ANNALS OF COMMUNICATIONS IN MATHEMATICS
Volume 3, Number 2 (2020), 132-138
ISSN: 2582-0818

ON RARELY FUZZY e^*-CONTINUOUS FUNCTIONS IN THE SENSE OF $\check{S}$OSTAK'S

E. ELAVARASAN*

ABSTRACT. In this paper, we introduce the concepts of rarely fuzzy e^*-continuous functions in the sense of $\check{S}$ostak's is introduced. Some interesting properties and characterizations of them are investigated. Also, some applications to fuzzy compact spaces are established.

1. INTRODUCTION

Kubiak [8] and $\check{S}$ostak [13] introduced the fundamental concept of a fuzzy topological structure, as an extension of both crisp topology and fuzzy topology [2], in the sense that not only the objects are fuzzified, but also the axiomatics. In [14, 15], $\check{S}$ostak gave some rules and showed how such an extension can be realized. Chattopadhyay et al., [4] have redefined the same concept under the name gradation of openness. Popa [11] introduced the notion of rarely continuity as a generalization of weak continuity [9] which has been further investigated by Long and Herrington [10] and Jafari [6] and [7]. Recently Vadivel et al. [17], [18] introduced the concept of fuzzy e^*-open, fuzzy e^*-closed sets and fuzzy e^* continuity in $\check{S}$ostak's fuzzy topological spaces. In this paper, we introduce the concepts of rarely fuzzy e^*-continuous functions in the sense of $\check{S}$ostak's. Some interesting properties and characterizations of them are investigated. Also, some applications to fuzzy compact spaces are established.

2. PRELIMINARIES

Throughout this paper, let X be a nonempty set, $I = [0,\ 1]$ and $I_0 = (0,\ 1]$. For $\lambda \in I^X$, $\overline{\lambda}(x) = \lambda$ for all $x \in X$. For $x \in X$ and $t \in I_0$, a fuzzy point x_t is defined by $x_t(y) = \begin{cases} t & \text{if } y = x \\ 0 & \text{if } y \neq x. \end{cases}$ Let $Pt(X)$ be the family of all fuzzy points in X. A fuzzy point $x_t \in \lambda$ iff $t < \lambda(x)$. All other notations and definitions are standard, for all in the fuzzy set theory.

2010 *Mathematics Subject Classification.* 54A40.

Key words and phrases. Rarely fuzzy e^*-continuous; fuzzy e^*-compact space; rarely fuzzy e^*-almost compact space; rarely fe^*-T_2-spaces.

Received: June 15, 2020. Accepted: July 15, 2020.

Definition 2.1. [13] A function $\tau : I^X \to I$ is called a fuzzy topology on X if it satisfies the following conditions:

(O1) $\tau(\overline{0}) = \tau(\overline{1}) = 1$,
(O2) $\tau(\bigvee_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \tau(\mu_i)$, for any $\{\mu_i\}_{i\in\Gamma} \subset I^X$,
(O3) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$, for any $\mu_1,\ \mu_2 \in I^X$.

The pair $(X,\ \tau)$ is called a fuzzy topological space (for short, fts). A fuzzy set λ is called an r-fuzzy open (r-fo, for short) if $\tau(\lambda) \geq r$. A fuzzy set λ is called an r-fuzzy closed (r-fc, for short) set iff $\overline{1} - \lambda$ is an r-fo set.

Theorem 2.1. [3] *Let (X, τ) be a fts. Then for each $\lambda \in I^X$ and $r \in I_0$, we define an operator $C_\tau : I^X \times I_0 \to I^X$ as follows: $C_\tau(\lambda,\ r) = \bigwedge\{\mu \in I^X : \lambda \leq \mu,\ \tau(\overline{1}-\mu) \geq r\}$. For $\lambda,\ \mu \in I^X$ and $r, s \in I_0$, the operator C_τ satisfies the following statements:*

(C1) $C_\tau(\overline{0}, r) = \overline{0}$,
(C2) $\lambda \leq C_\tau(\lambda,\ r)$,
(C3) $C_\tau(\lambda,\ r) \vee C_\tau(\mu,\ r) = C_\tau(\lambda \vee \mu,\ r)$,
(C4) $C_\tau(\lambda,\ r) \leq C_\tau(\lambda,\ s)$ *if* $r \leq s$,
(C5) $C_\tau(C_\tau(\lambda,\ r),\ r) = C_\tau(\lambda,\ r)$.

