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ROUGH APPROXIMATIONS OF INTERVAL ROUGH FUZZY IDEALS IN GAMMA-SEMIGROUPS

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ABSTRACT. In this paper, we introduce the notions of interval rough fuzzy Γ-ideals, bi-Γ-ideal, prime-Γ- ideal and prime-bi-Γ-ideal in Γ-semigroup and establish some interesting properties of these structures.

1. INTRODUCTION

In 1965, the concept of fuzzy sets was introduced by Zadeh[10] and paved the way to study uncertainty problems. Applications of fuzzy set theory have been found in various fields. Sen [7] definded the Γ-semigroup in 1986. The notion of rough set was introduced by Pawlak[4] as a new tool for reasoning about data. Rough set theory is a powerful tool to deal with imperfect data. The rough set theory is an extension of set theory. The main idea of rough set corresponds to the concepts of lower and upper approximations of a set. The lower approximation of a given set is the union of all equivalence classes which are subsets of the set, and the upper approximation is the union of all equivalence classes which have a nonempty intersection with the set. Zadeh[9] introduced the notion of interval-valued fuzzy sets as a generalization of fuzzy sets in 1975, ie., a fuzzy subset with an interval-valued membership function. Later by combining rough sets and fuzzy sets, the notion of rough fuzzy sets were introduced by Dubois and Prade [2]in 1990. The notion of interval valued rough fuzzy sets in semigroups was introduced by Subha et al.[5].

Throughout this paper let us denote S as Γ-semigroup, ∂ as complete congruence relation and Ω as interval valued fuzzy set.

2. PRELIMINARIES

In this section we list some concepts that are required in the development of our work.

Definition 2.1. Let S be a Γ semigroup and ∂ congruence relation on S. The pair (S, ∂) is called an approximation space. Let Ω be any nonempty subset of S. The sets

$$\partial^l(\Omega) = \{x \in S/[x]_\partial \subseteq \Omega\} \text{ and } \partial^u(\Omega) = \{x \in S/[x]_\partial \cap \Omega \neq \phi\}$$

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are called the lower and upper approximations of Ω. Then $\partial(\Omega) = (\partial^l(\Omega), \partial^u(\Omega))$ is called rough set in (S, ∂).

Definition 2.2. Let Ω be a fuzzy subset of S. The fuzzy subsets of S defined by

$$\partial^u(\Omega)(x) = \bigvee_{a\in[x]_\partial} \Omega(a) \text{ and } \partial^l(\Omega)(x) = \bigwedge_{a\in[x]_\partial} \Omega(a)$$

are called respectively, the upper and lower approximations of the fuzzy set Ω.
$\partial(\Omega) = (\partial^l(\Omega), \partial^u(\Omega))$ is called a rough fuzzy set of Ω with respect to ∂ if $\partial^l(\Omega) \neq \partial^u(\Omega)$.

Definition 2.3. Let Ω be an IF subset of S and $[\lambda_1, \lambda_2] \in D[0,1]$, we call $(\Omega, [\lambda_1, \lambda_2]) = \{x \in X : \Omega^-(x) \geq \lambda_1, \Omega^+(x) \geq \lambda_2\}$ and
$(\Omega, (\lambda_1, \lambda_2)) = \{x \in X : \Omega^-(x) > \lambda_1, \Omega^+(x) > \lambda_2\}$ the $[\lambda_1, \lambda_2]-$ level set of Ω and $(\lambda_1, \lambda_2)-$ level set of A, respectively.

Let S be the finite and nonempty set called the universe and assume that S is a Γ-semigroup. Let Ω be an IF subset of S and let ∂ be the complete congruence relation on S. Let $\partial^l(\Omega)$ and $\partial^u(\Omega)$ be the IF subset of S defined by,

$$\partial^l(\Omega)(x) = [\wedge\Omega^-(y); y \in [x]_\partial, \wedge\Omega^+(y); y \in [x]_\partial]$$
$$\partial^u(\Omega)(x) = [\vee\Omega^-(y); y \in [x]_\partial, \vee\Omega^+(y); y \in [x]_\partial]$$

Then $\partial(\Omega) = (\partial^l(\Omega), \partial^u(\Omega))$ is called an IRF set if $\partial^l(\Omega) \neq \partial^u(\Omega)$.

