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ON $e^{\star}$-REGULARITY AND $e^{\star}$-NORMALITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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ABSTRACT. In this paper, the concept of intuitionistic fuzzy e^*-open and e^*-closed sets to introduce some new types of intuitionistic fuzzy e^*-separation axioms, intuitionistic fuzzy e^*-T_i space (for $i = 3, 4$) and intuitionistic fuzzy e^*-regular and e^*-normal spaces are introduced, some theorems about them are investigated. Stronger forms of intuitionistic fuzzy e^*-regular and intuitionistic fuzzy e^*-normal spaces are introduced. Moreover, the relationships between these separation axioms and others are investigated.

1. INTRODUCTION

Ever since the introduction of fuzzy sets by Zadeh [32], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by Change [6]. Atanassov [1] introduced the notion of intuitionistic fuzzy sets, Coker [7] introduced the intuitionistic fuzzy topological spaces. Several authors [2, 15, 20, 26] introduced the concepts of fuzzy separaion axioms using the notion of fuzzy open set and Othman and Latha [20] by using the notions fo fuzzy regular open sets, fuzzy β-open sets, fuzzy α-open sets and fuzzy semi α-open sets respectively. Singal and Prakash [25] have introduced the concept of fuzzy pre-separation axioms. Qahtani and Al-Qubati [21] have introduced and studied new kinds of fuzzy pre-separation axioms. Several notions based on fuzzy pre-separation axioms have been studied. Some authors studied the concept of separation axioms in intuitionistic fuzzy topological spaces. In 2001, Bayhan and Coker [3] gave some characterizations of T_1 and T_2 separation axioms in intuitionistic topological spaces, they gave interrelations between several types of separationn axioms and some counter examples. In 2003, Lupianez [17] defined new notions of Hausdorffness in the intuitionistic fuzzy topological spaces. In 2005, Bayhan and Coker [5] studied pairwise separation axioms in double intuitionistic topological spaces. For more studies, we can find them in [4, 9, 18, 24]. The initiations of $e^{\star}$-open sets, $e^{\star}$-continuity and $e^{\star}$-compactness in topological spaces are due to Ekici [10, 11, 12, 13, 14]. Sobana et.al

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[29] were introduced the concept of fuzzy $e^\star$-open sets, fuzzy $e^\star$-continuity and fuzzy $e^\star$-compactness in intuitionistic fuzzy topological spaces (briefly., IFTS's). The main purpose of this paper is to introduce and study some new types of intuitionistic fuzzy e^*-separation axioms, which is intuitionistic fuzzy e^*-T_i space (for, $i = 3, 4$) and intuitionistic fuzzy e^*-regular and intuitionistic fuzzy e^*-normal spaces by using the concept of intuitionistic fuzzy e^*-open (resp. closed) sets. Also, we will introduce stronger forms of intuitionistic fuzzy e^*-regular and intuitionistic fuzzy e^*-normal spaces with relationships between these separation axioms and others

2. PRELIMINARIES

Definition 2.1. [1] Let X be a nonempty fixed set and I be the closed interval in $[0,\ 1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form $A = \{< x,\ \mu_A(x), \nu_A(x) >; x \in X\}$ where the mappings $\mu_A(x) : X \to I$ and $\nu_A(x) : X \to I$ denote the degree of membership (namely) $\mu_A(x)$ and the degree of non membership (namely) $\nu_A(x)$ for each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2. [1] Let A and B are intuitionistic fuzzy sets of the form $A = \{< x,\ \mu_A(x), \nu_A(x) >: x \in X\}$ and $B = \{< x,\ \mu_B(x), \nu_B(x) >: x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;
(ii) $\overline{A}\ (or A^c) = \{< x,\ \nu_A(x), \mu_A(x) >: x \in X\}$;
(iii) $A \cap B = \{< x,\ \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >: x \in X\}$;
(iv) $A \cup B = \{< x,\ \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >: x \in X\}$;
(v) $[\,]A = \{< x,\ \mu_A(x),\ 1 - \mu_A(x) >: x \in X\}$;
(vi) $\langle\,\rangle\, A = \{< x,\ 1 - \nu_A(x),\ \nu_A(x) >: x \in X\}$;

We will use the notation $A = \{< x,\ \mu_A,\ \nu_A >: x \in X\}$ instead of $A = \{< x,\ \mu_A(x), \nu_A(x) >: x \in X\}$.