Theorem 2.2. [3] *Let (X, τ) be a fts. Then for each $\lambda \in I^X$ and $r \in I_0$, we define an operator $I_\tau : I^X \times I_0 \to I^X$ as follows: $I_\tau(\lambda,\ r) = \bigvee\{\mu \in I^X : \mu \leq \lambda,\ \tau(\mu) \geq r\}$. For $\lambda,\ \mu \in I^X$ and $r, s \in I_0$, the operator I_τ satisfies the following statements:*

(I1) $I_\tau(\overline{1}, r) = \overline{1}$,
(I2) $I_\tau(\lambda,\ r) \leq \lambda$,
(I3) $I_\tau(\lambda,\ r) \wedge I_\tau(\mu,\ r) = I_\tau(\lambda \wedge \mu,\ r)$,
(I4) $I_\tau(\lambda,\ r) \leq I_\tau(\lambda,\ s)$ *if* $s \leq r$,
(I5) $I_\tau(I_\tau(\lambda,\ r),\ r) = I_\tau(\lambda,\ r)$.
(I6) $I_\tau(\overline{1} - \lambda, r) = \overline{1} - C_\tau(\lambda, r)$ *and* $C_\tau(\overline{1} - \lambda, r) = \overline{1} - I_\tau(\lambda,\ r)$

Definition 2.2. [12] Let $(X,\ \tau)$ be a fts, $\lambda \in I^X$ and $r \in I_0$. Then

(1) a fuzzy set λ is called r-fuzzy regular open (for short, r-fro) if $\lambda = I_\tau(C_\tau(\lambda, r), r)$.
(2) a fuzzy set λ is called r-fuzzy regular closed (for short, r-frc) if $\lambda = C_\tau(I_\tau(\lambda, r), r)$.

Definition 2.3. [16] Let $(X,\ \tau)$ be a fts. For $\lambda, \mu \in I^X$ and $r \in I_0$.

(1) The r-fuzzy δ-closure of λ, denoted by δ-$C_\tau(\lambda, r)$, and is defined by δ-$C_\tau(\lambda, r) = \bigwedge\{\mu \in I^X | \mu \geq \lambda, \mu$ is r-frc $\}$.
(2) The r-fuzzy δ-interior of λ, denoted by δ-$I_\tau(\lambda, r)$, and is defined by δ-$I_\tau(\lambda, r) = \bigvee\{\mu \in I^X | \mu \leq \lambda, \mu$ is r-fro $\}$.

Definition 2.4. [16] Let $(X,\ \tau)$ be a fts and $\lambda \in I^X,\ r \in I_0$. Then λ is called

(1) r-fuzzy e-open (for short, r-feo) if $\lambda \leq I_\tau(\delta\text{-}C_\tau(\lambda, r), r) \vee C_\tau(\delta\text{-}I_\tau(\lambda, r), r)$.
(2) r-fuzzy e-closed (for short, r-fec) if $\lambda \geq I_\tau(\delta\text{-}C_\tau(\lambda, r), r) \wedge C_\tau(\delta\text{-}I_\tau(\lambda, r), r)$.

Definition 2.5. [17] Let $(X,\ \tau)$ be a fts and $\lambda \in I^X,\ r \in I_0$. Then λ is called

(1) r-fuzzy e^*-open (for short, r-fe*o) if $\lambda \leq C_\tau(I_\tau(\delta\text{-}C_\tau(\lambda, r), r), r)$.
(2) r-fuzzy e^*-closed (for short, r-fe*c) if $\lambda \geq I_\tau(C_\tau(\delta\text{-}I_\tau(\lambda, r), r), r)$.

