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INTERVAL-VALUED FUZZY ALMOST BI-IDEALS OF NEAR-RINGS

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ABSTRACT. In this paper, we introduce the concept of interval valued fuzzy almost bi-ideals of near-rings. Which is a generalized concept of fuzzy almost bi-ideals of near-rings. We also characterize some properties and provide examples of interval-valued fuzzy almost bi-ideals of near-rings.

1. INTRODUCTION

In 1965, the definition of fuzzy subsets was introduced by Zadeh [28]. In 1971, Rosenfeld [18] extended the concept of fuzzy set theory to group theory and defined fuzzy group. Kuroki [12] studied various kinds of fuzzy ideals in semigroups and characterized them. Interesting research on fuzzy semigroups can be found in [11]. In 2011, Chon [3] also characterized the fuzzy bi-ideal generated by fuzzy subsets in semigroups. In 1980, Grosek and Satko [6] defined and studied the notion of left (respectively, right, two-sided) almost ideals of semigroups. Moreover, they characterized when a semigroup S contains no proper left (respectively, right, two-sided) almost ideals. In 1981, Bogdanovic [2] introduced the notion of almost bi-ideals in semigroups. Fuzzy ideals of rings were exhibited by W. Liu [15] and it has been studied by several authors [7, 12, 17]. The notions of fuzzy subnear-ring, fuzzy ideals of near-rings were ushered by Salah Abou-Zaid [1]. The concept of bi-ideals was applied to near-rings in [19]. Manikantan [16] introduced the notion of fuzzy bi-ideals of near-rings and discussed some of its properties. The notion of fuzzy almost bi-ideals of near-rings[27]. In this paper, we first give some properties of interval valued fuzzy almost bi-ideals of near-rings and introduce the notion of interval valued fuzzy almost bi-ideals by using the concepts of fuzzy almost bi-ideals and fuzzy ideals of near-rings. Moreover, we explore some characterization interval valued fuzzy almost bi-ideals of near-rings.

2. PRELIMINARIES

Definition 2.1. *A non-empty set N with two binary operations '$+'$ and '$.'$ is called a near-ring if*
(1) $(N, +)$ is a group,

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(2) $(N, .)$ *is a semigroup,*
(3) $x.(y + z) = x.y + x.z$*, for all* $x, y, z \in N$*.*
We use word 'near-ring' to mean 'left near-ring'. We denote xy *instead of* $x.y$*. Note that* $x0 = 0$ *and* $x(-y) = -xy$ *but in general* $0x \neq 0$ *for some* $x \in N$*.*

Definition 2.2. *An ideal* I *of a near-ring* N *is a subset of* N *such that*
(4) $(I, +)$ *is a normal subgroup of* $(N, +)$*,*
(5) $NI \subseteq I$*,*
(6) $((x + i)y - xy) \in I$ *for any* $i \in I$ *and* $x, y \in N$
Note that I *is a left ideal of* N *if* I *satisfies (4) and (5), and* I *is a right ideal of* N *if* I *satisfies (4) and (6).*

Definition 2.3. *A two sided* N*-subgroup of a near-ring* N *is a subset* H *of* N *such that*
(i) $(H, +)$ *is a subgroup of* $(N, +)$*,*
(ii) $NH \subset H$*,*
(iii) $HN \subset H$*.*
If H *satisfies (i) and (ii) then it is called a left* N*-subgroup of* N*. If* H *satisfies (i) and (iii) then it is called a right* N*-subgroup of* N*.*

Definition 2.4. *Let* N *be a near-ring. Given two subsets* A *and* B *of* N*, the product* $AB = \{ab | a \in A, b \in B\}$ *Also we define another operation* $'*'$ *on the class of subsets of* N *given by* $A * B = \{a(a' + b) - aa' | a, a' \in A, b \in B\}$*.*

Definition 2.5. *A subgroup* B *of* $(N, +)$ *is said to be a bi-ideal of* N *if* $BNB \cap BN * B \subseteq B$*.*

Definition 2.6. *Let* X *be any non-empty set. A mapping* $\lambda : X \to [0, 1]$ *is called a fuzzy subset of* X*.*

