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A NEW FORM OF GENERALIZED m-PF IDEALS IN BCK/BCI-ALGEBRAS

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ABSTRACT. In this paper, we introduce a new kind of an m-polar fuzzy ideal of a BCK/BCI-algebra called, an m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy ideal and investigate some of its properties. Ordinary ideals and m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy ideals are connected by means of level cut subset.

1. INTRODUCTION

BCK-algebras entered into mathematics in 1966 through the work of Imai and Iséki [12], and they have been applied to several domains of mathematics, such as group theory, topology, functional analysis and probability theory. Additionally, Iséki [13] initiated the idea of a BCI-algebra, which is a generalization of a BCK-algebra.

The idea of fuzzy sets was introduced by Zadeh [20] in 1965 to handle uncertainties in several real applications, and the idea of bipolar fuzzy sets on a universe X was introduced by Zhang[21] in 1994 as a generalization of fuzzy sets. The notion of m-polar fuzzy sets was presented by chen et al. [9] in 2014 as an extension of bipolar fuzzy sets. Bipolar fuzzy sets, m-polar fuzzy (m-PF) sets and several hybrid models of fuzzy sets play a prominent rule in several algebraic structures, such as BCK/BCI-algebras [3, 7, 4, 6, 18, 16, 17], hemirings [11], groups [10] and lie-subalgebras [1, 2]. In 1971, Rosenfeld [19] applied fuzzy sets to groups and proposed the concept of fuzzy subgroups. As a generalization of fuzzy subgroups, Bhakat and Das [8] initiated the notions of $(\in, \in \vee q)$-fuzzy subgroups by using the concept of fuzzy points and its "belongingness $(\in)$" and "quasi-coincidence (q)" with a fuzzy set. Jun [15] introduced a generalization of fuzzy ideals in BCK/BCI-algebras, called (α, β)-fuzzy ideals. After that, Jana et al. [14] proposed the concept of $(\in, \in \vee q)$-bipolar fuzzy ideals in BCK/BCI-algebras. Further, Al-Masarwah and Ahmad [5] presented the notion of m-polar $(\in, \in \vee q)$-fuzzy ideals in BCK/BCI-algebras as a generalization of m-polar fuzzy ideals.

This paper is a continuation of papers [4] and [5]. We introduce the notion of m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy ideals and investigate some of its properties. We discuss the relation between ordinary ideals and m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy ideals in BCK/BCI-algebras.

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2. Preliminaries

Some of the significant notions pertaining to BCK/BCI-algebras, m-PF sets, m-PF points and m-PF ideals that are useful for subsequent discussions are stated below. In what follows, let X be a BCK/BCI-algebra unless otherwise specified.

By a BCI-algebra, we mean an algebra $(X;*,0)$ of type (2, 0) satisfying the following axioms for all $x,y,z\in X$:

(1) $((x*y)*(x*z))*(z*y)=0$,
(2) $(x*(x*y))*y=0$,
(3) $x*x=0$,
(4) $x*y=0$ and $y*x=0$ imply $x=y$.

If a BCI-algebra X satisfies $0*x=0\ \forall x\in X$, then X is called a BCK-algebra. A partial ordering $\leq$ on a BCK/BCI-algebra X can be defined by $x\leq y$ if and only if $x*y=0$. Any BCK/BCI-algebra X satisfies the following axioms for all $x,y,z\in X$:

(1) $x*0=x$,
(2) $(x*y)*z=(x*z)*y$.

A non-empty subset J of X is said to be an ideal of X if for all $x,y\in X$:

(1) $0\in J$,
(2) $x*y\in J$ and $y\in J$ imply $x\in J$.

Definition 2.1 ([9]). An m-PF set $\widehat{\mathcal{H}}$ on $X(\neq\phi)$ is a function $\widehat{\mathcal{H}}:X\to[0,1]^m$, where

$$\widehat{\mathcal{H}}(x)=(p_1\circ\widehat{\mathcal{H}}(x),p_2\circ\widehat{\mathcal{H}}(x),...,p_m\circ\widehat{\mathcal{H}}(x))$$

is the membership value of every element $x\in X$ and $p_i\circ\widehat{\mathcal{H}}:[0,1]^m\to[0,1]$ is the i-th projection mapping for all $i=1,2,...,m$. The values $\widehat{0}=(0,0,...,0)$ and $\widehat{1}=(1,1,...,1)$ are the smallest and largest values in $[0,1]^m$, respectively.