Theorem 2.1. [5] *Let ∂ be a congruence relation on S. If Ω is an IF subset of S and $[\lambda_1, \lambda_2] \in D[0,1]$, then*
(i) $(\partial^l(\Omega), [\lambda_1, \lambda_2]) = \partial^l(\Omega, [\lambda_1, \lambda_2])$
(ii) $(\partial^u(\Omega), (\lambda_1, \lambda_2)) = \partial^u(\Omega, (\lambda_1, \lambda_2))$.

Theorem 2.2. [8] *Let ∂ be a complete congruence on S and Ω prime ideal of S. Then the following statements are true.*
(i) *If $\partial^l(\Omega) \neq \phi$, then Ω is a ∂-lower rough prime ideal of S.*
(ii) *Ω is a ∂-upper rough prime ideal of S.*

3. Interval Rough fuzzy Γ-Ideals, Interval Rough fuzzy Bi-Γ-Ideal and Interval Rough fuzzy Sub-Γ-semigroup ($IRFgI$, $IRFBgI$ and $IRFSgS$)

In this section we introduce the concept of IRF set of S. We also introduce the concept of $IRFgI$, $IRFSgS$ and $IRFBgI$ of S. Example of $IRFgI$ is discussed.
An IF subset of S is called an $IRFgI$ of S if it is both upper and lower $IRFgI$ of S.

Theorem 3.1. *If Ω is an $IFgI$ of S, then $\partial^u(\Omega)$ is an $IFgI$ of S.*

Proof. Assume that Ω is an IF left gI of S, then for all $x, y \in S$, we have $\Omega(x\gamma y) \geq \Omega(y)$.

$$\begin{aligned}\partial^u(\Omega)(x\gamma y) &= \bigvee_{p\gamma q\in[x\gamma y]_\partial} \Omega(p\gamma q)\\ &\geq \bigvee_{p\in[x]_\partial\gamma q\in[y]_\partial} \Omega(p\gamma q)\\ &\geq \bigvee_{q\in[y]_\partial} \Omega(q)\\ &= \partial^u(\Omega)(y)\end{aligned}$$

Hence $\partial^u(\Omega)$ is an $IFgI$ of S. □

Theorem 3.2. *If Ω is an $IFgI$ of S, then $\partial^u(\Omega)$ is an $IFgI$ of S.*

Proof. Similar to 3.1. □

Theorem 3.3. *If Ω is an $IFgI$ of S, then $\partial^u(\Omega)$ is $IRFgI$ of S.*

Proof. By applying Theorems 3.1 and 3.2 we get the result. □

Example 3.1. Let $S = \{e, a, b\}$ be a Γ-semigroup and $\Gamma = \{\gamma\}$ with the following multiplication table

γ	e	a	b
e	e	e	e
a	e	a	e
b	e	e	b

Let ∂ be a complete congruence relation S and the equivalence classes of S are $\{\{e\}, \{a, b\}\}$. Also an IF subset of S are defined by $\Omega(e) = [1, 1]$, $\Omega(a) = [.8, .9]$ and $\Omega(b) = [.7, .8]$. Then Ω is an $IRFgI$ of S.

Theorem 3.4. *Let Ω be an IF subset of S. Then Ω is an $IFgI$ of S if and only if for all $[\lambda_1, \lambda_2] \in D[0, 1]$ then $(\Omega, [\lambda_1, \lambda_2])(resp., (\Omega, (\lambda_1, \lambda_2)) \neq \phi, (\Omega, (\lambda_1, \lambda_2)))$ is a gI of S.*