Definition 2.6. [17] Let $(X,\ \tau)$ be a fts. For $\lambda, \mu \in I^X$ and $r \in I_0$.

(1) The r-fuzzy e^*-closure of λ, denoted by $e^*C_\tau(\lambda, r)$, and is defined by $e^*C_\tau(\lambda, r) = \bigwedge\{\mu \in I^X | \mu \geq \lambda, \mu$ is r-fe*c $\}$.
(2) The r-fuzzy e^*-interior of λ, denoted by $e^*I_\tau(\lambda, r)$, and is defined by $e^*I_\tau(\lambda, r) = \bigvee\{\mu \in I^X | \mu \leq \lambda, \mu$ is r-fe*o $\}$.

Definition 2.7. [18] Let $(X,\ \tau)$ and $(Y,\ \eta)$ be a fts's. Let $f:(X,\ \tau)\to(Y,\ \eta)$ be a function. Then f is called

(1) fuzzy e^*-continuous (for short, fe^*-continuous) iff $f^{-1}(\mu)$ is r-fe*o for each $\mu\in I^Y,\ r\in I_0$ with $\eta(\mu)\geq r$.
(2) fuzzy e^*-open (for short, fe^*-open) iff $f(\lambda)$ is r-fe*o for each $\lambda\in I^X,\ r\in I_0$ with $\tau(\lambda)\geq r$.
(3) fuzzy e^*-closed (for short, fe^*-closed) iff $f(\lambda)$ is r-fe*c for each $\lambda\in I^X,\ r\in I_0$ with $\tau(\overline{1}-\lambda)\geq r$.
(4) fuzzy e^*-irresolute (for short, fe^*-irresolute) iff $f^{-1}(\mu)$ is r-fe*c for each r-fe*c set $\mu\in I^Y$.

Definition 2.8. [1] Let (X,τ) be a fts and $r\in I_0$. For $\lambda\in I^X$, λ is called an r-fuzzy rare set if $I_\tau(\lambda,r)=\overline{0}$.

Definition 2.9. [1] Let $(X,\ \tau)$ and $(Y,\ \eta)$ be a fts's. Let $f:(X,\ \tau)\to(Y,\ \eta)$ be a function. Then f is called

(1) weakly continuous if for each $\mu\in I^Y$, where $\sigma(\mu)\geq r,\ r\in I_0,\ f^{-1}(\mu)\leq I_\tau(f^{-1}(C_\sigma(\mu,r)),r)$.
(2) rarely continuous if for each $\mu\in I^Y$, where $\sigma(\mu)\geq r,\ r\in I_0$, there exists an r-fuzzy rare set $\lambda\in I^Y$ with $\mu+C_\sigma(\lambda,r)\geq 1$ and $\rho\in I^X$, where $\tau(\rho)\geq r$ such that $f(\rho)\leq\mu\vee\lambda$.

Proposition 2.3. [1] *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fts's, $r\in I_0$ and $f:(X,\ \tau)\to(Y,\ \sigma)$ is fuzzy open and one-to-one, then f preserves r-fuzzy rare sets.*

3. RARELY FUZZY e^*-CONTINUOUS FUNCTIONS

Definition 3.1. Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be two fts's, and $f:(X,\ \tau)\to(Y,\ \sigma)$ be a function. Then f is called

(1) rarely fuzzy e-continuous (for short, rarely fe-continuous) if for each $\mu\in I^Y$, where $\sigma(\mu)\geq r,\ r\in I_0$, there exists an r-fuzzy rare set $\lambda\in I^Y$ with $\mu+C_\sigma(\lambda,r)\geq 1$ and a r-feo set $\rho\in I^X$ such that $f(\rho)\leq\mu\vee\lambda$.
(2) rarely fuzzy e^*-continuous (for short, rarely fe^*-continuous) if for each $\mu\in I^Y$, where $\sigma(\mu)\geq r,\ r\in I_0$, there exists an r-fuzzy rare set $\lambda\in I^Y$ with $\mu+C_\sigma(\lambda,r)\geq 1$ and a r-fe*o set $\rho\in I^X$ such that $f(\rho)\leq\mu\vee\lambda$.