Definition 2.7. *Let* μ *and* λ *be any two fuzzy subsets of* N*. Then* $\mu \cap \lambda$, $\mu \cup \lambda$, $\mu + \lambda$, $\mu\lambda$, $\mu \subseteq \lambda$ *and* $\mu * \lambda$ *are fuzzy subsets of* N *defined by:*
$(\mu \cap \lambda)(x) = min\{\mu(x), \lambda(x)\}$.
$(\mu \cup \lambda)(x) = max\{\mu(x), \lambda(x)\}$.
$\mu \subseteq \lambda$ if $\mu(x) \leq \lambda(x)$

$$(\mu + \lambda)(x) = \begin{cases} \sup\limits_{x=y+z} \{\min\{\mu(y),\ \lambda(z)\}\} & \text{if x is expressible as x = y+z,} \\ 0 & \text{otherwise.} \end{cases}$$

$$(\mu\lambda)(x) = \begin{cases} \sup\limits_{x=yz} \{\min\{\mu(y),\ \lambda(z)\}\} & \text{if x is expressible as x = yz,} \\ 0 & \text{otherwise.} \end{cases}$$

for all $x \in N$. Let λ be a fuzzy subset of N. The support of λ is define by support $\lambda := \{x \in N | \lambda(x) \neq 0\}$.

Definition 2.8. *Let* A *be a non-empty subset of* N*. The characteristic mapping of* A *is a fuzzy subset of* N *defined by*

$$C_A(x) = \begin{cases} 1 & \textit{if } x \in A, \\ 0 & \textit{if } x \notin A. \end{cases}$$

Definition 2.9. *Let s be any element in N. The characteristic mapping of s is a fuzzy subset of N defined by*

$$C_s(x) = \begin{cases} 1 & \text{if } x = s, \\ 0 & \text{if } x \notin s. \end{cases}$$

Definition 2.10. *A fuzzy subset λ of a near-ring N is called a fuzzy N-subgroup of N if*
(1) λ is a fuzzy subgroup of $(N, +)$
(2) $\lambda(xy) \geq \lambda(y)$
(3) $\lambda(xy) \geq \lambda(x)$, for all $x, y \in N$.
A fuzzy subset with (1) and (2) is called a fuzzy left N-subgroup of N, whereas a fuzzy subset with (1) and (3) is called a fuzzy right R-subgroup of N.

$$(\lambda{*}\mu)(x) = \begin{cases} \sup\limits_{x=a(b+c)-ab} \{\min\{\lambda(a),\ \mu(c)\}\} & \text{if x is expressible as x =a(b+c)-ab,} \\ \overline{0} & \text{otherwise.} \end{cases}$$

Definition 2.11. *A fuzzy subset λ of a near-ring N is called a fuzzy subnear-ring of N if $\forall\ x, y \in N$,*
(i) $\lambda(x - y) \geq min\{\lambda(x), \lambda(y)\}$
(ii) $\lambda(xy) \geq min\{\lambda(x), \lambda(y)\}$
If a fuzzy subset λ of a near-ring N satisfies the property (i), then letting $x = y$, we have $\lambda(0) \geq \lambda(x)\ \forall\ x \in N$.

Definition 2.12. *Let λ be a non-empty fuzzy subset of N. λ is a fuzzy ideal of N, if for all $x, y, i \in N$ and*
(1) $\lambda(x - y) \geq min\{\lambda(x), \lambda(y)\}$
(2) $\lambda(x) = \lambda(y + x - y)$
(3) $\lambda(xy) \geq \lambda(x)$
(4) $(\lambda(x(y + i) - xy) \geq \lambda(i))$ for any $x, y, i \in N$.
If λ satisfies (1), (2) and (3), then it is called a fuzzy right ideal of N. If λ satisfies (1), (2) and (4), then it is called a fuzzy left ideal of N, If λ is both fuzzy right as well as fuzzy left ideal of N, then λ is called a fuzzy ideal of N.

Definition 2.13. *A fuzzy subgroup λ of N is called a fuzzy bi-ideal of N if for all $x \in N$,*
$((\lambda \circ N \circ \lambda) \cap (\lambda \circ N) * \lambda))(x) \leq \lambda(x) \to (1)$
If N is zero symmetric, then (1) reduces $(\lambda \circ N \circ \lambda)(x) \subseteq \lambda(x)$.