Proof. Let us assume that Ω is an $IFgI$ of S. Assume $(\Omega, [\lambda_1, \lambda_2]) \neq \phi$.
Let $x \in (\Omega, [\lambda_1, \lambda_2])$, $y \in S$ and $\gamma \in \Gamma$. Then $\Omega(x) \geq [\lambda_1, \lambda_2]$. Since Ω is an $IFgI$ of S then $\Omega(x\gamma y) \geq \Omega(x) \wedge \Omega(y) \geq [\lambda_1, \lambda_2]$ implies $x\gamma y \in (\Omega, [\lambda_1, \lambda_2])$. Similarly $y\gamma x \in (\Omega, [\lambda_1, \lambda_2])$. Hence $(\Omega, [\lambda_1, \lambda_2])$ is an gI of S.
Conversely, let us take $[\lambda_1, \lambda_2] \in D[0, 1]$ if $(\Omega, [\lambda_1, \lambda_2]) \neq \phi$, then $(\Omega, [\lambda_1, \lambda_2])$ is an gI of S. Let $x, y \in S$ and $\gamma \in \Gamma$. Then we have two cases
Case(I): If $\Omega(x) \leq \Omega(y)$ and let $[\lambda_1, \lambda_2] = \Omega(x)$. Then $x \in (\Omega, [\lambda_1, \lambda_2])$.
By assumption $(\Omega, [\lambda_1, \lambda_2])$ is an gI of S. So $x\gamma y \in (\Omega, [\lambda_1, \lambda_2])$. Then

$$\Omega(x\gamma y) \geq [\lambda_1, \lambda_2] = \Omega(x) = \Omega(x) \wedge \Omega(y).$$

Case(II): If $\Omega(x) \geq \Omega(y)$. Let $[\lambda_1, \lambda_2] = \Omega(y)$. Then $y \in (\Omega, [\lambda_1, \lambda_2])$.
By assumption we've $(\Omega, [\lambda_1, \lambda_2])$ is an gI of S. So $x\gamma y \in (\Omega, [\lambda_1, \lambda_2])$

$$\Omega(x\gamma y) \geq [\lambda_1, \lambda_2] = \Omega(y) = \Omega(x) \wedge \Omega(y).$$

Hence from Case(I) and Case(II) we have $\Omega(x\gamma y) \geq \Omega(x) \wedge \Omega(y)$. Therefore Ω is an $IFgI$ of S. Similarly we prove for other case. □

Theorem 3.5. *Let Ω be an $IFgI$ of S. Then $\partial^u(\Omega)$ is an $IFgI$ of S.*

Proof. Assume that Ω is an $IFgI$ of S. Let $x, y \in S$ and $\gamma \in \Gamma$ then

$$\begin{aligned}
&\Omega(x\gamma y) \geq \Omega(x) \vee \Omega(y).\\
&\partial^u(\Omega)(x\gamma y) = \bigvee_{a \in [x\gamma y]_\partial} \Omega(a)\\
&\qquad = \bigvee_{a \in [x]_\partial \gamma [y]_\partial} \Omega(a)\\
&\qquad = \bigvee_{c \in [x]_\partial d \in [y]_\partial} \Omega(c\gamma d)\\
&\qquad \geq \left(\bigvee_{c \in [x]_\partial} \Omega(c) \right) \wedge \left(\bigvee_{d \in [y]_\partial} \Omega(d) \right)\\
&\qquad = \partial^u(\Omega)(x) \wedge \partial^u(\Omega)(y).
\end{aligned}$$

Hence the theorem. □

An IF subset of S is called an IRF sub-Γ-semigroup of S if it is both upper and lower IRF sub-Γ-semigroup of S.