Definition 2.3. [7] $0_\sim = \{< x,\ 0,\ 1 >; x \in X\}$ and $1_\sim = \{< x,\ 1,\ 0 >; x \in X\}$. Let $\alpha,\ \beta \in [0,\ 1]$ such that $\alpha + \beta \leq 1$. An intuitionistic fuzzy point $(IFP)_{p_{(\alpha,\ \beta)}}$ is intuitionistic fuzzy set defined by $p_{(\alpha,\ \beta)}(x)= \begin{cases} (\alpha,\ \beta) \text{ if x=p} \\ (0,\ 1) \text{ otherwise} \end{cases}$

Definition 2.4. [7] An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family T of intuitionistic fuzzy sets in X satisfying the following axioms:

(i) $0_\sim,\ 1_\sim \in T$;
(ii) $G_1 \cap G_2 \in T$, for any $G_1,\ G_2 \in T$;
(iii) $\cup G_i \in T$ for any arbitrary family $\{G_i; i \in J\} \subseteq T$.

In this paper by $(X,\ T)$ or simply by X we will denote the intuitionistic fuzzy topological space(IFTS). Each IFS which belongs to T is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.5. [16] Let $p_{(\alpha,\ \beta)}$ be an IFP in IFTS X. An IFS A in X is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(\alpha,\ \beta)}$ if there exists an IFOS B in X such that $p_{(\alpha,\ \beta)} \in B \subseteq A$.

Let X and Y are two non-empty sets and $f : (X,\ T) \to (Y,\ \sigma)$ be a function [7]. If $B = \{< y,\ \mu_B(y),\ \nu_B(y) >; y \in Y\}$ is an IFS in Y, then the pre-image of B under f is denoted and defined by $f^{-1}(B) = \{< x,\ f^{-1}(\mu_B(x)),\ f^{-1}(\nu_B(x)) >; x \in X\}$. Since

$\mu_B(x),\ \nu_B(x)$ are fuzzy sets, we explain that $f^{-1}(\mu_B(x)) = \mu_B(x)(f(x)), f^{-1}(\nu_B(x)) = \nu_B(x)(f(x))$.

Definition 2.6. [7] Let $(X,\ T)$ be an IFTS and $A = \{< x,\ \mu_A,\ \nu_A >; x \in X\}$ be an IFS in X. Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior of A are defined by

(i) $cl(A) = \bigcap\{C : C$ is an IFCS in $X$and $C \supseteq A\}$;
(ii) $int(A) = \bigcup\{D : D$ is an IFOS in $X$and $D \subseteq A\}$;

It can be also shown that $cl(A)$ is an IFCS, $int(A)$ is an IFOS in X and A is and IFCS in X if and only if $cl(A) = A$; A is an IFOS in X if and only if $int(A) = A$

Proposition 2.1. [7] *Let $(X,\ T)$ be an IFTS and $A,\ B$ be intuitionistic fuzzy sets in X. Then the following properties hold:*

(i) $cl(\overline{A}) = \overline{(int(A))},\ int(\overline{A}) = \overline{(cl(A))}$;
(ii) $int(A) \subseteq A \subseteq cl(A)$.

Definition 2.7. Let A be IFS in an IFTS (X, T). A is called an intuitionistic fuzzy regular open set [30] (briefly $IFROS$) if $A = intcl(A)$ and intuitionistic fuzzy regular closed set (briefly $IFRCS$) if $A = clint(A)$

Definition 2.8. [30] Let (X, T) be an IFTS and $A =< x, \mu_A(x), \nu_A(x) >$ be a IFS in X. Then the fuzzy δ closure of A are denoted and defined by $cl_\delta(A) = \cap\{K : K$ is an IFRCS in X and $A \subseteq K\}$ and $int_\delta(A) = \cup\{G : G$ is an IFROS in X and $G \subseteq A\}$.

