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SOME OPERATIONS OF FUZZY SETS IN UP-ALGEBRAS WITH RESPECT TO A TRIANGULAR NORM

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ABSTRACT. This paper aim is to apply the notions of the intersection and the union of any fuzzy set to UP-algebras. We investigate properties of the intersection and the union of T-fuzzy UP-subalgebras, T-fuzzy near UP-filters, T-fuzzy UP-filters, T-fuzzy UP-ideals, T-fuzzy strongly UP-ideals, anti-T-fuzzy UP-subalgebras, and anti-T-fuzzy near UP-filters of UP-algebras.

1. INTRODUCTION AND PRELIMINARIES

The branch of the logical algebra, a UP-algebra was introduced by Iampan [4] in 2017, and it is known that the class of KU-algebras [10] is a proper subclass of the class of UP-algebras. It have been examined by several researchers, for example, Somjanta et al. [16] introduced the notion of fuzzy sets in UP-algebras, the notion of intuitionistic fuzzy sets in UP-algebras was introduced by Kesorn et al. [7], Kaijae et al. [6] introduced the notions of anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras of UP-algebras, the notion of Q-fuzzy sets in UP-algebras was introduced by Tanamoon et al. [19], Sripaeng et al. [18] introduced the notion anti-Q-fuzzy UP-ideals and anti Q-fuzzy UP-subalgebras of UP-algebras, the notion of $\mathcal{N}$-fuzzy sets in UP-algebras was introduced by Songsaeng and Iampan [17], Senapati et al. [14, 15] applied cubic set and interval-valued intuitionistic fuzzy structure in UP-algebras, Romano [11] introduced the notion of proper UP-filters in UP-algebras, etc.

In this paper, we apply the notions of the intersection and the union of any fuzzy set to UP-algebras. We investigate properties of the intersection and the union of T-fuzzy UP-subalgebras, T-fuzzy near UP-filters, T-fuzzy UP-filters, T-fuzzy UP-ideals, T-fuzzy strongly UP-ideals, anti-T-fuzzy UP-subalgebras, and anti-T-fuzzy near UP-filters of UP-algebras.

Before we begin our study, we will introduce the definition of a UP-algebra.

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Definition 1.1. [4] An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra* where X is a nonempty set, $\cdot$ is a binary operation on X, and 0 is a fixed element of X (i.e., a nullary operation) if it satisfies the following axioms:

(UP-1): $(\forall x, y, z \in X)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,
(UP-2): $(\forall x \in X)(0 \cdot x = x)$,
(UP-3): $(\forall x \in X)(x \cdot 0 = 0)$, and
(UP-4): $(\forall x, y \in X)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y)$.

From [4], we know that the notion of UP-algebras is a generalization of KU-algebras (see [10]).

Example 1.2. [13] Let X be a universal set and let $\Omega \in \mathcal{P}(X)$ where $\mathcal{P}(X)$ means the power set of X. Let $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. Define a binary operation $\cdot$ on $\mathcal{P}_\Omega(X)$ by putting $A \cdot B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(X)$ where A^C means the complement of a subset A. Then $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 1 with respect to* Ω. Let $\mathcal{P}^\Omega(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$. Define a binary operation $*$ on $\mathcal{P}^\Omega(X)$ by putting $A * B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(X)$. Then $(\mathcal{P}^\Omega(X), *, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 2 with respect to* Ω. In particular, $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra and we shall call it the *power UP-algebra of type 1*, and $(\mathcal{P}(X), *, X)$ is a UP-algebra and we shall call it the *power UP-algebra of type 2*.

Example 1.3. [3] Let $\mathbb{N}$ be the set of all natural numbers with two binary operations $\circ$ and $\bullet$ defined by

$$(\forall x, y \in \mathbb{N})\left(x \circ y = \begin{cases} y & \text{if } x < y, \\ 0 & \text{otherwise} \end{cases}\right)$$

and

$$(\forall x, y \in \mathbb{N})\left(x \bullet y = \begin{cases} y & \text{if } x > y \text{ or } x = 0, \\ 0 & \text{otherwise} \end{cases}\right).$$

Then $(\mathbb{N}, \circ, 0)$ and $(\mathbb{N}, \bullet, 0)$ are UP-algebras.