Definition 2.2 ([4]). An m-PF set $\widehat{\mathcal{H}}$ of X is said to be an m-PF ideal if the assertions below are valid: for all $x,y\in X$,

(1) $\widehat{\mathcal{H}}(0)\geq\widehat{\mathcal{H}}(x)$,
(2) $\widehat{\mathcal{H}}(x)\geq\inf\{\widehat{\mathcal{H}}(x*y),\widehat{\mathcal{H}}(y)\}$.

That is,

(1) $p_i\circ\widehat{\mathcal{H}}(0)\geq p_i\circ\widehat{\mathcal{H}}(x)$,
(2) $p_i\circ\widehat{\mathcal{H}}(x)\geq\inf\{p_i\circ\widehat{\mathcal{H}}(x*y),p_i\circ\widehat{\mathcal{H}}(y)\}$

for all $i=1,2,...,m$.

Definition 2.3 ([4]). Let $\widehat{\mathcal{H}}$ be an m-PF set of X. Then, the set

$$\widehat{\mathcal{H}}_{\widehat{t}}=\{x\in X\mid\widehat{\mathcal{H}}(x)\geq\widehat{t}\}$$

is called the level cut subset of $\widehat{\mathcal{H}}$ for all $\widehat{t}\in(0,1]^m$.

An m-PF set $\widehat{\mathcal{H}}$ of X of the form

$$\widehat{\mathcal{H}}(y)=\begin{cases}\widehat{t}=(t_1,t_2,...,t_m)\in(0,1]^m, & \text{if } y=x\\ \widehat{0}=(0,0,...,0), & \text{if } y\neq x\end{cases}$$

is called an m-PF point, denoted by $x_{\widehat{t}}$, with support x and value $(t_1,t_2,...,t_m)=\widehat{t}$.

An m-PF point $x_{\widehat{t}}$

(1) Belongs to $\widehat{\mathcal{H}}$, denoted by $x_{\widehat{t}}\in\widehat{\mathcal{H}}$, if $\widehat{\mathcal{H}}(x)\geq\widehat{t}$ i.e., $p_i\circ\widehat{\mathcal{H}}(x)\geq t_i$ for each $i=1,2,...,m$,

(2) Is quasi-coincident with $\widehat{\mathcal{H}}$, denoted by $x_{\widehat{t}}q\widehat{\mathcal{H}}$, if $\widehat{\mathcal{H}}(x)+\widehat{t} > \widehat{1}$ i.e., $p_i \circ \widehat{\mathcal{H}}(x)+t_i > \widehat{1}$ for each $i = 1, 2, ..., m$.

We say that

(1) $x_{\widehat{t}}\overline{\alpha}\widehat{\mathcal{H}}$ if $x_{\widehat{t}}\alpha\widehat{\mathcal{H}}$ does not hold,
(2) $x_{\widehat{t}} \in \vee q\widehat{\mathcal{H}}$ if $x_{\widehat{t}} \in \widehat{\mathcal{H}}$ or $x_{\widehat{t}}q\widehat{\mathcal{H}}$,
(3) $x_{\widehat{t}} \in \wedge q\widehat{\mathcal{H}}$ if $x_{\widehat{t}} \in \widehat{\mathcal{H}}$ and $x_{\widehat{t}}q\widehat{\mathcal{H}}$.

Definition 2.4 ([4]). An m-PF set $\widehat{\mathcal{H}}$ of X is called an m-polar $(\in, \in \vee q)$-fuzzy ideal of X if the assertions below are valid: for all $x, y \in X$ and $\widehat{t}, \widehat{s} \in (0, 1]^m$,

(1) $x_{\widehat{t}} \in \widehat{\mathcal{H}}$ implies $0_{\widehat{t}} \in \vee q\widehat{\mathcal{H}}$,
(2) $(x * y)_{\widehat{t}} \in \widehat{\mathcal{H}}$ and $y_{\widehat{s}} \in \widehat{\mathcal{H}}$ imply $x_{\inf\{\widehat{t},\widehat{s}\}} \in \vee q\widehat{\mathcal{H}}$.