Remark. *(1) Every weakly continuous (resp. fuzzy continuous) function is rarely continuous* [1] *(resp. fuzzy e-continuous* [16]*) but converse is not true.*
(2) Every rarely continuous function [5] *is rarely $f\delta s$-continuous and rarely $f\delta p$-continuous function but converse is not true.*
(3) Every rarely $f\delta s$-continuous and rarely $f\delta p$-continuous function [5] *is rarely fe-continuous but converse is not true.*
(4) Every rarely fe-continuous function is rarely fe^*-continuous but converse is not true.

From the above definition and remark it is not difficult to conclude that the following diagram of implications is true.

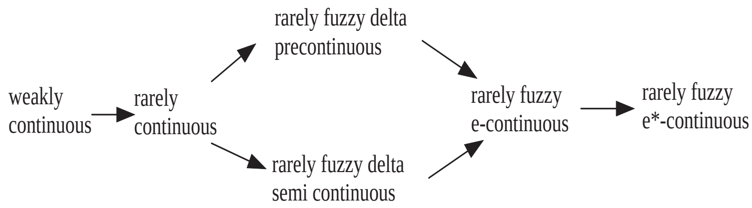

Diagram -I

Example 3.2. Let $X = \{a, b, c\} = Y$. Define $\lambda_1, \lambda_2 \in I^X,\ \lambda_3 \in I^Y$ as follows: $\lambda_1(a) = 0.4,\ \lambda_1(b) = 0.6,\ \lambda_1(c) = 0.5,\ \lambda_2(a) = 0.6,\ \lambda_2(b) = 0.4,\ \lambda_2(c) = 0.4,\ \lambda_3(a) = 0.6,\ \lambda_3(b) = 0.4,\ \lambda_3(c) = 0.5$. Define the fuzzy topologies $\tau,\ \sigma : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{10} & \text{if } \lambda = \lambda_1, \\ \frac{1}{10} & \text{if } \lambda = \lambda_2, \\ \frac{1}{10} & \text{if } \lambda = \lambda_1 \vee \lambda_2, \\ \frac{1}{10} & \text{if } \lambda = \lambda_1 \wedge \lambda_2, \\ 0 & \text{otherwise,} \end{cases} \quad \sigma(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{10} & \text{if } \lambda = \lambda_3, \\ 0 & \text{otherwise.} \end{cases}$$

Let $r = 1/10$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a,\ f(b) = a,\ f(c) = c$ and $\gamma \in I^Y$ be a $1/10$-fuzzy rare set defined by $\gamma(a) = 0.4,\ \gamma(b) = 0.5,\ \gamma(c) = 0.5$ and a r-feo set $\lambda_4 \in I^X$ is defined by $\lambda_4(a) = 0.6,\ \lambda_4(b) = 0.4,\ \lambda_4(b) = 0.5$, $f(\lambda_4) = (0.6,\ 0.4,\ 0.5) \leq \lambda_3 \vee \gamma = (0.6,\ 0.4,\ 0.5)$. Then f is rarely fe-continuous.