Example 1.4. [17] Let $A = \{0, 1, 2, 3, 4, 5, 6\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	0	0	2	3	2	3	6
2	0	1	0	3	1	5	3
3	0	1	2	0	4	1	2
4	0	0	0	3	0	3	3
5	0	0	2	0	2	0	2
6	0	1	0	0	1	1	0

Then $(A, \cdot, 0)$ is a UP-algebra.

For more examples of UP-algebras, see [1, 5, 12, 13].

In a UP-algebra $X = (X, \cdot, 0)$, the following assertions are valid (see [4, 5]).

$$(\forall x \in X)(x \cdot x = 0), \tag{1.1}$$
$$(\forall x, y, z \in X)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0), \tag{1.2}$$
$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0), \tag{1.3}$$
$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0), \tag{1.4}$$
$$(\forall x, y \in X)(x \cdot (y \cdot x) = 0), \tag{1.5}$$
$$(\forall x, y \in X)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x), \tag{1.6}$$
$$(\forall x, y \in X)(x \cdot (y \cdot y) = 0), \tag{1.7}$$
$$(\forall a, x, y, z \in X)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0), \tag{1.8}$$
$$(\forall a, x, y, z \in X)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0), \tag{1.9}$$
$$(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0), \tag{1.10}$$
$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0), \tag{1.11}$$
$$(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0), \text{ and} \tag{1.12}$$
$$(\forall a, x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0). \tag{1.13}$$

2. Fuzzy sets with respect to a t-norm in UP-algebras

Definition 2.1. [20] A *fuzzy set* A in a nonempty set X (or a *fuzzy subset* of X) is described by its membership function α_A. To every point $x \in X$, this function associates a real number $\alpha_A(x)$ in the unit interval $[0, 1]$. The number $\alpha_A(x)$ is interpreted for the point as a degree of belonging x to the fuzzy set A, that is, $A := \{(x, \alpha_A(x)) \mid x \in X\}$. We say that a fuzzy set A in X is *constant* if its membership function α_A is constant.

Definition 2.2. [8] A *triangular norm* (briefly, t-norm) is a binary operation T on the unit interval [0,1], i.e., a function $T : [0, 1] \times [0, 1] \to [0, 1]$ that satisfies the following axioms:

(T1): Boundary condition: $(\forall x \in [0, 1])(T(x, 1) = x)$,
(T2): Commutativity: $(\forall x, y \in [0, 1])(T(x, y) = T(y, x))$,
(T3): Associativity: $(\forall x, y, z \in [0, 1])(T(x, T(y, z)) = T(T(x, y), z))$, and
(T4): Monotonicity: $(\forall x, y, z \in [0, 1])(y \leq z \Rightarrow T(x, y) \leq T(x, z))$.

Let T be a t-norm. Then the following properties hold (see [2]).

$$(\forall x, y \in [0, 1])(T(x, y) \leq x \text{ and } T(x, y) \leq y), \tag{2.1}$$
$$(\forall x \in [0, 1])(T(x, 0) = 0), \tag{2.2}$$
$$(\forall a, b, x, y \in [0, 1])(x \leq a, y \leq b \Rightarrow T(x, y) \leq T(a, b)), \text{ and} \tag{2.3}$$
$$(\forall a, b, x, y \in [0, 1])(x \leq a, y \leq a \Rightarrow T(x, y) \leq a). \tag{2.4}$$

In what follows, let X denote a UP-algebra $(X, \cdot, 0)$ and T a t-norm unless otherwise specified.

Definition 2.3. [2] A fuzzy set A in X is called

(1) a *T-fuzzy UP-subalgebra* of X if $(\forall x, y \in X)(\alpha_A(x \cdot y) \geq T(\alpha_A(x), \alpha_A(y)))$.
(2) a *T-fuzzy near UP-filter* of X if
   (i) $(\forall x \in X)(\alpha_A(0) \geq \alpha_A(x))$, and
   (ii) $(\forall x, y \in X)(\alpha_A(x \cdot y) \geq T(\alpha_A(y), \alpha_A(y)))$.
(3) a *T-fuzzy UP-filter* of X if
   (i) $(\forall x \in X)(\alpha_A(0) \geq \alpha_A(x))$, and

(ii) $(\forall x, y \in X)(\alpha_A(y) \geq T(\alpha_A(x \cdot y), \alpha_A(x)))$.