Definition 2.9. [29] Let A be an IFS in an IFTS(X, T). A is called an intuitionistic fuzzy $e^\star$-open set (IF$e^\star$OS, for short) in X if $A \subseteq clintcl_\delta(A)$

Definition 2.10. [27] Let (X, T) be an IFTS and $A =< x, \mu_A, \nu_A >$ be an IFS in X. Then the intuitionistic fuzzy e^*-interior and intuitionistic fuzzy e^*-closure are defined and denoted by:

$$cl_{e^*}(A) = \cap\{K : K \text{ is an IF}e^*\text{CS in } X \text{ and } A \subseteq K\}$$

and

$$int_{e^*}(A) = \cup\{G : G \text{ is an IF}e^*\text{OS in } X \text{ and } G \subseteq A\}.$$

It is clear that A is an IFe^*CS (IFe^*OS) in X iff $A = cl_{e^*}(A)(A = int_{e^*}(A))$.

Definition 2.11. [7] Let (X, T) and (Y, S) be IFTS's. A function $f : (X, T) \to (Y, S)$ is called intuitionistic fuzzy continuous(resp., $e^\star$-continuous [29]) if $f^{-1}(B)$ is an IFOS(resp., IF$e^\star$OS)in X for every $B \in S$.

Definition 2.12. [28] A IFTS (X, T) is said to be a intuitionistic fuzzy stronger- $e^\star$-T_1 (briefly, $IFe^\star$-T_1s) if every IFP is an IF$e^\star$CS.

3. $e^\star$-Regularity in Intuitionistic Fuzzy Topologi-cal Spaces

Definition 3.1. An IFTS (X, T) is said to be intuitionistic fuzzy e^*-Uryshon (briefly, IFe^* - $T_{2\frac{1}{2}}$) if for every pair of IFP's $p = x_{(\alpha,\beta)}, q = y_{(\gamma,\eta)}$ with different supports, there exists an $IFe^*OS's$ M and N such that $(p \subseteq M, q \nsubseteq M)$ and $(q \subseteq N, p \nsubseteq N)$ and $IFe^*cl(M) \nsubseteq IFe^*cl(N)$.

Definition 3.2. An IFTS (X, T) is said to be intuitionistic fuzzy e^*-regular space (briefly, IFe^*Rs) if for every pair of IFP p and IFe^*CS N such that $p \nsubseteq N$ there exists an $IFe^*OS's$ M_1 and M_2 such that $(p \subseteq M_1, N \subseteq M_2$ and $M_1 \nsubseteq M_2)$.

Theorem 3.1. *If (X,T) be an IFe^*Rs, then for any IFP $p = x_{(\alpha,\beta)}$ and an IFOS N such that $p \subseteq N$, there exists an IFe^*OS M_1 such that $p \subseteq M_1 \subseteq IFe^*cl(M_1) \subseteq N$.*

Proof. Suppose that X be an IFe^*Rs. Let $N = \langle x, \mu_N(x), \gamma_N(x)\rangle$ be an IFOS of X and p be an IFP in X such that $p \subseteq N$. Then $N^c = \langle x, \gamma_N(x), \mu_N(x)\rangle$ is an IFe*CS in X . Since X is an IFe^*Rs, therefore there exist two IFe*OS's M_1 and M_2 such that $p \in M_1, N^c \subseteq M_2$ and $M_1 \nsubseteq M_2$. Now M_2^c is an IFe*CS such that $M_1 \subseteq M_2^c \subseteq N$, thus, $p \in M_1 \subseteq IFe^*cl(M_1)$ and $IFe^*cl(M_1) \subseteq M_2^c \subseteq N$, so $IFe^*cl(M_1) \subseteq N$. Hence $p \subseteq M_1 \subseteq IFe^*cl(M_1) \subseteq N$. □

Definition 3.3. A IFTS (X,T) is said to be an intuitionistic fuzzy e^*T_3 space (briefly, IFe^*T_3) if it is an IFe^*Rs as well as IFe^* - T_1s space.