Theorem 3.6. *If Ω is an IF sub-Γ-semigroup of S, then $\partial^u(\Omega)$ is an IF sub-Γ-semigroup of S.*

Proof. Assume that Ω is an IF sub-Γ-semigroup of S. Let $x, y \in S$ and $\gamma \in \Gamma$. Then $\Omega(x\gamma y) \geq \Omega(x) \wedge \Omega(y)$

$$\begin{aligned} \partial^u(\Omega)(x\gamma y) &= \bigvee_{p\gamma q \in [x\gamma y]_\partial} \Omega(p\gamma q) \\ &\geq \bigvee_{p\in[x]_\partial \gamma q\in[y]_\partial} (\Omega(p\gamma q) \\ &\geq \bigvee_{p\in[x]_\partial \gamma q\in[y]_\partial} (\Omega(p) \wedge \Omega(q)) \\ &\geq (\bigvee_{p\in[x]_\partial} \Omega(p)) \wedge (\bigvee_{q\in[x]_\partial} \Omega(q)) \\ &= \partial^u(\Omega)(x) \wedge \partial^u(\Omega)(y) \end{aligned}$$

This proves that $\partial^u(\Omega)$ is an IF sub-Γ-semigroup of S. Similarly we can prove for lower approximation. □

Theorem 3.7. *If Ω is an IF sub-Γ-semigroup of S, then $\partial^l(\Omega)$ is an IF sub-Γ-semigroup of S.*

By combining Theorem 3.6 and Theorem 3.7 we get the following theorem.

Theorem 3.8. *Let Ω be an IF sub-Γ-semigroup of S. Then Ω is an IRF sub-Γ-semigroup of S.*

Theorem 3.9. *Let Ω be an IF subset of S then Ω is an IF sub-Γ-semigroup of S if and only if $(\Omega, (\lambda_1, \lambda_2))$ is a sub-Γ-semigroup of S for all $[\lambda_1, \lambda_2] \in D[0, 1]$.*

Proof. Suppose Ω is an IF sub-Γ-semigroup of S. Let $x, y \in (\Omega, (\lambda_1, \lambda_2))$ and $\gamma \in \Gamma$ then

$$\Omega(x) > (\lambda_1, \lambda_2) \text{ and } \Omega(y) > (\lambda_1, \lambda_2) \text{ implies } \Omega(x\gamma y) > (\lambda_1, \lambda_2)$$

Thus $x\gamma y \in (\Omega, (\lambda_1, \lambda_2))$. Hence $(\Omega, (\lambda_1, \lambda_2))$ is a sub-Γ-semigroup of S.
Conversely, suppose that $(\Omega, (\lambda_1, \lambda_2))$ is a sub-Γ-semigroup of S for each $[\lambda_1, \lambda_2] \in D[0, 1]$. For any $x, y \in S$, and $\gamma \in \Gamma$. Let

$$\Omega(x) = [\lambda_1, \lambda_2] \text{ and } \Omega(y) = [\lambda_3, \lambda_4].$$

Then $\Omega(x) = (\lambda_1, \lambda_2) > (\lambda_1 \wedge \lambda_3, \lambda_2 \wedge \lambda_4)$ and $\Omega(y) = (\lambda_3, \lambda_4) > (\lambda_1 \wedge \lambda_3, \lambda_2 \wedge \lambda_4)$. Thus $x, y \in (\Omega, (\lambda_1 \wedge \lambda_3, \lambda_2 \wedge \lambda_4))$, since $(\lambda_1 \wedge \lambda_3, \lambda_2 \wedge \lambda_4) \in D[0, 1]$. By hypothesis $x\gamma y \in (\Omega, (\lambda_1 \wedge \lambda_3, \lambda_2 \wedge \lambda_4))$. Hence Ω is an IF sub-Γ-semigroup of S. □

Theorem 3.10. *Let Ω be an IF subset of S then Ω is an IF sub-Γ-semigroup of S if and only if $(\Omega, [\lambda_1, \lambda_2])$ is a sub-Γ-semigroup of S for all $[\lambda_1, \lambda_2] \in D[0, 1]$.*

Theorem 3.11. *If Ω is an $IFBgI$ of S, then $\partial^u(\Omega)$ is an $IFBgI$ of S.*

Proof. Assume that Ω is an $IFBgI$ of S. Then $\Omega(x\beta s\gamma y) \geq \Omega(x) \wedge \Omega(y)$ for all $x, y, s \in S$ and $\beta, \gamma \in \Gamma$.