3. m-POLAR $(\overline{\in}, \overline{\in} \vee \overline{q})$-FUZZY IDEALS

In this section, we define m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy ideals of X and discuss several results.

Definition 3.1. An m-PF set $\widehat{\mathcal{H}}$ of X is called an m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy ideal of X if the assertions below are valid: for all $x, y \in X$ and $\widehat{t}, \widehat{s} \in (0, 1]^m$,

(1) $0_{\widehat{t}}\overline{\in}\widehat{\mathcal{H}}$ implies $x_{\widehat{t}}\overline{\in} \vee \overline{q}\widehat{\mathcal{H}}$,
(2) $x_{\inf\{\widehat{t},\widehat{s}\}}\overline{\in}\widehat{\mathcal{H}}$ implies $(x * y)_{\widehat{t}}\overline{\in} \vee \overline{q}\widehat{\mathcal{H}}$ or $y_{\widehat{s}}\overline{\in} \vee \overline{q}\widehat{\mathcal{H}}$.

Example 3.2. Consider a BCK-algebra $X = \{0, a, b, c, d\}$ which is defined in Table 1:

TABLE 1. The operation "$*$".

$*$	0	**a**	**b**	**c**	**d**
0	0	0	0	0	0
a	a	0	0	0	0
b	b	a	0	0	0
c	c	b	a	0	0
d	d	c	b	a	0

Let $\widehat{\mathcal{H}}$ be a 3-PF set defined as:

$$\widehat{\mathcal{H}}(x) = \begin{cases} (0.91, 0.91, 0.97), & \text{if } x = 0 \\ (0.37, 0.37, 0.78), & \text{if } x = a \\ (0.50, 0.50, 0.50), & \text{if } x = b \\ (0.44, 0.44, 0.49), & \text{if } x = c \\ (0.25, 0.25, 0.28), & \text{if } x = d. \end{cases}$$

Clearly, $\widehat{\mathcal{H}}$ is a 3-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy ideal of X.

Theorem 3.1. *An m-PF set $\widehat{\mathcal{H}}$ of X is an m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy ideal of X if and only if for all $x, y \in X$:*

(i) $\sup\{\widehat{\mathcal{H}}(0), \widehat{0.5}\} \geq \widehat{\mathcal{H}}(x)$,
(ii) $\sup\{\widehat{\mathcal{H}}(x), \widehat{0.5}\} \geq \inf\{\widehat{\mathcal{H}}(x * y), \widehat{\mathcal{H}}(y)\}$.

Proof. Let $\widehat{\mathcal{H}}$ be an m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy ideal of X. Suppose there exists $x \in X$ such that $\sup\{\widehat{\mathcal{H}}(0), \widehat{0.5}\} < \widehat{t} = \widehat{\mathcal{H}}(x)$. Then,

$$\widehat{t} \in (0.5, 1]^m, 0_{\widehat{t}} \overline{\in} \widehat{\mathcal{H}} \text{ and } x_{\widehat{t}} \in \widehat{\mathcal{H}}.$$

By Definition 3.1 (1), we have $x_{\widehat{t}} \overline{\in} \vee \overline{q} \widehat{\mathcal{H}}$, i.e., $\widehat{\mathcal{H}}(x) < \widehat{t}$ or $\widehat{\mathcal{H}}(x) + \widehat{t} \leq \widehat{1}$. Since $\widehat{\mathcal{H}}(x) = \widehat{t}$, therefore $\widehat{t} \leq \widehat{0.5}$. This is a contradiction. Hence, $\sup\{\widehat{\mathcal{H}}(0), \widehat{0.5}\} \geq \widehat{\mathcal{H}}(x)$ for all $x \in X$. Suppose there exist $x, y \in X$ such that $\sup\{\widehat{\mathcal{H}}(x), \widehat{0.5}\} < \widehat{t} = \inf\{\widehat{\mathcal{H}}(x*y), \widehat{\mathcal{H}}(y)\}$. Then,

$$\widehat{t} \in (0.5, 1]^m, x_{\widehat{t}} \overline{\in} \widehat{\mathcal{H}} \text{ and } (x*y)_{\widehat{t}}, y_{\widehat{t}} \in \widehat{\mathcal{H}}.$$