Proposition 3.2. *Every subspace of IFe^*Rs is IFe^*Rs.*

Proof. Let (X,T) be a IFe^*Rs and Y be subspace of X. To prove that Y is an IFe^*-regular, where $T_Y = \{G_Y = \langle x, \mu_{G|Y}(x), \nu_{G|Y}(x)\rangle, x \in Y, G \in T\}$ and $G = \langle x, \mu_G(x), \nu_G(x)\rangle$. Let $p = x_{(\alpha,\beta)}$ be an IFP in Y , and N_Y be an IFe^*CS in Y such that $p \nsubseteq N_Y$. Since Y is a subspace of X , so $p \in X$ and there exist an IFe^*CS F in X such that the closed set generated by it for Y is F_Y. Since X is IFe^*Rs such that $p \nsubseteq F$, there exist two $IFe^*OS's$ M_1, M_2 such that $p \subseteq M_1 = \langle x, \mu_{M_1}(x), \nu_{M_1}(x)\rangle$ and $N \subseteq M_1 = \langle x, \mu_{M_2}(x), \nu_{M_2}(x)\rangle$ and $M_1 \nsubseteq M_2, M_{1Y} = \langle x, \mu_{M_1|Y}(x), \nu_{M_1|Y}(x)\rangle$ and $M_{2Y} = \langle x, \mu_{M_2|Y}(x), \nu_{M_2|Y}(x)\rangle$ are IFe^*OS in Y such that that $p \subseteq M_{1Y}, N \subseteq M_{2Y}$ and $M_{1Y} \nsubseteq M_{2Y}$.Hence Y is an IFe*Rs □

Theorem 3.3. *An IFTS (X,T) is said to be an IFe^*Rs if and only if for an IFP $p = x_{(\alpha,\beta)}$ and an IFCS N such that $p \nsubseteq N$, there exist two $IFe^*OS's$ M_1, M_2 such that $p \subseteq M_1, N \subseteq M_2$, and $IFe^*cl(M_1) \nsubseteq (IFe^*cl(M_2))$.*

Proof. Since $p \nsubseteq N$ and (X,T) is IFe^*Rs, there exist two $IFe^*OS's$ M_1, M_2 such that $p \subseteq N, N \subseteq M_2$ and $M_1 \nsubseteq M_2$ and by Theorem 3.1 $p \subseteq M_2 \subseteq IFe^*cl(M_2) \subseteq M_1$. Hence $IFe^*cl(M_2) \subseteq M_1$. Also, $N \subseteq M_2 \subseteq IFe^*cl(M_2)$. But since $IFe^*cl(M_1) \nsubseteq IFe^*cl(M_2)$, then $N \subseteq M_2 \subseteq IFe^*cl(M_2) \subseteq IFe^*cl(M_1) \subseteq M_1^c$ and hence $p \subseteq M_1, N \subseteq M_2$ and $IFe^*cl(M_1) \nsubseteq (IFe^*cl(M_2))$.

The converse is clear. □

Theorem 3.4. *Let (X,T) be an IFe^*Rs which is also IFT_0. Then (X,T) is IFe^*-$T_{2\frac{1}{2}}$.*

Proof. Let $p = x_{(\alpha,\beta)}$ and $q = x_{(\gamma,\eta)}$ are IFP's with different supports. Since (X,T) is an IFT_0 space, then there exists an IFOS N such that $p \subseteq N, q \nsubseteq N$ or $q \subseteq N, p \nsubseteq N$. Consider the part $p \subseteq N, q \nsubseteq N$. This implies that $p \nsubseteq N^c$ where N^c is an IFCS. Since (X,T) is an IFe^*Rs, so by Theorem 3.3, there exist an $IFe^*OS's$ M_1 and M_2 such that $p \subseteq M_1$and $N^c \subseteq M_2$ and $IFe^*cl(M_1) \nsubseteq (IFe^*cl(M_2))$ or $p \subseteq M_1, q \subseteq M_2$and $IFe^*cl(M_1) \nsubseteq (IFe^*cl(M_2))$. Hence (X,T) is $IFe^*T_{2\frac{1}{2}}$. □

Theorem 3.5. *If $f : (X,T) \to (Y,S)$ is a closed injective intuitionistic fuzzy e*-continuous mapping and (Y,S) is an $IFRS$, then (X,T) is an IFe^*Rs.*

Proof. Let p be an IFP and N be an IFCS in X . Then $f(p)$ is an IFP and $f(N)$ is an IFCS in Y . Since (Y,S) is an intuitionistic fuzzy regular space then there exist two IFOS's M_1 and M_2 such that $f(N) \subseteq M_2, f(p) \subseteq M_1$ and $M_1 \nsubseteq M_2$. It follows that $N \subseteq f^{-1}(M_2), p \subseteq f^{-1}(M_1)$ and $f^{-1}(M_1) \subseteq f^{-1}(M_2)$ where $f^{-1}(M_1)$ and $f^{-1}(M_2)$ are $IFe^*OS's$ in X . Hence (X,T) is an IFe^*Rs. □