Example 3.3. Let $X = \{a, b, c\} = Y$. Define $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in I^X,\ \lambda_5 \in I^Y$ as follows: $\lambda_1(a) = 0.3,\ \lambda_1(b) = 0.4,\ \lambda_1(c) = 0.5,\ \lambda_2(a) = 0.6,\ \lambda_2(b) = 0.5,\ \lambda_2(c) = 0.5,\ \lambda_3(a) = 0.6,\ \lambda_3(b) = 0.5,\ \lambda_3(c) = 0.4,\ \lambda_4(a) = 0.3,\ \lambda_4(b) = 0.4,\ \lambda_4(c) = 0.4,\ \lambda_5(a) = 0.7,\ \lambda_5(b) = 0.6,\ \lambda_5(c) = 0.4$. Define the fuzzy topologies $\tau,\ \sigma : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{10} & \text{if } \lambda = \lambda_1, \\ \frac{1}{10} & \text{if } \lambda = \lambda_2, \\ \frac{1}{10} & \text{if } \lambda = \lambda_3, \\ \frac{1}{10} & \text{if } \lambda = \lambda_4, \\ 0 & \text{otherwise,} \end{cases} \quad \sigma(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{10} & \text{if } \lambda = \lambda_5, \\ 0 & \text{otherwise.} \end{cases}$$

Let $r = 1/10$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a,\ f(b) = a,\ f(c) = c$ and $\gamma \in I^Y$ be a $1/10$-fuzzy rare set defined by $\gamma(a) = 0.4,\ \gamma(b) = 0.5,\ \gamma(c) = 0.6$ and a r-fe*o set $\lambda_6 \in I^X$ is defined by $\lambda_6(a) = 0.7,\ \lambda_6(b) = 0.6,\ \lambda_6(b) = 0.4$, $f(\lambda_6) = (0.7,\ 0.6,\ 0.4) \leq \lambda_5 \vee \gamma = (0.7,\ 0.6,\ 0.4)$. Then f is rarely fe^*-continuous but not rarely fe-continuous, because $\lambda_6 \in I^X$ is not r-feo set.

Definition 3.4. Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be two fts's, and $f : (X,\ \tau) \to (Y,\ \eta)$ be a function. Then f is called weakly fuzzy e^*-continuous (for short, weakly fe^*-continuous) if for each r-fe*o set $\mu \in I^Y$, $r \in I_0$, $f^{-1}(\mu) \leq I_\tau(f^{-1}(C_\sigma(\mu, r)), r)$.

Definition 3.5. A fts (X, τ) is said to be fe^*-$T_{1/2}$-space if every r-fe*o set $\lambda \in I^X,\ r \in I_0$ is r-fo set.

Proposition 3.1. *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fts's. If $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ is both fe^*-open, fe^*-irresolute and $(X,\ \tau)$ is fe^*-$T_{1/2}$ space, then it is weakly fe^*-continuous.*

Proof. Let $\lambda \in I^X,\ r \in I_0$ with $\tau(\lambda) \geq r$. Since f is fe^*-open $f(\lambda) \in I^Y$ is r-fe*o. Also, since f is fe^*-irresolute, $f^{-1}(f(\lambda)) \in I^X$ is r-fe*o set. Since $(X,\ \tau)$ is fe^*-$T_{1/2}$ space, every r-fe*o set is r-fo set, now, $\tau(f^{-1}(f(\lambda))) \geq r$.

Consider $f^{-1}(f(\lambda)) \leq f^{-1}(C_\sigma(f(\lambda), r))$ from which
$I_\tau(f^{-1}(f(\lambda)), r) \leq I_\tau(f^{-1}(C_\sigma(f(\lambda), r)), r)$.

Since $\tau(f^{-1}(f(\lambda))) \geq r$, $f^{-1}(f(\lambda)) \leq I_\tau(f^{-1}(C_\sigma(f(\lambda), r)), r)$, thus f is weakly fe^*-continuous. □

Definition 3.6. Let (X, τ) be a fts. A r-fuzzy e^*-open cover of (X, τ) is the collection $\{\lambda_i \in I^X,\ \lambda_i \text{ is } r\text{-fe}^*\text{o},\ i \in J\}$ such that $\bigvee_{i \in J} \lambda_i = \overline{1}$.