(4) a T-*fuzzy UP-ideal* of X if

(i) $(\forall x \in X)(\alpha_A(0) \geq \alpha_A(x))$, and

(ii) $(\forall x, y, z \in X)(\alpha_A(x \cdot z) \geq T(\alpha_A(x \cdot (y \cdot z)), \alpha_A(y)))$.

(5) a T-*fuzzy strongly UP-ideal* of X if

(i) $(\forall x \in X)(\alpha_A(0) \geq \alpha_A(x))$, and

(ii) $(\forall x, y, z \in X)(\alpha_A(x) \geq T(\alpha_A((z \cdot y) \cdot (z \cdot x)), \alpha_A(y)))$.

Burandate et al. [2] proved the generalization that the notion of T-fuzzy UP-ideals is a generalization of T-fuzzy strongly UP-ideals, the notion of T-fuzzy UP-filters is a generalization of T-fuzzy UP-ideals, the notion of T-fuzzy near UP-filters is a generalization of T-fuzzy UP-filters, and the notion of T-fuzzy UP-subalgebras is a generalization of T-fuzzy UP-filters. Moreover, the notion of T-fuzzy near UP-filters does not coincide with the notion of T-fuzzy UP-subalgebras.

Example 2.4. [2] Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	2
2	0	1	0	3	1
3	0	1	2	0	4
4	0	0	0	3	0

Let $\mathrm{T_{Luk}}$ be the Łukasiewicz t-norm defined by

$$(\forall x, y \in [0, 1])(\mathrm{T_{Luk}}(x, y) = \max\{x + y - 1, 0\}). \tag{2.5}$$

Define a fuzzy set A in X by

$$\alpha_A = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.7 & 0.4 & 0.2 & 0.5 & 0.3 \end{pmatrix}.$$

Then A is a $\mathrm{T_{Luk}}$-fuzzy UP-ideal of X. Since

$$\alpha_A(2) = 0.2 < 0.4 = \mathrm{T_{Luk}}(\alpha_A((2 \cdot 0) \cdot (2 \cdot 2)), \alpha_A(0)),$$

we have A is not a $\mathrm{T_{Luk}}$-fuzzy strongly UP-ideal of X.

Example 2.5. [2] Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	0	0	0
2	0	2	0	0	0
3	0	2	2	0	0
4	0	2	2	4	0

Define a fuzzy set A in X by

$$\alpha_A = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.8 & 0.7 & 0.9 & 0.9 & 0.9 \end{pmatrix}.$$

Then A is a $\mathrm{T_{Luk}}$-fuzzy UP-subalgebra of X (see $\mathrm{T_{Luk}}$ in Example 2.4). Since

$$\alpha_A(1) = 0.7 < 0.8 = \mathrm{T_{Luk}}(\alpha_A(4 \cdot 1), \alpha_A(4)),$$

we have A is not a $\mathrm{T_{Luk}}$-fuzzy UP-filter of X. Since $\alpha_A(0) < \alpha_A(2)$, we have A does not satisfy the condition: $(\forall x \in X)(\alpha_A(0) \geq \alpha_A(x))$.

Example 2.6. [2] Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	0	0	0
2	0	2	0	0	0
3	0	2	2	0	0
4	0	2	2	4	0

Let $\mathrm{T_{min}}$ be the Gödel t-norm defined by

$$(\forall x, y \in [0, 1])(\mathrm{T_{min}}(x, y) = \min\{x, y\}). \tag{2.6}$$

Define a fuzzy set A in X by

$$\alpha_A = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.1 & 0.3 & 0.2 & 0 \end{pmatrix}.$$

Then A is a $\mathrm{T_{min}}$-fuzzy UP-subalgebra of X (see $\mathrm{T_{min}}$ in Example 2.5). Since

$$\alpha_A(4 \cdot 3) = 0 < 0.2 = \mathrm{T_{min}}(\alpha_A(3), \alpha_A(3)),$$

we have A is not a $\mathrm{T_{min}}$-fuzzy near UP-filter of X.