$$\begin{aligned} \partial^u(\Omega)(x\beta s\gamma y) &= \bigvee_{r\in[x\beta s\gamma y]_\partial} \Omega(r) \\ &= \bigvee_{r\in[x]_\partial \beta [s]_\partial \gamma [y]_\partial} \Omega(r) \end{aligned}$$

$$= \bigvee_{pqt \in [x]_\partial \beta [s]_\partial \gamma [y]_\partial} \Omega(p\beta q\gamma t)$$
$$\geq \bigvee_{p \in [x]_\partial t \in [y]_\partial} [(\Omega(p) \wedge \Omega(t))]$$
$$= \partial^u(\Omega)(x) \wedge \partial^u(\Omega)(y) \tag{1}$$

From equation (1) and by Theorem 3.1 we obtain $\partial^u(\Omega)$ is an $IFBgI$ of S. □

Theorem 3.12. *If Ω is an $IFBgI$ of S, then $\partial^l(\Omega)$ is an $IFBgI$ of S.*

Theorem 3.13. *Let Ω be an IF subset of S. If Ω is an $IFBgI$ of S, then Ω is an $IRFBgI$ of S.*

Proof. Follows from Theorem 3.11 and 3.12 □

Theorem 3.14. *An IF subset of S is an $IFBgI$ of S if and only if $(\Omega, (\lambda_1, \lambda_2))$ is a BgI of S for all $[\lambda_1, \lambda_2] \in D[0, 1]$.*

Proof. Suppose that Ω is an $IFBgI$ of S. Let $[\lambda_1, \lambda_2] \in D[0, 1]$. Then Ω is an IF sub-Γ-semigroup of S. By Theorem 3.9 $(\Omega, (\lambda_1, \lambda_2))$ is a sub-Γ-semigroup of S. Let $t \in (\Omega, (\lambda_1, \lambda_2))\Gamma S\Gamma(\Omega, (\lambda_1, \lambda_2))$,$y \in S$ and $\alpha, \beta \in \Gamma$ such that $t = x\alpha y\beta z$. Since Ω is an $IFBgI$, $\Omega(x\alpha y\beta z) \geq \Omega(x) \wedge \Omega(z) > (\lambda_1, \lambda_2)$ implies $t \in (\Omega, (\lambda_1, \lambda_2))$.
Thus $(\Omega, (\lambda_1, \lambda_2))\Gamma S\Gamma(\Omega, (\lambda_1, \lambda_2)) \subseteq (\Omega, (\lambda_1, \lambda_2))$. Hence $(\Omega, (\lambda_1, \lambda_2))$ is a BgI of S.
Conversely, suppose that $(\Omega, (\lambda_1, \lambda_2))$ is a BgI of S, then $(\Omega, (\lambda_1, \lambda_2))$ is sub-Γ-semigroup of S. By Theorem 3.9 Ω is an IF sub-Γ-semigroup of S. For any $x, z \in S$ and $\alpha, \beta \in \Gamma$. Let $\Omega(x) = [\lambda_3, \lambda_4]$ and $\Omega(z) = [\lambda_5, \lambda_6]$ then $x, z \in (\Omega, (\lambda_3 \wedge \lambda_5, \lambda_4 \wedge \lambda_6))$.
Let $y \in S$ then $x\alpha y\beta z \in (\Omega, (\lambda_3 \wedge \lambda_5, \lambda_4 \wedge \lambda_6))$ implies that
$\Omega(x\alpha y\beta z) > (\lambda_3 \wedge \lambda_5, \lambda_4 \wedge \lambda_6) = \Omega(x) \wedge \Omega(z)$. Hence Ω is an $IFBgI$ of S. □

4. INTERVAL ROUGH FUZZY PRIME-Γ-IDEAL($IRFPgI$)

In this section we introduce the concept of $IRFPgI$ of S.