Definition 3.7. A fts (X, τ) is said to be a r-fuzzy e^*-compact space if every r-fuzzy e^*-open cover of (X, τ) has a finite sub cover.

Definition 3.8. A fts (X, τ) is said to be rarely fuzzy e^*-almost compact if for every r-fuzzy e^*-open cover $\{\lambda_i \in I^X,\ \lambda_i \text{ is } r\text{-fe}^*\text{o},\ i \in J\}$ of (X, τ), there exists a finite subset J_0 of J such that $\bigvee_{i \in J} \lambda_i \vee \rho_i = \overline{1}$ where $\rho_i \in I^X$ are r-fuzzy rare sets.

Proposition 3.2. *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fts's, $r \in I_0$ and $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ be rarely fe^*-continuous. If $(X,\ \tau)$ is r-fuzzy e^*-compact then $(Y,\ \sigma)$ is rarely fuzzy e^*-almost compact.*

Proof. Let $\{\lambda_i \in I^Y,\ i \in J\}$ be r-fuzzy e^*-open cover of (Y, σ). Then $\overline{1} = \bigvee_{i \in J} \lambda_i$. Since f is rarely fe^*-continuous, there exists a r-fuzzy rare sets $\rho_i \in I^Y$ such that $\lambda_i + C_\sigma(\rho_i, r) \geq \overline{1}$ and a r-fe*o set $\mu_i \in I^X$ such that $f(\mu_i) \leq \lambda_i \vee \rho_i$. Since (X, τ) is r-fuzzy e^*-compact, every fuzzy e^*-open cover of (X, τ) has a finite sub cover. Thus $\overline{1} \leq \bigvee_{i \in J_0} \mu_i$. Hence $\overline{1} = f(\overline{1}) = f(\bigvee_{i \in J_0} \mu_i) = \vee_{i \in J_0} f(\mu_i) \leq \bigvee_{i \in J_0} \lambda_i \vee \rho_i$. Therefore (Y, σ) is rarely fuzzy e^*-almost compact. □

Proposition 3.3. *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fts's, $r \in I_0$ and $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ be rarely fe-continuous. If $(X,\ \tau)$ is r-fuzzy e^*-compact then $(Y,\ \sigma)$ is rarely fuzzy e^*-almost compact.*

Proof. Since every rarely fe-continuous function is rarely fe^*-continuous, then proof follows immediately from the Proposition 3.2. □

Proposition 3.4. *Let $(X,\ \tau)$, $(Y,\ \sigma)$ and $(Z,\ \eta)$ be any fts's. If $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ be rarely fe^*-continuous, fe^*-open and $g : (Y,\ \sigma) \rightarrow (Z,\ \eta)$ is fuzzy open and one-to-one, then $g \circ f : (X,\ \tau) \rightarrow (Z,\ \eta)$ is rarely fe^*-continuous.*

Proof. Let $\lambda \in I^X$, $r \in I_0$ with $\tau(\lambda) \geq r$. Since f is fe^*-open $f(\lambda) \in I^Y$ with $\sigma(f(\lambda)) \geq r$. Since f is rarely fe^*-continuous and there exists a r-fuzzy rare set $\rho \in I^Y$ with $f(\lambda) + C_\sigma(\rho, r) \geq \overline{1}$ and a r-fe*o set $\mu \in I^X$ such that $f(\mu) \leq f(\lambda) \vee \rho$. By the proposition 2.3, $g(\rho) \in I^Z$ is also a r-fuzzy rare set. Since $\rho \in I^Y$ is such that $\rho < \gamma$ for all $\gamma \in I^Y$ with $\sigma(\gamma) \geq r$ and g is injective, it follows that $(g \circ f)(\lambda) + C_\eta(g(\rho), r) \geq \overline{1}$. Then $(g \circ f)(\mu) = g(f(\mu)) \leq g(f(\lambda) \vee \rho) \leq g(f(\lambda)) \vee g(\rho) \leq (g \circ f)(\lambda) \vee g(\rho)$. Hence the result. □