Definition 2.7. [2] A fuzzy set A in X is called

(1) an *anti-T-fuzzy UP-subalgebra* of X if $(\forall x, y \in X)(\alpha_A(x\cdot y) \leq T(\alpha_A(x), \alpha_A(y)))$.
(2) an *anti-T-fuzzy near UP-filter* of X if
  (i) $(\forall x \in X)(\alpha_A(0) \leq \alpha_A(x))$, and
  (ii) $(\forall x, y \in X)(\alpha_A(x \cdot y) \leq T(\alpha_A(y), \alpha_A(y)))$.
(3) an *anti-T-fuzzy UP-filter* of X if
  (i) $(\forall x \in X)(\alpha_A(0) \leq \alpha_A(x))$, and
  (ii) $(\forall x, y \in X)(\alpha_A(y) \leq T(\alpha_A(x \cdot y), \alpha_A(x)))$.
(4) an *anti-T-fuzzy UP-ideal* of X if
  (i) $(\forall x \in X)(\alpha_A(0) \leq \alpha_A(x))$, and
  (ii) $(\forall x, y, z \in X)(\alpha_A(x \cdot z) \leq T(\alpha_A(x \cdot (y \cdot z)), \alpha_A(y)))$.
(5) an *anti-T-fuzzy strongly UP-ideal* of X if
  (i) $(\forall x \in X)(\alpha_A(0) \leq \alpha_A(x))$, and
  (ii) $(\forall x, y, z \in X)(\alpha_A(x) \leq T(\alpha_A((z \cdot y) \cdot (z \cdot x)), \alpha_A(y)))$.

Burandate et al. [2] proved the generalization that the notion of anti-T-fuzzy near UP-filters is a generalization of anti-T-fuzzy UP-subalgebras. Moreover, the notions of anti-T-fuzzy strongly UP-ideals, anti-T-fuzzy UP-ideals, anti-T-fuzzy UP-filters, and anti-T-fuzzy UP-subalgebras coincide.

Theorem 2.1. [2] *If A is an anti-T-fuzzy UP-subalgebra of X, then A is constant.*

3. Main results

In this section, we investigate properties of the intersection and the union of T-fuzzy UP-subalgebras, T-fuzzy near UP-filters, T-fuzzy UP-filters, T-fuzzy UP-ideals, T-fuzzy strongly UP-ideals, anti-T-fuzzy UP-subalgebras, and anti-T-fuzzy near UP-filters of UP-algebras.

Definition 3.1. [9] Let $\mathscr{A}$ be a nonempty family of fuzzy sets in a nonempty set X. Define the *intersection* $\cap\mathscr{A}$ in X by its membership function $\alpha_{\cap\mathscr{A}}$ which defined as follows:

$$(\forall x \in X)(\alpha_{\cap\mathscr{A}}(x) = \inf\{\alpha_A(x)\}_{A\in\mathscr{A}}). \tag{3.1}$$

Define the *union* $\cup\mathscr{A}$ in X by its membership function $\alpha_{\cup\mathscr{A}}$ which defined as follows:

$$(\forall x \in X)(\alpha_{\cup\mathscr{A}}(x) = \sup\{\alpha_A(x)\}_{A\in\mathscr{A}}). \tag{3.2}$$

Theorem 3.1. *Let $\mathscr{A}$ be a nonempty family of T-fuzzy UP-subalgebras of X. Then $\cap\mathscr{A}$ is also a T-fuzzy UP-subalgebra of X.*

Proof. Let $x, y \in X$. Then $\alpha_A(x \cdot y) \geq T(\alpha_A(x), \alpha_A(y))$ for all $A \in \mathscr{A}$. Since $\inf\{\alpha_A(x)\}_{A\in\mathscr{A}} \leq \alpha_A(x)$ and $\inf\{\alpha_A(y)\}_{A\in\mathscr{A}} \leq \alpha_A(y)$ for all $A \in \mathscr{A}$, it follows from (2.3) that

$$(\forall A \in \mathscr{A})(T(\inf\{\alpha_A(x)\}_{A\in\mathscr{A}}, \inf\{\alpha_A(y)\}_{A\in\mathscr{A}}) \leq T(\alpha_A(x), \alpha_A(y)) \leq \alpha_A(x \cdot y)).$$