Definition 2.14. [10] An interval number $\overline{a}$ on [0,1] is a closed subinterval of [0,1], that is, $\overline{a} = [a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ where a^- and a^+ are the lower and upper end limits of $\overline{a}$ respectively. The set of all closed subintervals of $[0, 1]$ is denoted by $D[0, 1]$. We also identify the interval $[a, a]$ by the number $a \in [0, 1]$. For any interval numbers $\overline{a}_i = [a_i^-, a_i^+], \overline{b}_i = [b_i^-, b_i^+] \in D[0, 1], i \in I$, we define
$max^i\{\overline{a}_i, \overline{b}_i\} = [max^i\{a_i^-, b_i^-\}, max^i\{a_i^+, b_i^+\}]$,
$min^i\{\overline{a}_i, \overline{b}_i\} = [min^i\{a_i^-, b_i^-\}, min^i\{a_i^+, b_i^+\}]$,
$inf^i\overline{a}_i = \left[\bigcap\limits_{i\in I} a_i^-, \bigcap\limits_{i\in I} a_i^+\right], sup^i\overline{a}_i = \left[\bigcup\limits_{i\in I} a_i^-, \bigcup\limits_{i\in I} a_i^+\right]$

In this notation $\overline{0} = [0, 0]$ and $\overline{1} = [1, 1]$. For any interval numbers $\overline{a} = [a^-, a^+]$ and $\overline{b} = [b^-, b^+]$ on $[0, 1]$, define
(1) $\overline{a} \leq \overline{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$.
(2) $\overline{a} = \overline{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$.

(3) $\overline{a} < \overline{b}$ if and only if $\overline{a} \leq \overline{b}$ and $\overline{a} \neq \overline{b}$
(4) $k\overline{a} = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$.

Definition 2.15. [10] Let X be any set. A mapping $\overline{A} : X \to D[0,1]$ is called an interval-valued fuzzy subset (briefly, i-v fuzzy subset) of X where $D[0,1]$ denotes the family of all closed subintervals of $[0,1]$ and $\overline{A}(x) = [A^-(x),\ A^+(x)]$ for all $x \in X$, where A^- and A^+ are fuzzy subsets of X such that $A^-(x) \leq A^+(x)$ for all $x \in X$.

Note that $\overline{A}(x)$ is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset.

Definition 2.16. [10] A mapping $min^i : D[0,1] \times D[0,1] \to D[0,1]$ defined by $min^i(\overline{a}, \overline{b}) = [min\{a^-, b^-\},\ min\{a^+, b^+\}]$ for all $\overline{a}, \overline{b} \in D[0,1]$ is called an interval min-norm.

Definition 2.17. [10] A mapping $max^i : D[0,1] \times D[0,1] \to D[0,1]$ defined by $max^i(\overline{a}, \overline{b}) = [max\{a^-, b^-\},\ max\{a^+, b^+\}]$ for all $\overline{a}, \overline{b} \in D[0,1]$ is called an interval max-norm.

Let min^i and max^i be the interval min-norm and max-norm on $D[0,1]$ respectively. Then the following are true.
1. $min^i\{\overline{a}, \overline{a}\} = \overline{a}$ and $max^i\{\overline{a}, \overline{a}\} = \overline{a}$ for all $\overline{a} \in D[0,1]$.
2. $min^i\{\overline{a}, \overline{b}\} = min^i\{\overline{b}, \overline{a}\}$ and $max^i\{\overline{a}, \overline{b}\} = max^i\{\overline{b}, \overline{a}\}$ for all $\overline{a}, \overline{b} \in D[0,1]$.
3. If $\overline{a} \geq \overline{b} \in D[0,1]$, then $min^i\{\overline{a}, \overline{c}\} \geq min^i\{\overline{b}, \overline{c}\}$ and $max^i\{\overline{a}, \overline{c}\} \geq max^i\{\overline{b}, \overline{c}\}$ for all $\overline{c} \in D[0,1]$.