Proposition 3.5. *Let $(X,\ \tau)$, $(Y,\ \sigma)$ and $(Z,\ \eta)$ be any fts's and $r \in I_0$. If $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ be fe^*-open, onto and $g : (Y,\ \sigma) \rightarrow (Z,\ \eta)$ be a function such that $g \circ f : (X,\ \tau) \rightarrow (Z,\ \eta)$ is rarely fe^*-continuous, then g is rarely fe^*-continuous.*

Proof. Let $\lambda \in I^X$ and $\mu \in I^Y$ be such that $f(\lambda) = \mu$. Let $(g \circ f)(\lambda) = \gamma \in I^Z$ with $\eta(\gamma) \geq r$. Since $(g \circ f)$ is fe^*-continuous, there exists a r-fuzzy rare set $\rho \in I^Z$ with $\gamma + C_\eta(\rho, r) \geq \overline{1}$ and a r-fe*o set $\delta \in I^X$ such that $(g \circ f)(\delta) \leq \gamma \vee \rho$. Since f is fe^*-open, $f(\delta) \in I^Y$ is a r-fe*o set. Thus there exists a r-fuzzy rare set $\rho \in I^Z$ with $\gamma + C_\eta(\rho, r) \geq \overline{1}$ and a r-fe*o set $f(\delta) \in I^Y$ such that $g(f(\delta)) \leq \gamma \vee \rho$. Hence g is rarely fe^*-continuous. □

Proposition 3.6. *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fts's and $r \in I_0$. If $f : (X,\ \tau) \to (Y,\ \sigma)$ be rarely fe^*-continuous and $(X,\ \tau)$ is fe^*-$T_{1/2}$-space, then f is rarely continuous.*

Proof. The proof is trivial. □

Definition 3.9. A fts (X, τ) is said to be rarely fe^*-T_2-space if for each pair $\lambda,\ \mu \in I^X$ with $\lambda \neq \mu$ there exist r-fe*o sets $\rho_1,\ \rho_2 \in I^X$ with $\rho_1 \neq \rho_1$ and a r-fuzzy rare set $\gamma \in I^X$ with $\rho_1 + C_\tau(\gamma, r) \geq \overline{1}$ and $\rho_2 + C_\tau(\gamma, r) \geq \overline{1}$ such that $\lambda \leq \rho_1 \vee \gamma$ and $\mu \leq \rho_2 \vee \gamma$.

Proposition 3.7. *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two fts's and $r \in I_0$. If $f : (X,\ \tau) \to (Y,\ \sigma)$ be fe^*-open and injective and $(X,\ \tau)$ is rarely fe^*-T_2 space, then $(Y,\ \sigma)$ is also a rarely fe^*-T_2 space.*

Proof. $\lambda,\ \mu \in I^X$ with $\lambda \neq \mu$. Since f is injective, $f(\lambda) \neq f(\mu)$. Since $(X,\ \tau)$ is a rarely fe^*-T_2-space, there exist r-fe*o sets $\rho_1,\ \rho_2 \in I^X$ with $\rho_1 \neq \rho_1$ and a r-fuzzy rare set $\gamma \in I^X$ with $\rho_1 + C_\tau(\gamma, r) \geq \overline{1}$ and $\rho_2 + C_\tau(\gamma, r) \geq \overline{1}$ such that $\lambda \leq \rho_1 \vee \gamma$ and $\mu \leq \rho_2 \vee \gamma$. Since f is fe^*-open, $f(\rho_1),\ f(\rho_2) \in I^Y$ are r-fe*o sets with $f(\rho_1) \neq f(\rho_2)$. Since f is fe^*-open and one-to-one, $f(\gamma)$ is also a r-fuzzy rare set with $f(\rho_1) + C_\sigma(\gamma, r) \geq \overline{1}$ and $f(\rho_2) + C_\sigma(\gamma, r) \geq \overline{1}$ such that $f(\lambda) \leq f(\rho_1 \vee \gamma)$ and $f(\mu) \leq f(\rho_1 \vee \gamma)$. Thus $(Y,\ \sigma)$ is a rarely fe^*-T_2-space. □