Thus

$$\begin{aligned} T(\alpha_{\cap\mathscr{A}}(x), \alpha_{\cap\mathscr{A}}(y)) &= T(\inf\{\alpha_A(x)\}_{A\in\mathscr{A}}, \inf\{\alpha_A(y)\}_{A\in\mathscr{A}}) \\ &\leq \inf\{\alpha_A(x \cdot y)\}_{A\in\mathscr{A}} \\ &= \alpha_{\cap\mathscr{A}}(x \cdot y). \end{aligned}$$

Hence, $\cap\mathscr{A}$ is a T-fuzzy UP-subalgebra of X. □

The following example show that the union of T-fuzzy UP-subalgebras of X is not a T-fuzzy UP-subalgebra of X.

Example 3.2. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	2
3	0	1	0	0

Define two fuzzy sets A_1 and A_2 in X as follows:

$$\alpha_{A_1} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.9 & 0.7 & 0.1 & 0.1 \end{pmatrix}.$$

and

$$\alpha_{A_2} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.8 & 0.4 & 0.5 & 0.6 \end{pmatrix}.$$

Then A_1 and A_2 are $\mathrm{T_{min}}$-fuzzy UP-subalgebras of X (see $\mathrm{T_{min}}$ in Example 2.6). We thus obtain the union of A_1 and A_2 as follows:

$$\alpha_{A_1\cup A_2} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.9 & 0.7 & 0.5 & 0.6 \end{pmatrix}.$$

Since

$$\alpha_{A_1\cup A_2}(1 \cdot 3) = 0.5 < 0.6 = \mathrm{T_{min}}(\alpha_{A_1\cup A_2}(1), \alpha_{A_1\cup A_2}(3)).$$

Therefore, $\alpha_{A_1\cup A_2}(x)$ is not a $\mathrm{T_{min}}$-fuzzy UP-subalgebra of X.

Theorem 3.2. *Let $\mathscr{A}$ be a nonempty family of T-fuzzy near UP-filters of X. Then $\cap\mathscr{A}$ is also a T-fuzzy near UP-filter of X.*

Proof. Let $x \in X$. Since $\alpha_A(0) \geq \alpha_A(x)$ for all $A \in \mathscr{A}$, we have

$$\alpha_{\cap\mathscr{A}}(0) = \inf\{\alpha_A(0)\}_{A\in\mathscr{A}} \geq \inf\{\alpha_A(x)\}_{A\in\mathscr{A}} = \alpha_{\cap\mathscr{A}}(x).$$

Let $x, y \in X$. Then $\alpha_A(x \cdot y) \geq T(\alpha_A(y), \alpha_A(y))$ for all $A \in \mathscr{A}$. Since $\inf\{\alpha_A(y)\}_{A \in \mathscr{A}} \leq \alpha_A(y)$ for all $A \in \mathscr{A}$, it follows from (2.3) that

$$(\forall A \in \mathscr{A})(T(\inf\{\alpha_A(y)\}_{A \in \mathscr{A}}, \inf\{\alpha_A(y)\}_{A \in \mathscr{A}}) \leq T(\alpha_A(y), \alpha_A(y)) \leq \alpha_A(x \cdot y)).$$

Thus

$$\begin{aligned} T(\alpha_{\cap\mathscr{A}}(y), \alpha_{\cap\mathscr{A}}(y)) &= T(\inf\{\alpha_A(y)\}_{A \in \mathscr{A}}, \inf\{\alpha_A(y)\}_{A \in \mathscr{A}}) \\ &\leq \inf\{\alpha_A(x \cdot y)\}_{A \in \mathscr{A}} \\ &= \alpha_{\cap\mathscr{A}}(x \cdot y). \end{aligned}$$