Theorem 3.6. *Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy e^*-open mapping. If X is an intuitionistic fuzzy regular space then Y is IFe^*Rs.*

Theorem 3.7. *Every IFe^*Rs and IFT_2 space is an $IFe^\star T_{2\frac{1}{2}}$ space.*

Proof. Let (X,T) be an IFTS which is IFe^*Rs and IFT_2. Let p and q be two IFP's with different supports in X. Since X is an IFT_2, there exist two IFOS's M and N such that $p\subseteq M, q\nsubseteq M, q\subseteq N, p\nsubseteq N$ and $M\nsubseteq N$ or $IFe^*cl(M)\nsubseteq N$. So that $q\nsubseteq M$. Since $IFe^*cl(M)$ is an IFCS and (X,T) is an IFe^*Rs. So by Proposition 3.2, there exist an $IFe^*OS's$ M_1 and M_2 such that $IFe^*cl(M)\subseteq M_2, q\subseteq M_1$ and $IFe^*cl(M_1)\nsubseteq IFe^*cl(M_2)$. Since $p\subseteq IFe^*cl(M)$, we have $p\subseteq M_2, q\subseteq M_1$, and $IFe^*cl(M_1)\nsubseteq IFe^*cl(M_2)$. Hence the space (X,T) is $IFe^*T_{2\frac{1}{2}}$ space. □

Definition 3.4. An IFTS(X,T) is said to be

(i) intuitionistic fuzzy e^*-regular space (i) (briefly, $IFe^*Rs(i)$) if for an IFP p and an IFe^*CS N such that $p\nsubseteq N$, there exist two $IFe^*OS's$ M_1 and M_2 such that $N\subseteq M_2, p\subseteq M_1$ and $M_1\nsubseteq M_2$.

(ii) intuitionistic fuzzy e^*-regular space (ii) (briefly, $IFe^*Rs(ii)$) if for an IFP p and an IFe^*CS N such that $p\nsubseteq N$, there exist two IFOS's M_1 and M_2 such that $N\subseteq M_2, p\subseteq M_1$ and $M_1\nsubseteq M_2$.

Theorem 3.8. *An IFTS (X,T) is said to be*

(i) $IFe^*Rs(i)$ if for an IFP p and an IFe^*OS M_1 such that $p\subseteq M_1$, there exist an IFe^*OS M_2 such that $p\subseteq M_2\subseteq IFe^*cl(M_2)\subseteq M_1$.

(ii) $IFe^*Rs(ii)$ if for an IFP p and an IFe^*OS M_1 such that $p\subseteq M_1$, there exist an IFOS M_2 such that $p\subseteq M_2\subseteq IFe^*cl(M_2)\subseteq M_1$.

Proof. (i) Let M_1 be an IFe^*OS such that $p\subseteq M_1$, then $p\nsubseteq M_1^c$ and M_1^c is an IFe^*CS. Therefore, there exist two IFOS's N and K such that $p\subseteq N, M_1^c\subseteq K$ and $N\nsubseteq K$. Now, we can get $IFe^*cl(N)^c\subseteq (M_1^c)^c = M_1$ and $P\subseteq N\subseteq IFe^*cl(N)\subseteq M_1$. It is clear that $p\subseteq N\subseteq IFe^*int(IFe^*cl(N)\subseteq IFe^*cl(N)\subseteq M_1)$. Therefore, if we put $IFe^*int(IFe^*cl(N) = M_2$, then $p\subseteq N\subseteq M_2\subseteq IFe^*cl(M_2)\subseteq M_1$, where M_2 is an IFOS. The converse is clear.

(ii) The proof is similar to (i). □

Theorem 3.9. *An IFTS (X,T) is said to be*

(i) $IFe^*Rs(i)$ if for an IFP p and an IFe^*CS N such that $p\nsubseteq N^c$, there exist two $IFe^*OS's$ M_1 and M_2 such that $p\subseteq M_1, N\subseteq M_2$, and $IFe^*cl(M_1)\nsubseteq IFe^*cl(M_2)$.

(ii) $IFe^*Rs(ii)$ if for an IFP p and an IFe^*CS N such that $p\nsubseteq N$, there exist two IFOS's M_1 and M_2 such that $p\subseteq M_1, N\subseteq M_2$, and $IFe^*cl(M_1)\nsubseteq IFe^*cl(M_2)$.