It follows that $(x*y)_{\widehat{t}} \overline{q} \mathcal{H}$ or $y_{\widehat{t}} \overline{q} \mathcal{H}$. Then, $\widehat{\mathcal{H}}(x*y) + \widehat{t} \leq \widehat{1}$ or $\widehat{\mathcal{H}}(y) + \widehat{t} \leq \widehat{1}$. Since $\widehat{\mathcal{H}}(x*y) \geq \widehat{t}$ and $\widehat{\mathcal{H}}(y) \geq \widehat{t}$. It follows that $\widehat{t} \leq \widehat{0.5}$, a contradiction. Hence, $\sup\{\widehat{\mathcal{H}}(x), \widehat{0.5}\} \geq \inf\{\widehat{\mathcal{H}}(x*y), \widehat{\mathcal{H}}(y)\}$ for all $x, y \in X$.

Conversely, let $0_{\widehat{t}} \overline{\in} \widehat{\mathcal{H}}$. Then, $\widehat{\mathcal{H}}(0) < \widehat{t}$, either $\widehat{\mathcal{H}}(0) \geq \widehat{\mathcal{H}}(x)$ or $\widehat{\mathcal{H}}(0) < \widehat{\mathcal{H}}(x)$. If $\widehat{\mathcal{H}}(0) \geq \widehat{\mathcal{H}}(x)$, then $\widehat{\mathcal{H}}(x) < \widehat{t}$, and so $x_{\widehat{t}} \overline{\in} \widehat{\mathcal{H}}$. That is, $x_{\widehat{t}} \overline{\in} \vee \overline{q} \widehat{\mathcal{H}}$. If $\widehat{\mathcal{H}}(0) < \widehat{\mathcal{H}}(x)$, then by (i), $\widehat{\mathcal{H}}(x) \leq \widehat{0.5}$. We consider two cases:
Case (1). If $\widehat{\mathcal{H}}(x) < \widehat{t}$, then $x_{\widehat{t}} \overline{\in} \widehat{\mathcal{H}}$, and so $x_{\widehat{t}} \overline{\in} \vee \overline{q} \widehat{\mathcal{H}}$.
Case (2). If $\widehat{\mathcal{H}}(x) \geq \widehat{t}$, then $\widehat{t} \leq \widehat{\mathcal{H}}(x) \leq \widehat{0.5}$, it follows that $x_{\widehat{t}} \overline{q} \widehat{\mathcal{H}}$, and so $x_{\widehat{t}} \overline{\in} \vee \overline{q} \widehat{\mathcal{H}}$.
Again, let $x_{\inf\{\widehat{t}, \widehat{s}\}} \overline{\in} \widehat{\mathcal{H}}$ for $\widehat{t}, \widehat{s} \in (0, 1]$. Then, $\widehat{\mathcal{H}}(x) < \inf\{\widehat{t}, \widehat{s}\}$. We consider two cases:
Case (1). If $\widehat{\mathcal{H}}(x) \geq \widehat{0.5}$, then

$$\inf\{\widehat{\mathcal{H}}(x*y), \widehat{\mathcal{H}}(y)\} \leq \widehat{\mathcal{H}}(x) < \inf\{\widehat{t}, \widehat{s}\}.$$

Consequently, $\widehat{\mathcal{H}}(x*y) < \widehat{t}$ or $\widehat{\mathcal{H}}(y) < \widehat{s}$. That is, $(x*y)_{\widehat{t}} \overline{\in} \widehat{\mathcal{H}}$ or $y_{\widehat{s}} \overline{\in} \widehat{\mathcal{H}}$. Hence, $(x*y)_{\widehat{t}} \overline{\in} \vee \overline{q} \widehat{\mathcal{H}}$ or $y_{\widehat{s}} \overline{\in} \vee \overline{q} \widehat{\mathcal{H}}$.
Case (2). If $\widehat{\mathcal{H}}(x) < \widehat{0.5}$, then

$$\inf\{\widehat{\mathcal{H}}(x*y), \widehat{\mathcal{H}}(y)\} \leq \widehat{0.5}.$$