Definition 2.18. [10] Let $\overline{A}$ be an i-v fuzzy subset of a set X and $[t_1, t_2] \in D[0,1]$. Then the set $\overline{U}(\overline{A} : [t_1,\ t_2]) = \{x \in X | \overline{A}(x) \geq [t_1,\ t_2]\}$ is called the upper level set of $\overline{A}$.
Note that
$\overline{U}(\overline{A} : [t_1, t_2]) = \{x \in X | [A^-(x), A^+(x)] \geq [t_1, t_2]\}$
$= \{x \in X | A^-(x) \geq t_1\} \cap \{x \in X | A^+(x) \geq t_2\}$
$= (U(A^- : t_1)) \cap (U(A^+ : t_2))$.

Definition 2.19. [1] If $\overline{\lambda}$ and $\overline{\mu}$ are i-v fuzzy subsets of a near-ring N. Then $\overline{\lambda} \cap \overline{\mu}, \overline{\lambda} \cup \overline{\mu}, \overline{\lambda} + \overline{\mu}, \overline{\lambda}\overline{\mu}$ and $\overline{\lambda} * \overline{\mu}$ are i-v fuzzy subsets of N defined by,
$(\overline{\lambda} \cap \overline{\mu})(x) = \text{min}^i\{\overline{\lambda}(x), \overline{\mu}(x)\}$
$(\overline{\lambda} \cup \overline{\mu})(x) = \text{max}^i\{\overline{\lambda}(x), \overline{\mu}(x)\}$

$$(\overline{\lambda} + \overline{\mu})(x) = \begin{cases} sup^i_{x=y+z}\{\text{min}^i\{\overline{\lambda}(y),\ \overline{\mu}(z)\}\} & \text{if x is expressible as x = y+z,} \\ \overline{0} & \text{otherwise.} \end{cases}$$

$$(\overline{\lambda}\,\overline{\mu})(x) = \begin{cases} \text{sup}^i_{x=yz}\{\text{min}^i\{\overline{\lambda}(y),\ \overline{\mu}(z)\}\} & \text{if x is expressible as x = yz,} \\ \overline{0} & \text{otherwise.} \end{cases}$$

$$(\overline{\lambda}{*}\overline{\mu})(x) = \begin{cases} sup^i_{x=a(b+c)-ab}\{\text{min}^i\{\overline{\lambda}(a),\ \overline{\mu}(c)\}\} & \text{if x is expressible as x =a(b+c)-ab,} \\ \overline{0} & \text{otherwise.} \end{cases}$$

3. INTERVAL-VALUED FUZZY ALMOST BI-IDEALS (IVFABIS)

In this section, we define IVFABIs in NR and give some relationship between ABIs and IVFABIs of NR.

Definition 3.1. Let $\overline{\mu}$ be an Interval-valued fuzzy subset of N such that $\overline{\mu} \neq \overline{0}$. $\overline{\mu}$ is called an IVFABI of N if $\forall\ n \in N, (\overline{\mu}C_n\overline{\mu}) \cap (\overline{\mu}C_n * \overline{\mu}) \cap \overline{\mu} \neq 0$.

Theorem 3.1. *Let $\overline{\mu}$ be a IVFABI of N and $\overline{\nu}$ be an interval-valued fuzzy subset of $N \ni \overline{\mu} \subseteq \overline{\nu}$. Then $\overline{\nu}$ is a IVFABI of N.*

Proof. Let $\overline{\mu}$ is an IVFABI of N and $\overline{\nu}$ is an interval-valued fuzzy subset of N such that $\overline{\mu} \subseteq \overline{\nu}$. Then $\forall\ n \in N, (\overline{\mu}C_n\overline{\mu}) \cap (\overline{\mu}C_n * \overline{\mu}) \cap \overline{\mu} \subseteq (\overline{\mu}C_n\overline{\nu}) \cap (\overline{\nu}C_n\overline{\nu} * \overline{\mu}) \cap \overline{\nu}$ and $(\overline{\mu}C_n\overline{\mu} \cap \overline{\mu}C_n * \overline{\mu}) \cap \overline{\mu} \neq 0$. This implies $(\overline{\nu}C_n\overline{\nu} \cap \overline{\nu}C_n * \overline{\nu}) \cap \overline{\nu} \neq 0$. $\forall\ n \in N$. $\therefore \overline{\nu}$ is an IVFABI. □

Corollary 3.2. *Let $\overline{\mu}$ and $\overline{\nu}$ is an IVFABI of N. Then $\overline{\mu} \cup \overline{\nu}$ is an IVFABI of N.*