Hence, $\cap\mathscr{A}$ is a T-fuzzy near UP-filter of X. □

Theorem 3.3. *Let $\mathscr{A}$ be a nonempty family of T-fuzzy UP-filters of X. Then $\cap\mathscr{A}$ is also a T-fuzzy UP-filter of X.*

Proof. The proof of the first statement is similar to the proof of Theorem 3.2. Let $x, y \in X$. Then $\alpha_A(y) \geq T(\alpha_A(x \cdot y), \alpha_A(x))$ for all $A \in \mathscr{A}$. Since $\inf\{\alpha_A(x \cdot y)\}_{A \in \mathscr{A}} \leq \alpha_A(x \cdot y)$ and $\inf\{\alpha_A(x)\}_{A \in \mathscr{A}} \leq \alpha_A(x)$ for all $A \in \mathscr{A}$, it follows from (2.3) that

$$(\forall A \in \mathscr{A})(T(\inf\{\alpha_A(x \cdot y)\}_{A \in \mathscr{A}}, \inf\{\alpha_A(x)\}_{A \in \mathscr{A}}) \leq T(\alpha_A(x \cdot y), \alpha_A(x)) \leq \alpha_A(y)).$$

Thus

$$\begin{aligned} T(\alpha_{\cap\mathscr{A}}(x \cdot y), \alpha_{\cap\mathscr{A}}(x)) &= T(\inf\{\alpha_A(x \cdot y)\}_{A \in \mathscr{A}}, \inf\{\alpha_A(x)\}_{A \in \mathscr{A}}) \\ &\leq \inf\{\alpha_A(y)\}_{A \in \mathscr{A}} \\ &= \alpha_{\cap\mathscr{A}}(y). \end{aligned}$$

Hence, $\cap\mathscr{A}$ is a T-fuzzy UP-filter of X. □

The following example show that the union of T-fuzzy UP-filters of X is not a T-fuzzy UP-filter.

Example 3.3. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	1
3	0	0	0	0

Define two fuzzy sets A_1 and A_2 in X as follows:

$$\alpha_{A_1} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.7 & 0.3 & 0.4 & 0.3 \end{pmatrix}.$$

and

$$\alpha_{A_2} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.8 & 0.5 & 0.2 & 0.2 \end{pmatrix}.$$

Then A_1 and A_2 are $\mathrm{T_{min}}$-fuzzy UP-filter of X (see $\mathrm{T_{min}}$ in Example 2.6). We thus obtain the union of A_1 and A_2 as follows:

$$\alpha_{A_1 \cup A_2} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.8 & 0.5 & 0.4 & 0.3 \end{pmatrix}.$$

Since

$$\alpha_{A_1 \cup A_2}(3) = 0.3 < 0.4 = \mathrm{T_{min}}(\alpha_{A_1 \cup A_2}(1 \cdot 3), \alpha_{A_1 \cup A_2}(1)).$$

Therefore, $\alpha_{A_1\cup A_2}(x)$ is not a $\mathrm{T_{min}}$-fuzzy UP-filter of X.

Theorem 3.4. *Let $\mathscr{A}$ be a nonempty family of T-fuzzy UP-ideals of X. Then $\cap\mathscr{A}$ is also a T-fuzzy UP-ideal of X.*

Proof. The proof of the first statement is similar to the proof of Theorem 3.2. Let $x, y, z \in X$. Then $\alpha_A(x\cdot z) \geq T(\alpha_A(x\cdot(y\cdot z)), \alpha_A(y))$ for all $A \in \mathscr{A}$. Since $\inf\{\alpha_A(x\cdot(y\cdot z))\}_{A\in\mathscr{A}} \leq \alpha_A(x\cdot(y\cdot z))$ and $\inf\{\alpha_A(y)\}_{A\in\mathscr{A}} \leq \alpha_A(y)$ for all $A\in\mathscr{A}$, it follows from (2.3) that

$$(\forall A\in\mathscr{A})(T(\inf\{\alpha_A(x\cdot(y\cdot z))\}_{A\in\mathscr{A}}, \inf\{\alpha_A(y)\}_{A\in\mathscr{A}}) \leq T(\alpha_A(x\cdot(y\cdot z)), \alpha_A(y)) \leq \alpha_A(x\cdot z)).$$