Theorem 4.1. *Let Ω be an IF subset of S. Then Ω is an $IFPgI$ of S if and only if for all $[\lambda_1, \lambda_2] \in D[0, 1]$, $(\Omega, [\lambda_1, \lambda_2])$ is a PgI of S.*

Proof. Assume that Ω is an $IFPgI$ of S. Then Ω is an IF an gI of S. Assume $(\Omega, [\lambda_1, \lambda_2]) \neq \phi$. By Theorem 3.4 $(\Omega, [\lambda_1, \lambda_2])$ is a gI of S. Let $x, y \in S$ and $\gamma \in \Gamma$ then $x\gamma y \in \Omega_{[\lambda_1, \lambda_2]}$. Since Ω is an $IFPgI$ of S. $\Omega(x\gamma y) = \Omega(x)$ or $\Omega(x\gamma y) = \Omega(y)$ and $x \in (\Omega, [\lambda_1, \lambda_2])$ or $y \in (\Omega, [\lambda_1, \lambda_2])$ Therefore $(\Omega, [\lambda_1, \lambda_2])$ is a PgI of S.

Conversely, assume that for all $[\lambda_1, \lambda_2] \in D[0, 1]$, if $(\Omega, [\lambda_1, \lambda_2]) \neq \phi$, then $(\Omega, [\lambda_1, \lambda_2])$ is a PgI of S. Let $x, y \in S$ and $\gamma \in \Gamma$. By Theorem 3.4 Ω is an $IFgI$ of S. This implies $\Omega(x\gamma y) \geq \Omega(y)$ and $\Omega(x\gamma y) \geq \Omega(x)$. Let $\lambda_1 = \Omega(x\gamma y)$. Thus $x\gamma y \in (\Omega, [\lambda_1, \lambda_2])$. Since $(\Omega, [\lambda_1, \lambda_2])$ is a PgI of S, $x \in (\Omega, [\lambda_1, \lambda_2])$ or $y \in (\Omega, [\lambda_1, \lambda_2])$. This implies that $\Omega(x) \geq [\lambda_1, \lambda_2] = \Omega(x\gamma y)$ or $\Omega(y) \geq [\lambda_1, \lambda_2] = \Omega(x\gamma y)$. Hence $\Omega(x\gamma y) = \Omega(x)$ or $\Omega(x\gamma y) = \Omega(y)$. Hence Ω is an $IFPgI$ of S. □

Theorem 4.2. *Let Ω be an IF subset of S. Then Ω is an $IFPgI$ of S if and only if for all $[\lambda_1, \lambda_2] \in D[0, 1]$, $(\Omega, (\lambda_1, \lambda_2))$ is a PgI of S.*

Theorem 4.3. *Let Ω be an $IFPgI$ of S. Then Ω is an $IRFPgI$ of S.*

Proof. By applying Theorems 3.4, 4.2, 2.1 and 2.2 we get the result. □

5. INTERVAL ROUGH FUZZY PRIME-BI-Γ-IDEAL ($IRFPBgI$)

In this section we introduce the notion of $IRFPBgI$ of S also discuss some property of this ideal.

Theorem 5.1. *Let Ω be an IF subset of S. Then Ω is an $IFPBgI$ of S if and only if $(\Omega, [\lambda_1, \lambda_2])(resp., (\Omega, (\lambda_1, \lambda_2)) \neq \phi)$ is a $PBgI$ of S for every $[\lambda_1, \lambda_2] \in D[0, 1]$.*

Proof. Suppose that Ω is an $IFPBgI$ of S. Then Ω is an $IFBgI$ of S. Assume that $(\Omega, [\lambda_1, \lambda_2]) \neq \phi$. By Theorem 3.14 $(\Omega, [\lambda_1, \lambda_2])$ is a BgI of S. Let $x, y, a \in S$ and $\gamma, \beta \in \Gamma$ such that $x\gamma a\beta y \in (\Omega, [\lambda_1, \lambda_2])$. Since Ω ia an $IFPBgI$ of S, we have $\Omega(x\gamma a\beta y) = \Omega(x)$ or $\Omega(x\gamma a\beta y) = \Omega(x)$. Thus $x \in (\Omega, [\lambda_1, \lambda_2])$ or $y \in (\Omega, [\lambda_1, \lambda_2])$. Hence $(\Omega, [\lambda_1, \lambda_2])$ is a $PBgI$ of S.