Assume $(x*y)_{\widehat{t}} \in \widehat{\mathcal{H}}$ or $y_{\widehat{s}} \in \widehat{\mathcal{H}}$. Then, $\widehat{t} \leq \widehat{\mathcal{H}}(x*y) \leq \widehat{0.5}$ or $\widehat{s} \leq \widehat{\mathcal{H}}(y) \leq \widehat{0.5}$. Thus, $\widehat{\mathcal{H}}(x*y) + \widehat{t} \leq \widehat{0.5} + \widehat{0.5} = \widehat{1}$ or $\widehat{\mathcal{H}}(y) + \widehat{s} \leq \widehat{0.5} + \widehat{0.5} = \widehat{1}$. It follows that $(x*y)_t \overline{q} \widehat{\mathcal{H}}$ or $y_s \overline{q} \widehat{\mathcal{H}}$ and so $(x*y)_{\widehat{t}} \overline{\in} \vee \overline{q} \widehat{\mathcal{H}}$ or $y_{\widehat{s}} \overline{\in} \vee \overline{q} \widehat{\mathcal{H}}$. Hence, $\widehat{\mathcal{H}}$ is an m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy ideal of X. □

Theorem 3.2. *Any m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy ideal $\widehat{\mathcal{H}}$ of X satisfies: for all $x, y \in X$,*

(1) $x \leq y \Rightarrow \sup\{\widehat{\mathcal{H}}(x), \widehat{0.5}\} \geq \widehat{\mathcal{H}}(y)$,
(2) $x * y \leq z \Rightarrow \sup\{\widehat{\mathcal{H}}(x), \widehat{0.5}\} \geq \inf\{\widehat{\mathcal{H}}(y), \widehat{\mathcal{H}}(z)\}$.

Proof. (1) Suppose that $x \leq y$ for all $x, y \in X$. Then, $x * y = 0$. We have

$$\begin{aligned} \sup\{\widehat{\mathcal{H}}(x), \widehat{0.5}\} &\geq \inf\{\widehat{\mathcal{H}}(x*y), \widehat{\mathcal{H}}(y)\} \\ &= \inf\{\widehat{\mathcal{H}}(0), \widehat{\mathcal{H}}(y)\} \\ &\geq \widehat{\mathcal{H}}(y). \end{aligned}$$

(2) Assume that $x * y \leq z$ hold in X. Then,

$$\begin{aligned} \sup\{\widehat{\mathcal{H}}(x*y), \widehat{0.5}\} &\geq \inf\{\widehat{\mathcal{H}}((x*y)*z), \widehat{\mathcal{H}}(z)\} \\ &= \inf\{\widehat{\mathcal{H}}(0), \widehat{\mathcal{H}}(z)\} \\ &\geq \widehat{\mathcal{H}}(z). \end{aligned}$$

Since $\widehat{\mathcal{H}}$ is an m-polar $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy ideal of X , we have

$$\sup\{\widehat{\mathcal{H}}(x), \widehat{0.5}\} \geq \inf\{\widehat{\mathcal{H}}(x*y), \widehat{\mathcal{H}}(y)\}.$$

Now,

$$\begin{aligned}\sup\{\widehat{\mathcal{H}}(x),\widehat{0.5},\widehat{0.5}\} &\geq \sup\{\inf\{\widehat{\mathcal{H}}(x*y),\widehat{\mathcal{H}}(y)\},\widehat{0.5}\}\\ \sup\{\widehat{\mathcal{H}}(x),\widehat{0.5}\} &\geq \inf\{\sup\{\widehat{\mathcal{H}}(x*y),\widehat{0.5}\},\sup\{\widehat{\mathcal{H}}(y),\widehat{0.5}\}\}\\ &\geq \inf\{\widehat{\mathcal{H}}(z),\widehat{\mathcal{H}}(y)\}\\ &= \inf\{\widehat{\mathcal{H}}(y),\widehat{\mathcal{H}}(z)\}.\end{aligned}$$

□

Theorem 3.3. *An m-PF set $\widehat{\mathcal{H}}$ of X is an m-polar $(\overline{\in},\overline{\in}\vee\overline{q})$-fuzzy ideal of X if and only if $\widehat{\mathcal{H}}_{\widehat{t}}\neq\phi$ is an ideal of X for all $\widehat{t}\in(0.5,1]^m$.*