4. CONCLUSIONS

Šostak's fuzzy topology has been recently of major interest among fuzzy topologies. In this paper, we have introduced rarely fuzzy e^*-continuous functions in fuzzy topological spaces of Šostak's. We have also introduced fuzzy e^*-compact space, rarely fuzzy e^*-almost compact space, rarely fe^*-T_2-spaces and some properties and characterizations of them are investigated.

5. ACKNOWLEDGEMENTS

The author would like to thank from the anonymous reviewers for carefully reading of the manuscript and giving useful comments, which will help us to improve the paper.

REFERENCES

[1] B. Amudhambigai, M. K. Uma and E. Roja, On rarely $\widetilde{g}$-continuous functions in smooth fuzzy topological spaces, The Journal of Fuzzy Mathematics., 20(2) (2012), 433–442.
[2] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24 (1968), 182–190.
[3] K. C. Chattopadhyay and S. K. Samanta, Fuzzy topology, Fuzzy Sets and Systems, 54 (1993), 207–212.
[4] K. C. Chattopadhyay, R. N. Hazra and S. K. Samanta, Gradation of openness, Fuzzy Sets and Systems, 49(2) (1992), 237–242.
[5] E. Elavarasan and A. Vadivel On rarely fuzzy e-continuous functions in the sense of sostak's (submitted).
[6] S. Jafari, A note on rarely continuous functions, Univ. Bacau. Stud. Cerc. St. Ser. Mat., 5, 29–34.
[7] S. Jafari, On some properties of rarely continuous functions, Univ. Bacau. Stud. Cerc. St. Ser. Mat., 7, 65–73.
[8] T. Kubiak, On fuzzy topologies, Ph.D. Thesis, A. Mickiewicz, Poznan, (1985).

[9] N. Levine, Decomposition of continuity in topological spaces, Amer. Math. Monthly., 60, 44–46.
[10] P. E. Long and L. L. Herrington, properties of rarely continuous functions, Glasnik Math., 17(37), 147–153.
[11] V. Popa, sur certain decompositionde la continuite dans les espaces topologiques, Glasnik Mat. Setr III., 14(34) 359–362.
[12] Seok Jong Lee and Eun Pyo Lee, Fuzzy r-regular open sets and fuzzy almost r-continuous maps, Bull. Korean Math. Soc., 39(3) (2002), 441–453.
[13] A. P. Šostak, On a fuzzy topological structure, Rend. Circ. Matem. Palermo Ser II, 11 (1986), 89–103.
[14] A. P. Šostak, Basic structures of fuzzy topology, J. Math. Sci., 78(6) (1996), 662–701.
[15] A. P. Šostak, Two decades of fuzzy topology : Basic ideas, Notion and results, Russian Math. Surveys, 44(6) (1989), 125-186.
[16] D. Sobana and V. Chandrasekar and A. Vadivel, Fuzzy e-continuity in Šostak's fuzzy topological spaces, AIP Conference Proceedings 21770:020090, December 2019.
[17] A. Vadivel, B. Vijayalakshmi and R. Prabhu, Fuzzy e^*-open sets in Šostak's fuzzy topological spaces (submitted).
[18] A. Vadivel, B. Vijayalakshmi and R. Prabhu, Fuzzy e^*-continuity in Šostak's fuzzy topological spaces (submitted).

E. ELAVARASAN
DEPARTMENT OF MATHEMATICS, SHREE RAGHAVENDRA ARTS AND SCIENCE COLLEGE (AFFILIATED TO THIRUVALLUVAR UNIVERSITY), KEEZHAMOONGILADI, CHIDAMBARAM-608102, TAMIL NADU, INDIA.
Email address: maths.aras@gmail.com