Thus

$$\begin{aligned} T(\alpha_{\cap\mathscr{A}}(x\cdot(y\cdot z)), \alpha_{\cap\mathscr{A}}(y)) &= T(\inf\{\alpha_A(x\cdot(y\cdot z))\}_{A\in\mathscr{A}}, \inf\{\alpha_A(y)\}_{A\in\mathscr{A}}) \\ &\leq \inf\{\alpha_A(x\cdot z)\}_{A\in\mathscr{A}} \\ &= \alpha_{\cap\mathscr{A}}(x\cdot z). \end{aligned}$$

Hence, $\cap\mathscr{A}$ is a T-fuzzy UP-ideal of X. □

The following example show that the union of T-fuzzy UP-ideals of X is not a T-fuzzy UP-ideal.

Example 3.4. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	1
3	0	0	0	0

Define two fuzzy sets A_1 and A_2 in X as follows:

$$\alpha_{A_1} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.7 & 0.3 & 0.4 & 0.3 \end{pmatrix}.$$

and

$$\alpha_{A_2} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.8 & 0.5 & 0.2 & 0.2 \end{pmatrix}.$$

Then A_1 and A_2 are $\mathrm{T_{min}}$-fuzzy UP-ideal of X (see $\mathrm{T_{min}}$ in Example 2.6). We thus obtain the union of A_1 and A_2 as follows:

$$\alpha_{A_1\cup A_2} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.8 & 0.5 & 0.4 & 0.3 \end{pmatrix}.$$

Therefore, $\alpha_{A_1\cup A_2}(x)$ is not a $\mathrm{T_{min}}$-fuzzy UP-ideal of X because

$$\alpha_{A_1\cup A_2}(0\cdot 3) = 0.3 < 0.4 = \mathrm{T_{min}}(\alpha_{A_1\cup A_2}(0\cdot(2\cdot 3)), \alpha_{A_1\cup A_2}(2)).$$

Theorem 3.5. *Let $\mathscr{A}$ be a nonempty family of T-fuzzy strongly UP-ideals of X. Then $\cap\mathscr{A}$ is also a T-fuzzy strongly UP-ideal of X.*

Proof. The proof of the first statement is similar to the proof of Theorem 3.2. Let $x, y, z \in X$. Then $\alpha_A(x) \geq T(\alpha_A((z\cdot y)\cdot(z\cdot x)), \alpha_A(y))$ for all $A\in\mathscr{A}$. Since $\inf\{\alpha_A((z\cdot y)\cdot(z\cdot x))\}_{A\in\mathscr{A}} \leq \alpha_A((z\cdot y)\cdot(z\cdot x))$ and $\inf\{\alpha_A(y)\}_{A\in\mathscr{A}} \leq \alpha_A(y)$ for all $A\in\mathscr{A}$, it follows from (2.3) that

$$(\forall A\in\mathscr{A})(T(\inf\{\alpha_A((z\cdot y)\cdot(z\cdot x))\}_{A\in\mathscr{A}}, \inf\{\alpha_A(y)\}_{A\in\mathscr{A}}) \leq T(\alpha_A((z\cdot y)\cdot(z\cdot x)), \alpha_A(y)) \leq \alpha_A(x)).$$

Thus

$$\begin{aligned}T(\alpha_{\cap\mathscr{A}}((z\cdot y)\cdot(z\cdot x)),\alpha_{\cap\mathscr{A}}(y)) &= T(\inf\{\alpha_A((z\cdot y)\cdot(z\cdot x))\}_{A\in\mathscr{A}},\inf\{\alpha_A(y)\}_{A\in\mathscr{A}})\\ &\leq \inf\{\alpha_A(x)\}_{A\in\mathscr{A}}\\ &= \alpha_{\cap\mathscr{A}}(x).\end{aligned}$$

Hence, $\cap\mathscr{A}$ is a T-fuzzy strongly UP-ideal of X. □

Theorem 3.6. *Let $\mathscr{A}$ be a nonempty family of anti-T-fuzzy UP-subalgebras (is also anti-T-fuzzy near UP-filters) of X. Then $\cup\mathscr{A}$ is also an anti-T-fuzzy UP-subalgebra of X.*