Proof. Assume that $\widehat{\mathcal{H}}$ is an m-polar $(\overline{\in},\overline{\in}\vee\overline{q})$-fuzzy ideal of X and $\widehat{t}\in(0.5,1]^m$. Suppose $x\in\widehat{\mathcal{H}}_{\widehat{t}}$. Then, $\widehat{\mathcal{H}}(x)\geq\widehat{t}$. Now,

$$\sup\{\widehat{\mathcal{H}}(0),\widehat{0.5}\}\geq\widehat{\mathcal{H}}(x)\geq\widehat{t}.$$

Thus, $\widehat{\mathcal{H}}(0)\geq\widehat{t}$. Hence, $0\in\widehat{\mathcal{H}}_{\widehat{t}}$. Let $x*y,y\in\widehat{\mathcal{H}}_{\widehat{t}}$. Then, $\widehat{\mathcal{H}}(x*y)\geq\widehat{t}$ and $\widehat{\mathcal{H}}(y)\geq\widehat{t}$. Now,

$$\sup\{\widehat{\mathcal{H}}(x),\widehat{0.5}\}\geq\inf\{\widehat{\mathcal{H}}(x*y),\widehat{\mathcal{H}}(y)\}\geq\widehat{t}.$$

Thus, $\widehat{\mathcal{H}}(x)\geq\widehat{t}$, that is, $x\in\widehat{\mathcal{H}}_{\widehat{t}}$. Therefore, $\widehat{\mathcal{H}}_{\widehat{t}}$ is an ideal of X.

Conversely, assume $\widehat{\mathcal{H}}_{\widehat{t}}\neq\phi$ is an ideal of X. Let $x\in X$ be such that $\sup\{\widehat{\mathcal{H}}(0),\widehat{0.5}\}<\widehat{\mathcal{H}}(x)$. Choose $\widehat{t}\in(0.5,1]^m$ such that

$$\sup\{\widehat{\mathcal{H}}(0),\widehat{0.5}\}<\widehat{t}\leq\widehat{\mathcal{H}}(x).$$

Then, $\widehat{\mathcal{H}}(0)<\widehat{t}$ and $x\in\widehat{\mathcal{H}}_{\widehat{t}}$. Since $\widehat{\mathcal{H}}_{\widehat{t}}$ is an ideal of X, we have $0\in\widehat{\mathcal{H}}_{\widehat{t}}$, and so $\widehat{\mathcal{H}}(0)\geq\widehat{t}$, a contradiction. Hence, $\sup\{\widehat{\mathcal{H}}(0),\widehat{0.5}\}\geq\widehat{\mathcal{H}}(x)$ for all $x\in X$. Assume $x,y\in X$ such that $\sup\{\widehat{\mathcal{H}}(x),\widehat{0.5}\}<\inf\{\widehat{\mathcal{H}}(x*y),\widehat{\mathcal{H}}(y)\}$. Choose $\widehat{t}\in(0.5,1]^m$ such that

$$\sup\{\widehat{\mathcal{H}}(x),\widehat{0.5}\}<\widehat{t}\leq\inf\{\widehat{\mathcal{H}}(x*y),\widehat{\mathcal{H}}(y)\}.$$

Then, $\widehat{\mathcal{H}}(x)<\widehat{t}$. Since $x*y,y\in\widehat{\mathcal{H}}_{\widehat{t}}$ and $\widehat{\mathcal{H}}_{\widehat{t}}$ is an ideal of X, so $x\in\widehat{\mathcal{H}}_{\widehat{t}}$. That is, $\widehat{\mathcal{H}}(x)\geq\widehat{t}$. This is a contradiction. Thus, $\sup\{\widehat{\mathcal{H}}(x),\widehat{0.5}\}\geq\inf\{\widehat{\mathcal{H}}(x*y),\widehat{\mathcal{H}}(y)\}$ for all $x,y\in X$. Hence, $\widehat{\mathcal{H}}$ is an m-polar $(\overline{\in},\overline{\in}\vee\overline{q})$-fuzzy ideal of X. □

4. Conclusions

In this work, first we have introduced the notion of m-polar $(\overline{\in},\overline{\in}\vee\overline{q})$ fuzzy ideals and investigated some of its properties. Then, we have discussed the relation between ordinary ideals and m-polar $(\overline{\in},\overline{\in}\vee\overline{q})$ fuzzy ideals in BCK/BCI-algebras.

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