Proof. Since $\overline{\mu} \subseteq \overline{\mu} \cup \overline{\nu}$ by Theorem 3.1, $\overline{\mu} \cup \overline{\nu}$ is an IVFABI of N. □

Example 3.2. Consider Z_5 under the usual addition. Let $\overline{\mu} : Z_5 \to D[0,1]$ be defined by $\overline{\mu}(\overline{0}) = 0, \overline{\mu}(\overline{1}) = 0.5, \overline{\mu}(\overline{2}) = 0, \overline{\mu}(\overline{3}) = 0.1, \overline{\mu}(\overline{4}) = 0.1$ and $\overline{\nu} \to D[0,1]$ be defined by $\overline{\nu}(\overline{0}) = 0, \overline{\nu}(\overline{1}) = 0.2, \overline{\nu}(\overline{3}) = 0$ and $\overline{\nu}(\overline{4}) = 0.2$. We have $\overline{\mu}$ and $\overline{\nu}$ are IVFABIs of Z_5 but $\overline{\mu} \cap \overline{\nu}$ is not an IVFABI of Z_5.

Theorem 3.3. *Let $B(\neq \emptyset) \subseteq N$ of N. Then B is an ABI of N iff $\overline{C}_B$ is an IVFABI of N.*

Proof. Let B is an ABI of N. Then $BnB \cap Bn * B \cap B \neq 0\ \forall\ n \in N$. Thus there $\exists\ x \in BnB, x \in Bn * B$ and $x \in B$. So $(\overline{C}_B\overline{C}_n\overline{C}_B)(x) = \overline{1}, (\overline{C}_B\overline{C}_n * \overline{C}_B)(x) = \overline{1}$ and $\overline{C}_B(x) = 1$. Hence $(\overline{C}_B\overline{C}_n\overline{C}_B) \cap (\overline{C}_B\overline{C}_n * \overline{C}_B) \cap \overline{C}_B \neq \overline{0}$ for all $n \in N$. $\therefore \overline{C}_B$ is an IVFABI of N.

Conversely, assume that $\overline{C}_B$ is an IVFABI of N. Let $n \in N$. Then $((\overline{C}_B\overline{C}_n\overline{C}_B) \cap (\overline{C}_B\overline{C}_n * \overline{C}_B)) \cap \overline{C}_B \neq \overline{0}$. Then $\exists\ x \in N \ni [((\overline{C}_B\overline{C}_n\overline{C}_B) \cap (\overline{C}_B\overline{C}_n * \overline{C}_B)) \cap \overline{C}_B](x) \neq \overline{0}$. Hence $x \in BnB \cap (Bn * B) \cap B$. So, $BnB \cap (Bn * B) \cap B \neq 0\ \forall\ n \in N$. Consequently, B is an ABI of N. □

Theorem 3.4. *Let $\overline{\mu}$ be an interval valued fuzzy subset of N. Then $\overline{\mu}$ is an IVFABI of N iff supp μ is an ABI of N.*