Proof. Similar to that of Theorem 3.8. □

Theorem 3.10. *Let f be an injective, intuitionistic fuzzy e^*-irresolute mapping from an IFTS (X,T) into an IFTS (Y,S). Then X is e^*Rs if Y is e^*Rs.*

Theorem 3.11. *Let (X,T) be an $IFe^*Rs(i)$ which is also IFe^*T_0. Then (X,T) is $IFe^*T_{2\frac{1}{2}}$.*

Proof. Let p and q are IFP's with different supports. Since (X,T) is a IFe^*T_0 space, then there exists an IFe^*OS M such that $p\nsubseteq M, q\nsubseteq M$ or $q\subseteq M, p\nsubseteq M$. Consider the part $p\nsubseteq M, q\nsubseteq M$. This implies that $p\subseteq (M^c)^c$, where M^c is an IFe^*CS. Since (X,T) is

$IFe^{*}Rs(i)$, so by Theorem 3.9, there exist $IFe^{*}OS's$ N and K such that $p \subseteq N, M^c \subseteq K$ and $IFe^{*}cl(N) \nsubseteq IFe^{*}cl(K)$, or $p \subseteq N, q \subseteq K$ and $IFe^{*}cl(N) \nsubseteq IFe^{*}cl(K)$. Hence (X,T) is an $IFe^{*}T_{2\frac{1}{2}}$. □

Theorem 3.12. *Let (X,T) be an $IFe^{*}Rs(ii)$ which is also $IFe^{*}T_0$. Then (X,T) is $IFe^{*}T_{2\frac{1}{2}}$.*

Theorem 3.13. *Let (X,T) be an $IFe^{*}Rs(ii)$ which is also $IFe^{*}T_2$. Then (X,T) is $IFe^{*}T_{2\frac{1}{2}}$.*

Definition 3.5. An IFTS (X,T) is said to be

(i) intuitionistic Fuzzy $e^{*}T_3$ space (i) (briefly, $IFe^{*}T_3s(i)$) if it is $IFe^{*}Rs(i)$ as well as $IFe^{\star}$-T_1s space.
(ii) intuitionistic Fuzzy $e^{*}T_3$ space (i) (briefly, $IFe^{*}T_3s(ii)$) if it is $IFe^{*}Rs(ii)$ as well as $IFe^{\star}$-T_1s space.

4. $e^{\star}$-NORMALITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

Definition 4.1. An IFTS (X,T) is said to be

(i) intuitionistic fuzzy e^{*}-normal space (briefly, $IFe^{*}Ns$) if for every pair of IFCS's N_1 and N_2 such that $N_1 \nsubseteq N_2$, there exist two $IFe^{*}OS's$ M_1 and M_2 such that $N_1 \subseteq M_1, N_2 \subseteq M_2$ and $M_1 \nsubseteq M_2$.
(ii) intuitionistic fuzzy $e^{*}T_4$ spac (briefly, $IFe^{*}T_4s$)e if it is an $IFe^{*}Ns$ as well as $IFe^{\star}$-T_1s space.

Theorem 4.1. *An IFTS (X,T) is an $IFe^{*}Ns$ if and only if for every IFCS N_1 and every IFOS N_2 such that $N_1 \subseteq N_2$ there exists an $IFe^{*}OS$ M such that $N_1 \subseteq M \subseteq IFe^{*}cl(M) \subseteq N_2$.*

Proof. Let (X,T) be an $IFe^{*}Ns$ and let $N_1 \subseteq N_2$, where N_1 is a IFCS and N_2 is an IFOS, then $(N_2)^c$ is an IFCS but (X,T) be an $IFe^{*}Ns$ hence there exist two $IFe^{*}OS's$ M_1 and M_2 such that $N_1 \subseteq M_1, (N_2)^c \subseteq M_2$ and $M_1 \nsubseteq M_2$, therefore $N_1 \subseteq M \subseteq (M_2)^c \subseteq N_2$ that implies $IFe^{*}cl(N_1) \subseteq IFe^{*}cl(M) \subseteq IFe^{*}cl(M_2)^c \subseteq IFe^{*}cl(N_2)$, then $N_1 \subseteq IFe^{*}cl(N_1) \subseteq IFe^{*}cl(M_1) \subseteq (M_2)^c \subseteq N_2$. Hence $N_1 \subseteq M_1 \subseteq IFe^{*}cl(M_1) \subseteq N_2$.The converse is clear. □