Proof. By Theorem 2.1, we have $\alpha_A(x) = \alpha_A(0)$ for all $x \in X$. Then $\alpha_{\cup\mathscr{A}}(x) = \sup\{\alpha_A(x)\}_{A\in\mathscr{A}} = \alpha_A(0)$. Let $x, y \in X$. Then

$$\begin{aligned}\alpha_{\cup\mathscr{A}}(x\cdot y) &= \alpha_A(0)\\ &= \alpha_A(0\cdot 0) && \text{((UP-2))}\\ &\leq T(\alpha_A(0),\alpha_A(0)) && \text{(Definition 2.7 (2) (ii))}\\ &= T(\alpha_{\cup\mathscr{A}}(x),\alpha_{\cup\mathscr{A}}(y)).\end{aligned}$$

Hence, $\cup\mathscr{A}$ is an anti-T-fuzzy UP-subalgebra of X. □

Theorem 3.7. *Let $\mathscr{A}$ be a nonempty family of anti-T-fuzzy UP-subalgebras (is also anti-T-fuzzy near UP-filters) of X. Then $\cap\mathscr{A}$ is also an anti-T-fuzzy UP-subalgebra of X.*

Proof. By Theorem 2.1, we have $\alpha_A(x) = \alpha_A(0)$ for all $x \in X$. Then $\alpha_{\cap\mathscr{A}}(x) = \inf\{\alpha_A(x)\}_{A\in\mathscr{A}} = \alpha_A(0)$. Let $x, y \in X$. Then

$$\begin{aligned}\alpha_{\cap\mathscr{A}}(x\cdot y) &= \alpha_A(0)\\ &= \alpha_A(0\cdot 0) && \text{((UP-2))}\\ &\leq T(\alpha_A(0),\alpha_A(0)) && \text{(Definition 2.7 (2) (ii))}\\ &= T(\alpha_{\cap\mathscr{A}}(x),\alpha_{\cap\mathscr{A}}(y)).\end{aligned}$$

Hence, $\cap\mathscr{A}$ is an anti-T-fuzzy UP-subalgebra of X. □

Theorem 3.8. *Let $\mathscr{A}$ be a nonempty family of anti-T-fuzzy near UP-filters of X. Then $\cup\mathscr{A}$ is also an anti-T-fuzzy near UP-filter of X.*

Proof. Let $x \in X$. Since $\alpha_A(0) \leq \alpha_A(x)$ for all $A \in \mathscr{A}$, we have

$$\alpha_{\cup\mathscr{A}}(0) = \sup\{\alpha_A(0)\}_{A\in\mathscr{A}} \leq \sup\{\alpha_A(x)\}_{A\in\mathscr{A}} = \alpha_{\cup\mathscr{A}}(x).$$

Let $x, y \in X$. Then $\alpha_A(x\cdot y) \leq T(\alpha_A(y),\alpha_A(y))$ for all $A \in \mathscr{A}$. Since $\sup\{\alpha_A(y)\}_{A\in\mathscr{A}} \geq \alpha_A(y)$ for all $A \in \mathscr{A}$, it follows from (2.3) that

$$(\forall A \in \mathscr{A})(T(\sup\{\alpha_A(y)\}_{A\in\mathscr{A}},\sup\{\alpha_A(y)\}_{A\in\mathscr{A}}) \geq T(\alpha_A(y),\alpha_A(y)) \geq \alpha_A(x\cdot y)).$$

Thus

$$\begin{aligned}T(\alpha_{\cup\mathscr{A}}(y),\alpha_{\cup\mathscr{A}}(y)) &= T(\sup\{\alpha_A(y)\}_{A\in\mathscr{A}},\sup\{\alpha_A(y)\}_{A\in\mathscr{A}})\\ &\geq \sup\{\alpha_A(x\cdot y)\}_{A\in\mathscr{A}}\\ &= \alpha_{\cup\mathscr{A}}(x\cdot y).\end{aligned}$$

Hence, $\cup\mathscr{A}$ is an anti-T-fuzzy near UP-filter of X. □

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