Proof. Let $\overline{\mu}$ be an IVFABI of N. Let $n \in N$. Then $\overline{\mu}(C_n\overline{\mu}) \cap (\overline{\mu}C_n * \overline{\mu}) \cap \overline{\mu} \neq \overline{0}$. Hence for each $n \in N$, there $\exists\ x \in N \ni [(\overline{\mu}C_n\overline{\mu}) \cap (\overline{\mu}C_n * \overline{\mu}) \cap \overline{\mu}](x) \neq 0$. So there $\exists\ y_1, y_2, y_3 \in N$ such that $x = y_1ny_2$ and $x = y_1n(y_2n + y_3) - (y_1n)(y_2n), \overline{\mu}(x) \neq \overline{0}, \overline{\mu}(y_1) \neq \overline{0}, \overline{\mu}(y_2) \neq 0$ and $\overline{\mu}(y_3) \neq \overline{0}$ That is $x, y_1, y_2, y_3 \in supp\overline{\mu}$. Thus $(C_{supp\overline{\mu}}C_nC_{supp}\overline{\mu})(x) \neq \overline{0}$. Therefore $((C)_{supp\overline{\mu}}C_nC_{supp\overline{\mu}}) \cap (C)_{supp\overline{\mu}}C_n * C_{supp\overline{\mu}} \neq \overline{0}$. Hence $C_{supp\overline{\mu}}$ be a IVFABI of N. By Theorem 3.3, $supp\mu$ be an ABI of N.
Conversely, assume that $supp\mu$ be an ABI of N. By Theorem 3.3, $C_{supp\overline{\mu}}$ be a IVFABI of N. Then $(C_{suppf}C_nC_{supp\overline{\mu}}) \cap (C_{supp\overline{\mu}}C_n * C_{supp\overline{\mu}}) \cap C_{supp\overline{\mu}} \neq \overline{0}$ for all $n \in N$. Then there $\exists\ x \in N \ni [(C_{supp\overline{\mu}}C_nC_{supp\overline{\mu}}) \cap (C_{supp\overline{\mu}}C_n * C_{supp\overline{\mu}}) \cap C_{supp\overline{\mu}}](x) \neq \overline{0}$. Hence $\left((C_{supp\overline{\mu}}C_nC_{supp\overline{\mu}}) \cap (C_{supp\overline{\mu}}C_n) * C_{supp\overline{\mu}} \cap C_{supp\overline{\mu}}\right)(x) \neq \overline{0}$ and $C_{supp\overline{\mu}}(x) \neq \overline{0}$. Then there $\exists\ y_1, y_2 \in N$ such that $x = y_1ny_2, \overline{\mu}(x) \neq \overline{0}, \overline{\mu}(y_1) \neq \overline{0}$ and $\overline{\mu}(y_2) \neq \overline{0}$. This means $(\overline{\mu}C_n\overline{\mu}) \cap (\overline{\mu}C_n * \overline{\mu}) \cap \overline{\mu} \neq \overline{0}$. $\therefore \overline{\mu}$ is an IVFABI of N. □

4. MINIMAL INTERVAL VALUED FUZZY ALMOST BI-IDEALS (MIVFABIS)

In this section, we define MAIVFBIs in NR and study relationship between support and MAFBIs of NRs.

Definition 4.1. A IVFABI $\overline{\mu}$ is called minimal if for each IVFABI $\overline{\nu}$ of N such that $\overline{\mu} \subseteq \overline{\mu}$, we have $supp\overline{\nu} = supp\overline{\mu}$.

Theorem 4.1. *Let $B(\neq \emptyset) \subseteq N$. Then B is a MABI of N iff $\overline{C}_B$ is a MIVFABI of N.*

Proof. Let B is a MABI of N. By Theorem 3.3, $\overline{C}_B$ is a IVFABI of N. Let $\overline{g}$ be a IVFABI of $N \ni \overline{g} \subseteq \overline{C}_B$. Then $supp\overline{g} \subseteq supp\overline{C}_B = B$. Since $\overline{g} \subseteq \overline{C}_{supp\overline{g}}$, we have $(\overline{g}\overline{C}_B\overline{g}) \cap (\overline{g}C_B * \overline{g}) \cap \overline{g} \subseteq (\overline{C}_{supp\overline{g}}C_B C_{supp\overline{g}}) \cap (\overline{C}_{supp\overline{g}}\overline{C}_B * \overline{C}_{supp\overline{g}}) \cap \overline{C}_{supp\overline{g}}$. Thus $\overline{C}_{supp\overline{g}}$ is an IVFABI of N. By Theorem 3.3, $suppg$ is an ABI of N. Since B is minimal, $suppg = B = suppC_B$. $\therefore C_B$ is minimal.
Conversely, assume that $\overline{C}_B$ is an MIVFABI of N. Let B' be an ABI of $N \ni B' \subseteq B$. Then $\overline{C}_{B'}$ is an IVFABI of N such that $C_{B'} \subseteq C_B$. Hence $B' = suppC_{B'} = suppC_B = B$. $\therefore$ B is minimal. □

Corollary 4.2. *N has no proper ABI of N iff $\forall$ IVFABI $\overline{\mu}$ of N, $supp\overline{\mu} = N$.*

5. Conclusions

In this study the notion of interval valued fuzzy almost bi-ideals and minimal interval valued fuzzy almost bi-ideals of near-rings have been presented and some properties of these ideals are derived.

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