Theorem 4.2. *If $f : (X,T) \to (Y,S)$ is closed injective, intuitionistic fuzzy e^{*}-continuous mapping and (Y,S) is a intuitionistic fuzzy normal space, then (X,T) is an $IFe^{*}Ns$.*

Theorem 4.3. *If $f : (X,T) \to (Y,S)$ be an intuitionistic fuzzy e^{*}-open mapping. Then (X,T) is a intuitionistic fuzzy normal space, if (Y,S) is an $IFe^{*}Ns$.*

Theorem 4.4. *Every $IFe^{*}Ns$ and $IFTs$ space is $IFe^{*}Rs$.*

Proof. Let (X,T) be an $IFe^{*}Ns$ and $IFTs$ space. Let p and N be respectively, an IFP and $IFe^{*}CS$ in X and $p \nsubseteq N$. Since (X,T) is IFTS, then p is an $IFe^{*}CS$ and since (X,T) is an $IFe^{*}Ns$, there exist $IFe^{*}OS's$ M_1 and M_2 such that $p \subseteq M_1, N \subseteq M_2$ and $M_1 \nsubseteq M_2$. Hence (X,T) is an $IFe^{*}Rs$. □

Definition 4.2. An IFTS (X,T) is said to be

(i) intuitionistic fuzzy e^{*}-normal space (i) (briefly, $IFe^{*}Ns(i)$) if for every pair of $IFe^{*}CS's$ N_1 and N_2 such that $N_1 \nsubseteq N_2$, there exist two $IFe^{*}OS's$ M_1 and M_2 such that $N_1 \subseteq M_1, N_2 \subseteq M_2$ and $M_1 \nsubseteq M_2$.

(ii) intuitionistic fuzzy e^*-normal space (ii) (briefly, $IFe^*Ns(ii)$) if for every pair of $IFe^*CS's$ N_1 and N_2 such that $N_1 \nsubseteq N_2$, there exist two IFOS's M_1 and M_2 such that $N_1 \subseteq M_1, N_2 \subseteq M_2$ and $M_1 \nsubseteq M_2$.

Theorem 4.5. *(1) Every $IFe^*Ns(i)$ and IFe^*Ts space is $IFe^*Rs(i)$.*
(2) Every $IFe^*Ns(ii)$ and IFe^*Ts space is $IFe^*Rs(ii)$.

Proof. (1) Let (X, T) be an $IFe^*Ns(i)$ and IFe^*Ts space. Let p and N be respectively, an IFP and an IFe^*CS in X and $p \nsubseteq N$. Since (X, T) is IFe^*Ts, then p is an IFe^*CS and since (X, T) is $IFe^*Ns(i)$, there exist an $IFe^*OS's$ M_1 and M_2 such that $p \subseteq M_1, N \subseteq M_2$ and $M_1 \nsubseteq M_2$. Hence (X, T) is an $IFe^*Rs(i)$.

Proof for case (2) is similarly follow. □

Definition 4.3. An IFTS (X, T) is said to be

(i) intuitionistic fuzzy e^*T_4 space (i) (briefly, $IFe^T_4s(i)$) if it is $IFe^*Ns(i)$ as well as $IFe^\star$-T_1s space.

(ii) intuitionistic fuzzy e^*T_4 space (ii) (briefly, $IFe^T_4s(ii)$) if it is $IFe^*Ns(ii)$ as well as $IFe^\star$-T_1s space.

Theorem 4.6. *(1) Every $IFe^*T_4s(i)$ is $IFe^*T_3s(i)$*
(2) Every $IFe^*T_4s(ii)$ is $IFe^*T_3s(ii)$

Proof. Strictly follow from Theorem 4.5 □

5. CONCLUSION

In this paper, we have introduced and studied the concepts of intuitionistic fuzzy e^*-open and e^*-closed sets to some new types of intuitionistic fuzzy e^*-separation axioms, intuitionistic fuzzy e^*-T_i space (for $i = 3, 4$) and intuitionistic fuzzy e^*-regular and e^*-normal spaces are introduced, some theorems about them are investigated. Stronger forms of intuitionistic fuzzy e^*-regular and intuitionistic fuzzy e^*-normal spaces are introduced. Moreover, the relationships between these separation axioms and others are investigated.

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