall x, y \in A)((x \cdot y) \sqcup (y \cdot x) \in F)$.

The following example shows that a UP-filter in a meet-commutative UP algebra does not have to be a prime UP-filter of the third kind.

Example 3.6. Let A be as in Example 3.2. Then $A = (A, \cdot, 0)$ is a meet-commutative UP-algebra. Subset $F := \{0\}$ is a UP-filter in A but it is not a prime UP-filter of the third kind in A because, for example, we have $(a \cdot b) \sqcup (b \cdot a) = c \notin F$. The subset $G; = \{0, b\}$ is not a prime UP-filter of the third kind in A also because, for example, $(a \cdot b) \sqcup (b \cdot a) = c \notin G$ holds. Subset $\{0, c\}$ is a prime UP filter of the third kind in A.

The following example shows that a UP-filter in a meet-commutative algebra can be a prime UP-filter of the third kind and neither a UP-filter of the first kind nor a prime UP-filter of the second kind.

Example 3.7. Let $A = \{1, a, b, c, d\}$ and operations '$\cdot$' on A as follows:

$\cdot$	0	a	b	c	d
0	0	a	b	c	d
a	0	0	0	0	0
b	0	b	0	0	0
c	0	c	c	0	d
d	0	d	d	c	0

Then $(A, \cdot, 0)$ is a meet-commutative UP-algebra. Here it is $(c \cdot d) \cdot d = d \cdot d = 0$, and $(d \cdot c) \cdot c = c \cdot c = 0$. So $c \sqcup d = 0$. Subset $F := \{0\}$ is a UP-filter in A. Obviously, this filter is not a prime UP-filter of the first kind because $c \sqcup d = 0 \in F$ but $c \notin F$ and $d \notin F$. It can be shown by direct verification that F is a prime UP-filter of the third type in A. Also, this UP-filter is not a prime UP-filter of the second type, because for $x = c$ and $y = d$ we have $x \cdot y = c \cdot d = d \notin F$ and $y \cdot x = d \cdot c = c \notin F$.

The following theorems show some of the basic properties of prime UP-filters of the third kind.

Theorem 3.1. *Any prime UP-filter of the second kind in a meet-commutative UP-algebra A is a prime UP-filter of the third kind in A.*

Proof. Let a UP-filter F satisfy the condition (PF2) and let $x, y \in A$ be arbitrary elements. Then $x \cdot y \in F$ or $y \cdot x \in F$. Since the inequalities $x \cdot y \leqslant ((y \cdot x) \cdot (x \cdot y)) \cdot (x \cdot y)$ and $y \cdot x \leqslant ((x \cdot y) \cdot (y \cdot x)) \cdot (y \cdot x)$ are valid formulas according to the claim (6) in the Proposition 1.8 in [1], we conclude that $(x \cdot y) \sqcup (y \cdot x) \in F$ according to (F-2). Hence (PF3) holds. $\square$

Theorem 3.2 (Extension property for prime UP-filters of the third kind)**.** *Let A be a meet-commutative UP-algebra and let F and G be UP-filter in A such that $F \subseteq G$. If F is a prime UP-filter of the third kind, then G is a prime UP-filter of the third kind also.*

Proof. Since F is a prime UP-filter of the third kind of A that satisfies the condition (PF3), it follows that the UP-filter G also satisfies the condition (PF3). Therefore, G is a prime UP-filter of the third kind in A. $\square$

Theorem 3.3. *If the order relation in a meet-commutative UP-algebra A is a linear relation, then each UP-filter of A is a prime UP-filter of the third kind of A.*

Proof. The proof of this theorem is obtained by combining Theorem 3.3 in [10] and Theorem 3.1. $\square$

Example 3.8. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	0	0
b	0	c	0	0
c	0	b	c	0

Then $A = (A, \cdot, 0)$ is a meet-commutative UP-algebra. Subsets $\{0\}$ and $\{0, c\}$ are UP-filters in A. Due to the linearity of the order relation in A, both UP-filters are prime UP-filters of the first / second / third kind in A.

The following theorem gives one sufficient condition that every UP-filter in a meet-commutative UP-algebra be a prime UP-filter of the third kind.

Theorem 3.4. *If a meet-commutative UP-algebra A satisfies the following condition*
(U) $(\forall x, y \in A)((x \cdot y) \sqcup (y \cdot x) = 0)$,
then any UP-filter in A is a prime UP-filter of the third kind in A.

Proof. Let $x, y \in A$ be such that $(x \cdot y) \sqcup (y \cdot x) = 0$. Then $(x \cdot y) \sqcup (y \cdot x) \in F$ for any UP-filter in a meet-commutative UP-algebra A by (F-1). So, the UP-filter F is a prime UP-filter of the third kind in A. □

At the end of this section, we give an example of a meet-commutative UP-algebra that satisfies condition (U).

Example 3.9. Let $A = \{0, a, b, c, d\}$ as in Example 3.7. Then $(A, \cdot, 0)$ is a meet-commutative UP-algebra where the order relation $\leqslant$ is not linear. Indeed, the elements c and d are not comparable but $(b \cdot c) \sqcup (c \cdot b) = 0 \sqcup c = 0$ is valid. Thus, this UP-algebra satisfies the condition (U).

It is quite justified to ask the questions:

Question 1: What kind is a meet-commutative UP-algebra if the prime UP-filters of the first kind and the prime UP-filters of the third kind in it are coincide?

Question 2: What kind is a meet-commutative UP-algebra if the prime UP-filters of the second kind and the prime UP-filters of the third kind in it are coincide?

4. Conclusion and further work

The concept of meet-commutative UP-algebras was introduced in the article [11] by Sawika et al. In such UP-algebra, the concept of prime UP-filter (of the first kind) was introduced by Muhiuddin et al. ([5]). In paper [10], the concept of prime UP-filter of the second kind is introduced and the relationship of these two types of prime UP-filters in these UP-algebras is considered. In this paper, the author introduce the concept of prime UP-filter of the third kind of a meet-commutative UP-algebra and he observed the connections between these three types of prime UP-filters in such algebras. For a meet-commutative UP-algebra A that satisfies condition (U), the term 'pre-linear meet-commutative UP-algebra' can be used by looking at the MTL-algebra (for example, [13]). In future work, a pre-linear meet-commutative UP-algebra could be observed, among other things.

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