Conversely, suppose that for all $[\lambda_1, \lambda_2] \in D[0, 1]$, if $(\Omega, [\lambda_1, \lambda_2]) \neq \phi$, and $(\Omega, [\lambda_1, \lambda_2])$ is a $PBgI$ of S. Let $x, a, y \in S$ and $\gamma, \beta \in \Gamma$. By Theorem 3.14 Ω is an $IFPBgI$ of S. Then, we have $\Omega(x\gamma a\beta y) = \Omega(x) \wedge \Omega(y)$.
Let $[\lambda_1, \lambda_2]$ = $\Omega(x\gamma a\beta y)$ and $x\gamma a\beta y \in (\Omega, [\lambda_1, \lambda_2])$. Since $(\Omega, [\lambda_1, \lambda_2])$ is a $PBgI$ of S, we have $x \in (\Omega, [\lambda_1, \lambda_2])$ or $y \in (\Omega, [\lambda_1, \lambda_2])$ which implies

$\Omega(x) \geq [\lambda_1, \lambda_2] = \Omega(x\gamma a\beta y)$ or $\Omega(x) \geq [\lambda_1, \lambda_2] = \Omega(x\gamma a\beta y)$.

Hence Ω is an $IFPBgI$ of S. □

A nonempty set IF set Ω is called an $IRFPBgI$ of S if it is both lower and upper $IRFPBgI$ of S.

Theorem 5.2. *Let Ω be an $IFPBgI$ of S. Then Ω is an $IRFPBgI$ of S.*

Theorem 5.3. *Let Ω be an $IFPBgI$ of S. Then Ω is a ∂-lower $IRFPBgI$ of S if and only if for all $[\lambda_1, \lambda_2] \in D[0, 1]$ $\underline{\partial}(\Omega, [\lambda_1, \lambda_2]) \neq \phi$, and $(\Omega, [\lambda_1, \lambda_2])$ is a ∂-lower rough $PBgI$ of S.*

Proof. Suppose that Ω is a ∂-lower $IRFPBgI$ of S. $\underline{\partial}(\Omega)$ is a $IFPBgI$ of S. By Theorem 5.1 $(\underline{\partial}(\Omega), [\lambda_1, \lambda_2])$ is a $PBgI$ of S. By Theorem 2.1 $\underline{\partial}(\Omega, [\lambda_1, \lambda_2])$=$(\underline{\partial}(\Omega), [\lambda_1, \lambda_2])$ and this implies $\underline{\partial}(\Omega, [\lambda_1, \lambda_2])$ is a $PBgI$ of S. Hence $(\Omega, [\lambda_1, \lambda_2])$ is a ∂-lower rough $PBgI$ of S. Similarly, we can obtain the converse part. □

Theorem 5.4. *Let Ω be an $IFPBgI$ of S. Then Ω is a ∂-upper $IRFPBgI$ of S if and only if for all*
$[\lambda_1, \lambda_2] \in D[0, 1]$ $(\Omega, (\lambda_1, \lambda_2)) \neq \phi$, and $(\Omega, (\lambda_1, \lambda_2))$ is a ∂-upper rough $PBgI$ of S.

Proof. The proof is similar to Theorem 5.3 and follows from Theorem 2.1 we can obtain the proof easily. □

6. CONCLUSION

Fuzzy set theory and rough set theory take into account two distinct aspects of uncertainty that can be experienced in real world problems in many fields. The fusion of fuzzy set and rough set lead to various models. This paper is deliberated to built up a relation between rough sets, fuzzy sets and interval-valued fuzzy sets. In the present paper, we use Γ-semigroup instead of universe set, and introduced the notion of interval-valued rough fuzzy ideals, interval-valued fuzzy bi-ideals and interval-valued fuzzy prime ideals. Also, we believe, this paper will turn out to be more useful in the theory of rough sets and fuzzy sets.

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